The demand for bonds with negative nominal interest rate:
capital gain approach

Takeshi Hoshikawa

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Takeshi Hoshikawa  
Faculty of Economics, Kindai University  
hoshikawa@kindai.ac.jp

Abstract  
This paper shows the demand for bonds with negative nominal interest rate even if the cost of holding money is zero. Capital gain from bond trading can lead to negative nominal interest rate when capital gain from bond price change exceeds the loss from negative interest rate. In this case, the monetary policy of central bank is important to keep bond price high.

1. NEGATIVE INTEREST RATE AND HOLDING MONEY COST  
This paper examines the demand for bonds with negative nominal interest rate by approaching capital gain from bond trading. Nominal interest rates usually have a zero lower bound. If interest rate is positive, lender gets the interest. If interest rate is negative lender gets the loss. Therefore, there is no demand for bonds with negative interest rate, because investors prefer money to the bonds. Central banks control the interest rate by open market operations, but it is difficult to lower the interest rate when interest rate is at the zero lower bound. If we remove the zero lower bound restriction, central bank gets more flexibility in monetary policy.

As of April 1, 2016, European Central Bank sets interest rate -0.4% on the deposit facility, Danmarks Nationalbank's interest rate on certificates of deposit is -0.65% and Swiss National Bank's interest rate on sight deposit account balances is -0.75%, Bank of Japan apply a negative interest rate of -0.1% to part of financial institutions. Riksbanken which is the central bank of Sweden, sets repo rate -0.5% as policy rate. It is important to consider the negative interest rate.

Previous studies\(^2\) considered the negative nominal interest rate when the cost of

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\(^2\) Ilgmann and Menner (2011) and IMF (2017) provided good surveys about negative interest rate.
holding money is not zero (Buiter and Panigirtzoglou 2003, Buiter 2010, Mankiw 2009, Ilgmann and Menner 2011, Kimball 2015, Agarwal and Kimball 2015, IMF 2017). If holding money cost is high, negative interest rate can be exist. The holding money costs are explained by costs of storage, or by taxing currency, or by depreciation of currency. It is thought to be a good method for considering negative interest rate. This paper try to find another method and shows the possibility of existence of negative nominal interest rate even if the cost of holding money is zero. Section 2 shows why investors buy the bond with negative interest rate. Section 3 is discussion about negative interest rate.

2. WHY INVESTORS BUY THE BOND WITH NEGATIVE INTEREST RATE?

Investors buy the bond with negative nominal interest rate when the bond has higher expected return than money. Interest rates are usually main part of return of holding bonds. Capital gain is also important part of return of the bond. This paper does not assume the money holding cost. This paper explains the demand for bonds exist when bonds have the higher expected return than money.

This paper makes the following assumptions:

ASSUMPTION $A_1$: The nominal return of holding money is zero.

ASSUMPTION $A_2$: The cost of holding money is zero.

ASSUMPTION $A_3$: There exist demand for bond if the expected return of holding bond is higher than the return of holding money.

ASSUMPTION $A_4$: The bond price $P_t > 0$ with the maturity $N$ is determined as shown below:

$$P_t = \sum_{n=1}^{N} \frac{C_{t+n}}{(1+i)^n}$$

(1)

Here $P_t$ is the price of a coupon bond or discount bond at time $t$, $C_{t+n}$ denote the cash flow at time $t+n$, $i$ is nominal interest rate. The bond price is determined by the present value of all future cash flows.

ASSUMPTION $A_5$: The expected holding period return $r$ of bond is the following:

$$r_t = \frac{C_{t+1} + E_t(P_{t+1}) - P_t}{P_t}$$

(2)

Here $E_t(P_{t+1})$ is the conditional expectation of the bond price. The income gain ($C_{t+1}/P_t$) and capital gain ($E_t(P_{t+1}) - P_t)/P_t$) compose the expected holding period return.

ASSUMPTION $A_6$: Central bank announce and will purchase the bond at the price
$P_{t+1}$ at time t+1 as monetary policy. This paper assume that market participants believe that the bond price of next period will be the announced price, $E_t(P_{t+1}) = P_{t+1}$.

The paper shows that there are demands for bond with negative interest rate by central bank's higher price bond purchase policy. It is following proposition.

**Proposition.** We assume the assumptions $A_1$--$A_6$, and assume that nominal interest rate is negative such as $i < 0$. Central bank purchases the bond at the enough high price $P_{t+1}$ at time t+1 to create positive capital gain for investors. If capital gain is larger than income gain $(E_t(P_{t+1}) - P_t)/P_t > C_{t+1}/P_t$, and expected holding return is positive $r > 0$. Then there exists the demand for bonds with negative nominal interest rate.

**Proof**
We assume the assumptions $A_1$--$A_6$, nominal interest rate is negative ($i < 0$) and expected holding return is positive $r > 0$. It means,

$$A_1 \land A_2 \land A_3 \land A_4 \land A_5 \land A_6 \land (i < 0) \land (r > 0)$$

Here $A_j$ is assumption j ($j = 1, 2, ..., 6$). When $r > 0$, bond has the higher expected return than money. This set is a subset in $A_3 \land (r > 0)$. If there is no demand for the bond, it is in contradiction with assumption $A_3$. ■

For example, a buyer paid 101 dollar for a discount bond with a face value of 100 dollar. In this case, the buyer usually will have 1 dollar loss. If the buyer can sell the bond to central bank for 102 dollar at next month, then bonds have the higher expected return than money. It indicates the possibility of existence of the demand for bonds with negative nominal interest rate, and increasing the monetary policy flexibility. This paper suggests the higher price bond purchasing policy as an unconventional monetary policy. The central bank plays important role, because there is no buyer of the bond when interest rate is negative.

In Japan, the auction result of the three-month No. 489 Treasury Discount Bills (T-Bills) was negative yield on October 23, 2014. The T-Bills are a discount bond which also called a zero coupon bond, and the yield at the average price was 0.0037%, and weighted average price per 100 yen was 100.001 yen. The buyer paid 100.001 yen for receiving 100 yen at the three month later.

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3 Ministry of Finance Japan website.
http://www.mof.go.jp/english/jgbs/auction/calendar/etbill/etbillresul/eresul20141023.htm
3. DISCUSSION

Related to negative interest rate, this paper discusses three topics which are the loss of central bank, the impact on private bank profit and the foreign factor. The higher price bond purchasing policy leads central bank to loss. It means this policy is temporary and cannot conduct long time. However it will be covered partly by seigniorage which is the difference between the value of money and the cost of money.

Even if bond trading with negative interest rate is profitable, negative interest rate put pressure on the profits of private banks because private banks are difficult to charge negative deposit rate for customers who try to withdraw money from banks. It may have monetary tightening effect by reducing bank deposit and bank loan. One possible plan is central bank lending with restriction. For example, if central bank lends private banks at -3% interest rate, and private banks should lend under -1% which is restricted interest rate for educational loan, housing loan, or refinancing. Private banks gain profits from differences in interest rates. The lending with restriction make private banks prohibiting money holding.

In case of Japan, foreign capital flows are important source of negative interest rate\(^4\). It can be explained using covered interest parity. Covered interest parity equation is following.

\[
i = i^* + \frac{F - S}{S}
\]

Here \(i\) is interest rate in Japan, \(i^*\) is interest rate in the U.S., \(S\) is the dollar yen spot exchange rate, \(F\) is the forward exchange rate. We assume that Japan is small economy and variables are exogenous except \(i\) for simplicity. If foreign interest rate \(i^*\) is not high (for example, 2%) and \((F \cdot S)/S\) is negative (for example, -3%), the right hand of equation will be negative (-1%). The right hand of equation means the Japanese yen conversion cost by foreign financial institutes. Japanese domestic interest rate \(i\) on the left hand of equation will be negative. It indicates foreign factor is also important. Some countries which conduct negative interest policy in Europe pay attention to the foreign factors such as exchange rate and capital flows.

The holding money cost is important for negative interest rate as previous studies. Capital gain also has important role same as in stock or real estate market. Negative interest rate have an income redistribution effect from lender to borrower. If lenders are richer than borrowers, negative interest rate may reduce inequality. Negative interest rate may also cause inefficient investment and we have to consider whether it is

\(^4\) See Bank of Japan (2005).
inefficient monetary policy or not. We have to consider about negative interest rate.


