Aggregate Demand Deficiency, Labor Unions, and Long-run Stagnation

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Abstract

Using a money-in-the-utility-function model, we present long-run stagnation where insatiable demand for money secularly causes deficient aggregate demand and thereby unemployment in the presence of nominal wage stickiness attributable to union wage setting. In this long-run stagnation, generous unemployment benefits reduce unemployment. Moreover, paradoxically, unemployment declines if labor unions give more weight to nominal wage gains compared with employment increases.

Keywords: Aggregate Demand, Labor Union, Long-run Stagnation, Phillips Curve, Unemployment Benefit

JEL Classification Codes: E12, E24, J51, J65

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1 Introduction

Recently, there have been growing concerns about poor economic performances of advanced economies such as Japan, the United States, and Europe. Summers (2014, 2015) considers that these economies have fallen into “secular stagnation” and that aggregate demand deficiencies give rise to such stagnation. Why do deficient aggregate demand and consequently unemployment arise at all? The answer may be in the following statement by Keynes (1936, Chapter 17, p. 235):

Unemployment develops, that is to say, because people want the moon;—men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off.

Krugman (1999) introduces a static money-in-the-utility-function (MIUF) model where the price level is assumed to be fixed. He shows that if the rigid price level is high and thus the real money supply is low, then excess demand for real money balances arises, which generates a deficiency of consumption (aggregate demand) and unemployment. He quotes the above statement by Keynes and mentions that this consequence is consistent with the statement.\(^1\) However, in his own words, the simple static model is a “small but essentially accurate model of an economy” but is “at best a crude approximation to a dynamic model in which behavior results from plans that are based on expec-

\(^1\)In addition to the statement, Krugman (1999) quotes the following from Keynes (1936, p. 235): “There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control.”
tations about the future.” Moreover, in the static model, the consumption
deficiency disappears because of the Pigou effect if the price level drops so
as to close the demand–supply gap. This seems to be inconsistent with the
experience of the Japanese economy since the early 1990s, which is the most
typical example of secular stagnation. Although the price level has dropped
(deflation has occurred), aggregate demand has not been stimulated and the
Japanese economy has secularly stagnated.

A dynamic MIUF model developed by Ono (1994, 2001) is able to explain
this experience of the Japanese economy. In the dynamic model, he assumes
that the marginal utility of money does not approach zero but remains at
a positive level even when real money holdings increase to infinity. Because
of this assumption, even if deflation persistently expands real money bal-
ances, households insatiately want to save money. Consequently, household
consumption is chronically insufficient to maintain full employment. In this
manner, the Pigou effect does not arise and an economy remains in a stag-
nation steady state where insufficient aggregate demand and unemployment
continue to exist. Ono (1994, 2001) argues that the mechanism generating
such long-run stagnation in his model is consistent with that discussed by
Keynes (1936, Chapter 17).

However, Ono’s (1994, 2001) model has a drawback in that stickiness
of nominal wages and prices lacks a microeconomic foundation. Therefore,

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2 There are many studies that extend Ono’s model. For instance, an early one is Mat-
suzaki (2003) and a recent one is Hashimoto (2015).

3 Constructing a dynamic model where status seeking causes people to hold money,
Murota and Ono (2011) obtain a stagnation steady state similar to that of Ono (1994,
2001) and also argue that the stagnation mechanism is essentially the same as Keynes’s
statement quoted at the opening of the introduction.
while we adopt Ono’s assumption of insatiable desire for money in order to analyze long-run stagnation, we give a microeconomic foundation to nominal wage stickiness (the Phillips curve), which is the first purpose of the present paper. Considering the microeconomic foundation is naturally important because, as is indeed shown in the present paper, some policies affect an economy through shifting the Phillips curve. To obtain microeconomically founded stickiness of nominal wages, we modify the union wage setting of Raurich et al. (2006) and Greiner (2013), by assuming that labor unions are concerned not with a rise in real wages but with that in nominal wages because of money illusion. Due to this assumption, in contrast with Raurich et al. (2006) and Greiner (2013), union wage setting gives rise to nominal wage stickiness characterized by a Phillips curve.

The second purpose is to examine the effects of an increase in unemployment benefits in long-run stagnation. Many economists have investigated its effect on unemployment, but the direction and magnitude appear to be inconclusive. For example, Nickell et al. (2005) and Bassanini and Duval (2009) find that it is related to high unemployment. Meanwhile, for instance, Howell and Rehm (2009) counter the view that it worsens unemployment. Recently, studies on its effect have increased because the U.S. government expanded unemployment benefits in order to protect the livelihood of people who lost their jobs because of the Great Recession in the United States. According to

4Raurich et al. (2006) and Greiner (2013) develop non-monetary, endogenous growth models and investigate relationships between fiscal policies and economic growth under real wage stickiness attributable to union wage setting.

5See, e.g., Kahneman et al. (1986), Blinder and Choi (1990), and Shafir et al. (1997) for evidence for money illusion. See, e.g., Miao and Xie (2013) and Vaona (2013) for theoretical studies that analyze money illusion in dynamic general equilibrium models.
some of these studies (e.g., Valletta and Kuang, 2010; Rothstein, 2011), the expanded benefits aggravated unemployment but the effect was small.

An increase in unemployment benefits is generally deemed to worsen unemployment through two channels, labor supply and labor demand, as follows (see, e.g., Howell and Rehm, 2009, pp. 62–63). While it urges unemployed workers to remain unemployed and to receive the increased benefits, it reduces firms’ labor demand and aggravates unemployment because it induces labor unions to call for wage increases. We examine effects through labor demand in two steady states with and without an aggregate demand deficiency. Considering the presence of demand deficiency is important because unemployment benefits are usually increased in serious recessions or stagnation where demand deficiencies expand unemployment, such as in the Great Recession in the United States.

We first set out a steady state without deficient aggregate demand. In this steady state, unemployment is attributable only to nominal wage setting by labor unions, and an increase in unemployment benefits raises the nominal wage and deteriorates unemployment through the demand-side channel mentioned above. We next treat a steady state where insatiable demand for money permanently creates a demand deficiency and therefore worsens unemployment, leading to price deflation and nominal wage deflation. In the latter steady state, we obtain a result opposite to that of the former steady state. An increase in unemployment benefits causes labor unions to claim an increase in the nominal wage, which moderates nominal wage deflation and thence price deflation. Since this moderation of price deflation increases the cost of holding money, household consumption (aggregate demand) is
stimulated and unemployment is reduced. In sum, we show that the effects of unemployment benefits depend on the presence or absence of aggregate demand deficiency.

The third purpose is to analyze what happens if labor unions place more importance on raising the nominal wage compared with increasing employment. This change in union behavior is naturally considered to aggravate unemployment because it puts upward pressure on the nominal wage. This is indeed the case of the steady state without demand deficiency. By contrast, in the steady state with demand deficiency, it leads to a reduction in unemployment because the upward pressure on the nominal wage boosts household consumption by mitigating price deflation and making money holding disadvantageous. In this manner, paradoxically, in long-run stagnation with deficient demand, aiming to increase the nominal wage rather than employment results in expanding employment. This can be regarded as an example of a paradox arising in models with Keynesian features, such as the “paradox of thrift,” “paradox of toil,” and “paradox of flexibility” (Eggertsson and Krugman, 2012). Finally, we discuss the relationship between the implication of this paradoxical result and the experience of the Japanese economy during the long-run stagnation since the 1990s.

Several studies are close to the present paper. Sugawara (2009) develops a two-period overlapping generations version of Ono’s (1994, 2001) model and analyzes the effects of unemployment benefits in long-run stagnation. He finds that generous unemployment benefits increase voluntary unemploy-

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6See also Eggertsson (2010) for the paradox of toil and Ono and Ishida (2014) for these paradoxes in long-run stagnation.
ment but decrease involuntary unemployment. However, in contrast with the present paper, he as well as Ono assumes a simple nominal wage adjustment without a microeconomic foundation. Using a new Keynesian model with labor market frictions, Albertini and Poirier (2014) show that the effects of unemployment benefits depend on an economic condition. In their model, an increase in unemployment benefits pushes up wages, which decreases unemployment if the nominal interest rate hits the zero lower bound but increases unemployment otherwise. In this regard, however, whereas Albertini and Poirier (2014) deal with a short-run slump, we focus on long-run stagnation such as that of the Japanese economy since the 1990s. Ono and Ishida (2014) and Murota (2016) consider microeconomic foundations for nominal wage stickiness and analyze long-run stagnation. However, unlike the present paper, they neither consider wage bargaining between labor unions and firms nor investigate the effects of unemployment benefits. Furthermore, no study cited in this paragraph examines what happens if labor unions regard nominal wage increases as more important.

The remainder of the present paper is organized as follows. Section 2 develops a MIUF model and shows nominal wage setting by labor unions. Section 3 presents two steady states without and with aggregate demand deficiency and analyzes the effects of an increase in unemployment benefits and of a change in union behavior. Section 4 concludes.

7 Using fair wage models, Ono and Ishida (2014) examine the effects of fiscal and monetary expansions and Murota (2016) explores the effects of an employment subsidy in addition to these expansions. As in the present paper, Murota (2016) shows that the effects of these policies hinge on the absence or presence of aggregate demand deficiency.
2 Model

We first describe firm behavior, household behavior, and the government in a MIUF model. We then show that union wage setting gives rise to nominal wage stickiness and derive the dynamic system of the model.

2.1 Firm, Household, and Government

A representative firm produces a commodity according to the following Cobb-Douglas production function:

\[ y_t = A k_t^{\alpha} l_t^{1-\alpha}, \]

where \( y_t \) is production of the commodity, \( A \) is total factor productivity, \( k_t \) is physical capital, \( l_t \) is labor, and \( \alpha \) is a constant \((0 < \alpha < 1)\). As usual, the firm’s profit maximization yields

\[ r_t = \alpha A (k_t/l_t)^{\alpha-1}, \quad (1) \]

\[ w_t = (1-\alpha) A (k_t/l_t)^{\alpha}, \quad (2) \]

where \( r_t \) is the real rental rate and \( w_t \) is the real wage rate.

A representative household maximizes the following lifetime utility:

\[ \int_0^\infty [u(c_t) + v(m_t)] \exp(-\rho t) dt, \]

where \( c_t \) is consumption, \( m_t \) is real money holdings, and \( \rho \ (> 0) \) is the subjective discount rate, and \( u(\cdot) \) and \( v(\cdot) \) satisfy

\[ u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad u'(0) = \infty, \quad u'(\infty) = 0; \]

\[ v'(\cdot) > 0, \quad v''(\cdot) < 0, \quad v'(0) = \infty, \quad v'(\infty) = 0. \quad (3) \]
However, the last assumption in (3), \( v'(\infty) = 0 \), is abandoned in Subsection 3.2. The budget constraint in real terms is

\[
\dot{a}_t = r_t k_t - \pi_t m_t + w_t l_t + \beta w_t (l^F - l_t) - c_t - \tau_t,
\]

(4)

where \( \pi_t (\equiv \dot{P}_t / P_t) \) is the rate of change in a price \( P_t \), \( \tau_t \) is a lump-sum tax, and \( a_t \) is total asset holdings:

\[
a_t = k_t + m_t.
\]

(5)

In addition, \( l^F \) is the labor endowment, which the household inelastically supplies, and \( l_t \) is the amount of employment. Hence \( l^F - l_t \) denotes unemployment. As shown by (4), the household earns labor income \( w_t l_t \) and receives unemployment benefits \( \beta w_t (l^F - l_t) \), where \( \beta \) is the replacement rate (0 < \( \beta \) < 1).

Substituting (5) into (4) to eliminate \( k_t \), we obtain the following current-value Hamiltonian:

\[
\mathcal{H}_t = u(c_t) + v(m_t) + \lambda_t [r_t a_t - (r_t + \pi_t) m_t + w_t l_t + \beta w_t (l^F - l_t) - c_t - \tau_t],
\]

where \( \lambda_t \) is a co-state variable. The first-order conditions are

\[
\begin{align*}
    u'(c_t) &= \lambda_t, \\
    v'(m_t) - \pi_t \lambda_t &= r_t \lambda_t, \\
    \dot{\lambda}_t - \rho \lambda_t &= -r_t \lambda_t,
\end{align*}
\]

(6)

and the transversality condition is

\[
\lim_{t \to \infty} \lambda_t a_t \exp(-\rho t) = 0.
\]

From (6), we obtain

\[
\rho + \eta(c_t) \frac{\dot{c}_t}{c_t} = r_t = -\pi_t + \frac{v'(m_t)}{u'(c_t)},
\]

(7)
where \( \eta(c_t) \equiv -u''(c_t)c_t/u'(c_t) \). Equation (7) governs the consumption-saving decision and the portfolio choice between capital and money.\(^8\) If the rate of change in the price \( \pi_t \) rises, the marginal benefit of money denoted by the right-hand side of (7) declines.

The government keeps the nominal money stock \( M_t \) constant:

\[
M_t = \overline{M},
\]

which produces the following law of motion for real money balances \( m_t \) (\( = \overline{M}/P_t \)):

\[
\frac{\dot{m}_t}{m_t} = -\pi_t.
\] (8)

The budget equation of the government is

\[
\beta w_t(l^F - l_t) = \tau_t.
\] (9)

2.2 Nominal Wage Setting

Following Raurich et al. (2006) and Greiner (2013), we consider wage setting by labor unions. However, we differ from them in some ways, the most important being that because of money illusion labor unions are concerned not about an increase in the real wage but about that in the nominal wage. Due to this assumption, nominal wage stickiness appears in the present paper whereas Raurich et al. (2006) and Greiner (2013) derive real wage stickiness.

The objective function of a representative labor union is

\[
\gamma \log (W_t - W^R_t) + \log l_t,
\] (10)

where \( W_t \) is a nominal wage, \( W^R_t \) is a nominal reference wage, and \( \gamma \) is a positive constant. \( \gamma \) denotes the degree of importance placed on a rise in the

\(^8\)See Ono (1994, 2001) for a detailed discussion of this equation.
nominal wage relative to an increase in employment, and $W^R_t$ is defined as follows:

$$W^R_t = \theta \int_{-\infty}^{t} Z_s \exp(-\theta(t-s))ds, \quad (11)$$

where $\theta$ is a positive constant and $Z_s$ is average nominal income defined such that

$$Z_s = \frac{W_s l_s + \beta W_s(l^F - l_s)}{l^F}. \quad (12)$$

The reason that the objective of the labor union depends on the difference between the nominal wage and the nominal reference wage, $W_t - W^R_t$, is that money illusion influences judgments of fairness, which is supported by empirical studies such as Kahneman et al. (1986), Blinder and Choi (1990), and Shafir et al. (1997).\(^9\)

Given the reference wage $W^R_t$, the labor union sets the nominal wage $W_t$ to maximize (10) subject to the labor demand curve derived by substituting $w_t = W_t/P_t$ into (2):

$$l_t = [(1 - \alpha)AP_l]^{\frac{1}{\alpha}}k_t W_t^{\frac{1}{\alpha}}. \quad (13)$$

The first-order condition for this maximization problem is

$$W_t = \frac{W^R_t}{1 - \alpha \gamma}. \quad (14)$$

From (11), (12), and (14), we obtain the following adjustment process for the nominal wage:

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{W}^R_t}{W^R_t} = \theta \left( \frac{Z_t}{W^R_t} - 1 \right) = \theta \left( \frac{l_t + \beta(l^F - l_t)}{l^F(1 - \alpha \gamma)} - 1 \right) = \Phi \left( \frac{l_t - l^T}{l^F} \right), \quad (15)$$

\(^9\)Kahneman et al. (1986, pp. 731–732) state: “Although the real income change is approximately the same in the two problems, the judgments of fairness are strikingly different. A wage cut is coded as a loss and consequently judged unfair. A nominal raise which does not compensate for inflation is more acceptable because it is coded as a gain to the employee, relative to the reference wage.”
where \( \Phi \) and \( \bar{I} \) are constants defined such that

\[
\Phi \equiv \frac{\theta(1 - \beta)}{1 - \alpha \gamma} > 0, \quad 0 < \bar{I} \equiv l^F \left( 1 - \frac{\alpha \gamma}{1 - \beta} \right) < l^F. \quad (16)
\]

Note that \( \bar{I} > 0 \) because \( \alpha, \beta, \) and \( \gamma \) are assumed to be small enough to satisfy \( \alpha \gamma + \beta < 1 \) and that (14) is valid, \( \Phi > 0, \) and \( \bar{I} < l^F \) (i.e., \( 1 - \alpha \gamma > 0 \) and \( 1 - \beta > 0 \)) under this assumption. Following Greiner (2013), we call \( \bar{I} \) “the normal level of employment,” but \( \bar{I} \) is a value such that the nominal wage stays constant rather than the real wage.

From (15), the rate of change in the nominal wage \( \dot{W}_t/W_t \) is positively related to employment \( l_t \):

\[
\frac{d(\dot{W}_t/W_t)}{dl_t} = \frac{\Phi}{l^F} > 0,
\]

which implies the following. An increase in employment \( l_t \) boosts the average income compared with the reference wage:

\[
\frac{d(Z_t/W_t^{R})}{dl_t} = \frac{d}{dl_t} \left( l_t + \beta(l^F - l_t) \right) / l^F(1 - \alpha \gamma) = \frac{1 - \beta}{l^F(1 - \alpha \gamma)} > 0,
\]

which raises the rate of change in the reference wage \( \dot{W}_t/W_t^{R} \). Therefore, the labor union claims an increase in the nominal wage and the rate of change in the nominal wage \( \dot{W}_t/W_t \) also rises. Moreover, from (15), the effect of generous unemployment benefits on the rate of change in the nominal wage is positive:

\[
\frac{\partial(\dot{W}_t/W_t)}{\partial \beta} = \frac{\theta(l^F - l_t)}{l^F(1 - \alpha \gamma)} > 0. \quad (17)
\]

Given \( l_t \), a rise in the replacement rate \( \beta \) increases \( Z_t/W_t^{R} \) and thence \( \dot{W}_t/W_t^{R} \). This increase in the reference wage also prompts the labor union
to call for a higher nominal wage, which results in boosting $\dot{W}_t/W_t$. Similarly, from (15), if the labor union regards a nominal wage increase as more important, the rate of change in the nominal wage naturally rises:

$$\frac{\partial (\dot{W}_t/W_t)}{\partial \gamma} = \frac{\alpha \theta [l_t + \beta (l^F - l_t)]}{l^F (1 - \alpha \gamma)^2} > 0. \quad (18)$$

Note that arranging (15) yields a Phillips curve:$$\frac{\dot{W}_t}{W_t} = -\Phi\left(\frac{l^F - l_t}{l^F}\right) + \Phi\left(l^F - \bar{l}\right),$$

where the rate of change in the nominal wage $\dot{W}_t/W_t$ is negatively related to the unemployment rate $(l^F - l_t)/l^F$. Thus, equations (17) and (18) imply that changes in the replacement rate and the degree of importance attached to a wage increase $\gamma$ affect the economy by shifting the Phillips curve.

### 2.3 Dynamics

The dynamic system of the economy consists of four dynamic equations for $c_t$, $k_t$, $m_t$, and $l_t$. From (1) and (7), the dynamic equation for $c_t$ is

$$\dot{c}_t = \eta (c_t)^{-1} [\alpha A(k_t/l_t)^{\alpha - 1} - \rho]. \quad (19)$$

From (1), (2), (4), (5), (8), and (9), the dynamic equation for $k_t$ is

$$\dot{k}_t = Ak_t^{\alpha} l_t^{1 - \alpha} - c_t. \quad (20)$$

Since from (1) and (7) the rate of change in the price $\pi_t$ is given by

$$\pi_t = \frac{v'(m_t)}{u'(c_t)} - \alpha A(k_t/l_t)^{\alpha - 1}, \quad (21)$$

Murota (2016) derives a similar Phillips curve in an efficiency wage model where a previous nominal wage is a reference wage.
from (8) the dynamic equation for \(m_t\) is

\[
\frac{\dot{m_t}}{m_t} = -\pi_t = -\frac{v'(m_t)}{u'(c_t)} + \alpha A(k_t/l_t)^{\alpha-1}.
\] (22)

From (13), (15), (20), and (21), we derive the dynamic equation for \(l_t\):

\[
\dot{l_t} = \dot{k_t} - \frac{\dot{W_t}}{k_t} = \frac{A k_t^\alpha l_t^{1-\alpha} - c_t}{k_t} - \alpha^{-1} \left[ \Phi \left( l_t - \frac{L}{l^F} \right) + \frac{v'(m_t)}{u'(c_t)} + \alpha A(k_t/l_t)^{\alpha-1} \right].
\] (23)

3 Steady States

In this section, as in Murota (2016), we first present the steady state where unemployment is caused only by nominal wage stickiness and then set out the steady state where unemployment additionally arises because of an aggregate demand deficiency.

3.1 Steady State without Deficient Aggregate Demand

From (19), (20), (22), and (23) where \(\dot{c} = 0, \dot{k} = 0, \dot{m} = 0, \text{ and } \dot{l} = 0\), we obtain

\[
\alpha A(k^*/l^*)^{\alpha-1} = \rho,
\] (24)

\[
c^* = A(k^*)^\alpha (l^*)^{1-\alpha},
\] (25)

\[
\rho = \frac{v'(m^*)}{u'(c^*)},
\] (26)

\[
\pi^* = \frac{\dot{W^*}}{W^*} = \Phi \left( \frac{l^* - \bar{I}}{l^F} \right) = 0,
\] (27)

where the asterisk denotes endogenous variables in this steady state.
The existence of this steady state is easily proved. From (27), the level of employment equals the normal level given in (16):

$$l^* = \bar{l} = t^F \left(1 - \frac{\alpha\gamma}{1 - \beta}\right) < t^F,$$

which implies that the union wage setting is the cause of unemployment:

$$t^F - l^* = \frac{\alpha\gamma}{1 - \beta} t^F > 0.$$  

Given $l^*=\bar{l}$, from (24) and (25), we obtain $k^*$ and then $c^*$:

$$k^* = \left(\frac{\rho}{\alpha A}\right)^{\frac{1}{\alpha - 1}} \bar{l}, \quad c^* = A \left(\frac{\rho}{\alpha A}\right)^{\frac{\alpha}{\alpha - 1}} \bar{l}.$$  

(29)

Since $c^*$ is obtained, $m^*$ is determined by (26) due to the assumptions for the marginal utility of money in (3).

Differentiating (28) with respect to $\beta$ yields

$$\frac{dl^*}{d\beta} = -\frac{\alpha\gamma t^F}{(1 - \beta)^2} < 0.$$  

(30)

As implied by (17), a rise in the replacement rate $\beta$ encourages the labor union to call for an increase in the nominal wage, which results in a decrease in employment $l^*$.  

Moreover, from (28), if the labor union puts more value

\[\text{11}\]

\[\text{11}\]From (26), (29), and (30), if $\beta$ rises, capital, consumption, and real money balances decrease:

$$\frac{dk^*}{d\beta} < 0, \quad \frac{dc^*}{d\beta} < 0, \quad \frac{dm^*}{d\beta} < 0.$$  

Taking into account that the price $P^*$ satisfies $m^* = \overline{M}/P^*$, from the third property $dm^*/d\beta < 0$, we find

$$\frac{dP^*}{d\beta} > 0.$$  

Since the nominal wage and the price both rise, the real wage remains unchanged. This is also obvious from (2) and (24), where the capital–labor ratio $k^*/l^*$ and thus the real wage are independent of $\beta$.  

15
on a rise in the nominal wage compared with an increase in employment (i.e., \( \gamma \) rises), then employment naturally decreases:

\[
\frac{dl^*}{d\gamma} = -\frac{\alpha}{1 - \beta} F < 0.
\]

(31)

This is simply because an increase in \( \gamma \) as well as in \( \beta \) pushes up the nominal wage, as shown by (18).

### 3.2 Steady State with Deficient Aggregate Demand

To analyze unemployment created by deficient aggregate demand, discarding \( v'(\infty) = 0 \) in (3) and following Ono (1994, 2001), we assume that the marginal utility of money has a positive lower bound \( v' \) as follows:\(^\text{12}\)

\[
\lim_{m \to \infty} v'(m) = v' > 0,
\]

which implies that the household insatiately wants to save money even when real money holdings increase to infinity. As shown by Ono (2001, Figure 4), this assumption yields the same money demand function as in the case of the Keynesian liquidity trap.

If the lower bound \( v' \) is so high that the following inequality holds:

\[
\rho < \frac{v'}{u'(e^*)},
\]

(32)

then we do not have the value of \( m^* \) satisfying (26). Therefore, in this case, the steady state exhibited in Subsection 3.1 does not exist. Alternatively, from (19), (20), (22), and (23), we obtain the following steady state:

\[
\alpha A(k/l)^{a-1} = \rho,
\]

(33)

\(^\text{12}\)See Ono et al. (2004) for an empirical support for this assumption and Murota and Ono (2015, p. 598) for a discussion of the validity of this assumption.
\[ c = Ak^\alpha l^{1-\alpha}, \tag{34} \]
\[ \frac{\dot{m}}{m} = -\pi = -\Phi \left( \frac{l-l^*}{l^F} \right) > 0, \tag{35} \]
\[ \rho = -\Phi \left( \frac{l-l^*}{l^F} \right) + \frac{v'}{u'(c)}, \tag{36} \]

where \( m \) continues to increase (\( \dot{m}/m > 0 \)) while \( c, k, \) and \( l \) remain with respective finite values (\( \dot{c} = 0, \dot{k} = 0, \) and \( \dot{l} = 0 \)).

Following Ono (1994, 2001), we examine the unique existence of this steady state. From (33) and (34), \( l \) is given as a function of \( c \):

\[ l = A^{-1} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1-\alpha}} c. \tag{37} \]

Substituting (37) into (36) yields

\[ \rho = -\Phi \frac{l^F}{l^F} \left[ A^{-1} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1-\alpha}} c - l^* \right] + \frac{v'}{u'(c)} \equiv f(c). \tag{38} \]

From (32), \( f(c^*) \) is larger than \( \rho \):\(^{13}\)

\[ f(c^*) = -\Phi \frac{l^F}{l^F} \left[ A^{-1} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1-\alpha}} c^* - l^* \right] + \frac{v'}{u'(c^*)} = \frac{v'}{u'(c^*)} > \rho. \]

Hence, if \( f(0) \) is smaller than \( \rho \):

\[ f(0) = \frac{\Phi l^*}{l^F} < \rho, \]

and if \( f(c) \) is increasing in \( c \):

\[ f'(c) = -\frac{\Phi A^{-1}}{l^F} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1-\alpha}} - \frac{v'u''(c)}{[u'(c)]^2} > 0, \tag{39} \]

\(^{13}\)From the second equation of (29), the first term equals zero:

\[ -\Phi \frac{l^F}{l^F} \left[ A^{-1} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1-\alpha}} c^* - l^* \right] = -\Phi \left( \frac{l-l^*}{l^F} \right) = 0. \]
then the value of $c$ satisfying (38), denoted by $\tilde{c}$, is uniquely determined so as to lie between 0 and $c^*$:

$$0 < \tilde{c} < c^*.$$ 

See Figure 1 for this determination of $\tilde{c}$.

As in Murota (2016), we show that the consumption deficiency ($\tilde{c} < c^*$) exacerbates unemployment and generates deflation. Taking $\tilde{c} < c^*$ into account, from (28), the second equation of (29), and (37), we find that the level of employment in this steady state, denoted by $\tilde{l}$, is less than $l^* (= \bar{l})$:

$$\tilde{l} = A^{-1}\left(\frac{\rho}{\alpha A}\right)^{\tilde{c}} < A^{-1}\left(\frac{\rho}{\alpha A}\right)^{c^*} = \bar{l} = l^*, \quad (40)$$

which implies that because of Keynesian unemployment newly created, total unemployment increases:

$$l^F - \tilde{l} > l^F - l^*.$$ 

Note that Keynesian unemployment is $l^* - \tilde{l}$, which equals total unemployment in this steady state ($l^F - \tilde{l}$) minus unemployment attributable to the union wage setting ($l^F - l^*$). From (35) and (40), the rate of change in the price in this steady state, denoted by $\tilde{\pi}$, is negative:

$$\tilde{\pi} = \Phi\left(\frac{\tilde{l} - \bar{l}}{l^F}\right) < 0,$$

which permanently increases real money balances, as shown by (35).\footnote{Despite the persistent increase in real money balances, the transversality condition is satisfied due to the presence of $\mu'$ (the insatiable desire for money). See Ono (1994, 2001) for this property.}

Moreover, from (23) with $\dot{k} = 0$ and $\dot{l} = 0$, the nominal wage in this steady state, $\tilde{W}$, continues to decline in synchrony with the price:

$$\frac{\dot{W}}{W} = \tilde{\pi} < 0. \quad (41)$$
The mechanism that creates the demand deficiency, unemployment, and deflation is essentially the same as that of Ono (1994, 2001). As shown by (32), if \( c = c^* \), the marginal benefit of money exceeds the time preference rate.\(^{15}\) Therefore, the household wants to save more money and cuts down consumption from \( c^* \) to \( \tilde{c} \). This consumption deficiency \((c^* - \tilde{c})\) gives rise to Keynesian unemployment \((l^* - \tilde{l})\) and deflation \((\tilde{\pi} < 0)\). The present mechanism of demand deficiency and unemployment is also similar to Keynes’s (1936, p. 235) statement quoted at the outset of the introduction and that of the static model of Krugman (1999).

Now let us analyze the effect of unemployment benefits in this steady state. By substituting (16) into (38), we obtain

\[
\rho = -A^{-1}\theta(1-\beta)\left(\frac{\rho}{\alpha A}\right)\tau^\omega \tilde{c} + \theta\left(1 - \frac{\beta}{1-\alpha\gamma}\right) + \frac{\varphi'}{u'(\tilde{c})},
\]

which yields\(^{16}\)

\[
\frac{d\tilde{c}}{d\beta} = \frac{\theta(l^F - \tilde{l})}{l^F(1-\alpha\gamma)f'(\tilde{c})} > 0.
\]

Differentiating (40) and taking (43) into account, we obtain the following result opposite to (30) in Subsection 3.1:

**Proposition 1.** In the steady state with deficient aggregate demand, generous unemployment benefits reduce unemployment:

\[
\frac{d\tilde{l}}{d\beta} = A^{-1}\left(\frac{\rho}{\alpha A}\right)\tau^\omega \frac{d\tilde{c}}{d\beta} > 0.
\]

\(^{15}\)The time preference rate \( \rho \) implies the degree of preference for consumption. As \( \rho \) is lower, the household more strongly prefers saving to consumption.

\(^{16}\)Totally differentiating (42), we derive

\[
\left[ -A^{-1}\theta(1-\beta)\left(\frac{\rho}{\alpha A}\right)\tau^\omega \tilde{c} - \frac{\varphi''}{u'(\tilde{c})} \right]d\tilde{c} = \frac{\theta}{l^F(1-\alpha\gamma)}\left[ l^F - A^{-1}\left(\frac{\rho}{\alpha A}\right)\tau^\omega \tilde{c} \right]d\beta.
\]

Applying the first equation of (16), (39), and (40) into this equation produces (43).
As implied by (17), an increase in the replacement rate $\beta$ moderates nominal wage deflation because it induces the labor union to claim an increase in the nominal wage.\textsuperscript{17} Accordingly, from (41), price deflation is also moderated. Since from (7) this moderation of price deflation reduces the marginal benefit of holding money, the household consumes more ($d\tilde{c}/d\beta > 0$) and employment is created ($d\tilde{l}/d\beta > 0$).\textsuperscript{18} Comparing Proposition 1 with the result of (30) reveals that the presence or absence of aggregate demand deficiency crucially influences the effect of unemployment benefits.

Next, we investigate what happens if the labor union considers a rise in the nominal wage to be more important. Totally differentiating (42) and using the first equation of (16) and (39), we derive

$$\frac{d\tilde{c}}{d\gamma} = \frac{\alpha \theta}{(1 - \alpha \gamma)^2 f'(\tilde{c})} \left[ A^{-1}(1 - \beta) \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1 - \gamma}} \tilde{c} + \beta \right] > 0. \quad (44)$$

From (40) and (44), we obtain a paradoxical result opposite to (31):

**Proposition 2.** In the steady state with deficient aggregate demand, if labor unions give more weight to a rise in the nominal wage relative to an increase in employment, then employment increases:

$$\frac{d\tilde{l}}{d\gamma} = A^{-1} \left( \frac{\rho}{\alpha A} \right)^{\frac{\alpha}{1 - \gamma}} \frac{d\tilde{c}}{d\gamma} > 0.$$

A rise in $\gamma$ affects employment through a mechanism similar to that of a rise in $\beta$. From (18) and (41), the moderation of nominal wage deflation caused

\textsuperscript{17}The mechanism that moderates nominal wage deflation differs from that of Sugawara (2009). In his model, an increase in unemployment benefits increases voluntary unemployment, which reduces excess labor supply and thereby mitigates nominal wage deflation under a sluggish nominal wage adjustment without a microeconomic foundation.

\textsuperscript{18}From (33) and (34), the increase in consumption ($d\tilde{c}/d\beta > 0$) also boosts the capital stock in this steady state, $\tilde{k}$:

$$\frac{d\tilde{k}}{d\beta} = \frac{\alpha}{\rho} \cdot \frac{d\tilde{c}}{d\beta} > 0.$$
by a rise in $\gamma$ leads to that of price deflation, which stimulates consumption. Thus, paradoxically, when the labor union seeks a rise in the nominal wage rather than an increase in employment, unemployment is reduced.

Proposition 2 seems to offer an explanation of the recent experience of the Japanese economy. In Japan, labor unions have become reluctant to claim increases in wages during long-run stagnation triggered by the collapse of the bubble in the early 1990s, and nominal wages have declined since the late 1990s. At the same time, Japan’s stagnation has become serious; aggregate demand deficiency, unemployment, and deflation have worsened and persisted. This experience of the Japanese economy may be consistent with the case where $\gamma$ is low in the present stagnation steady state. If the labor union gives less importance to a rise in the nominal wage compared with an increase in employment ($\gamma$ is low), then demand deficiency, Keynesian unemployment, and deflation of the nominal wage and price all become exacerbated.

4 Conclusion

We show long-run stagnation where insatiable money demand creates insufficient aggregate demand and Keynesian unemployment in a MIUF model

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19 For example, Noda and Hirano (2013, p. 97) state: “However, since the burst of the bubble economy, wage increase rates at Shunto have declined and the presence of unions has decreased with the obsolescence of the system. Due to a long-stagnant economy and globally intense competition, most unions avoid making demands for higher wages, and instead, are more concerned with the ability of companies to secure their employment.”

20 According to Kodama et al. (2015, Figure 1), Japan’s average nominal wage has consecutively dropped since the late 1990s.

21 See, e.g., Nishizaki et al. (2014, Figure 4) and Murota (2016, Figure 1) for demand deficiency and Murota and Ono (2012, Figures 1 and 2) for unemployment and deflation, respectively.
where union wage setting causes nominal wage stickiness. We examine the effect of an increase in unemployment benefits through labor demand and find that the direction of its effect depends on the absence or presence of aggregate demand deficiency. If there is no demand deficiency, it pushes up the nominal wage, reduces labor demand, and worsens unemployment. By contrast, if aggregate demand is insufficient and deflation of the nominal wage and price arises, it increases labor demand and decreases unemployment, because it moderates price deflation by doing nominal wage deflation and therefore stimulates consumption (aggregate demand).

The effect of a change in union behavior also depends on the absence or presence of aggregate demand deficiency. We analyze the case where the labor union places more importance on nominal wage gains. If demand deficiency does not exist, a rise in the nominal wage caused by such a change decreases labor demand and increases unemployment. However, if demand deficiency exists and deflation of the nominal wage and price occurs, the change in union behavior mitigates nominal wage deflation and consequently price deflation, which boosts consumption and reduces unemployment. The results obtained in the present paper suggest that stopping price deflation by pushing up nominal wages is important for an economy to pull out of long-run stagnation attributable to deficient aggregate demand.
References


Figure 1: The existence of the unique value of $\tilde{c}$