Effects of an Employment Subsidy in Long-run Stagnation

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Abstract

We develop a money-in-the-utility-function model with two features. One is that a Phillips curve relationship between nominal wages and unemployment appears because of efficiency wages. The other is that as in the Japanese economy since the early 1990s, unemployment attributable to aggregate demand deficiency arises even in the long run. We analyze the effect of an employment subsidy in this long-run stagnation and show that an increase in the subsidy may worsen aggregate demand deficiency and unemployment.

Keywords: Aggregate demand, Efficiency wage, Employment subsidy, Long-run stagnation, Phillips curve

JEL Classification Codes: E12, E24

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1 Introduction

Ono (1994, 2001) develops a money-in-the-utility-function (MIUF) model with stickiness of prices or nominal wages and an insatiable desire for money. He shows that because of the insatiable desire for money, an economy falls into a stagnation steady state where a persistent aggregate demand deficiency occurs and permanently creates involuntary unemployment and deflation. In sum, he argues that persistent demand deficiency can be the cause of long-run stagnation. His argument is against the orthodox view of contemporary macroeconomics that an aggregate demand deficiency exists only in the short run even if such a deficiency arises.

However, the Japanese economy appears to have been in such long-run stagnation as presented by Ono (1994, 2001) since the early 1990s. In fact, aggregate demand deficiency, high unemployment, and deflation have been almost persistently observed. For example, as shown by Figure 1, Japan’s output gap has been negative since 1993 except for 1996, 1997, and 2007 (see also Nishizaki et al., 2014, Figure 4).¹ Nevertheless, few economists have considered that this prolonged demand deficiency is the cause of Japan’s long-run stagnation and has secularly created unemployment and deflation, probably because the view that demand deficiency persists has not been orthodox. An exception was Yoshiyasu Ono, who has actively argued that the prolonged demand deficiency causes Japan’s long-run stagnation (e.g., Ono, 2010).

Ono’s argument has gradually prevailed. For example, several economists

¹See, e.g., Ono (2010, Figure 2.1) and Murota and Ono (2012, Figure 1) for unemployment and Murota and Ono (2012, Figure 2) for deflation.
have analyzed stagnation steady states with aggregate demand deficiency in frameworks that extend Ono’s model (e.g., Matsuzaki, 2003; Johdo, 2009; Hashimoto, 2011). By constructing models that are somewhat different from Ono’s model in the assumption for the marginal utility of money, Murota and Ono (2011, 2012) obtain similar stagnation steady states. Meanwhile, recently, Summers (2014, 2015) points out the possibility that the U.S. economy has lapsed into “secular stagnation,” the cause of which is aggregate demand deficiency. Summers’s view receives much attention and studies on secular stagnation are rapidly increasing (e.g., Eggertsson and Mehrotra, 2014).²

We consider long-run stagnation attributable to aggregate demand deficiency such as in the Japanese economy since the 1990s. For this purpose, following Ono (1994, 2001), we develop a MIUF model, which is presented in Section 2. However, this model differs from Ono’s model in stickiness of nominal wages or prices. Whereas Ono simply assumes sluggish adjustments of nominal wages and prices without microeconomic foundations,³ we characterize nominal wage stickiness by a Phillips curve that appears because of an efficiency wage.

In a typical early contribution to efficiency wage theory, Solow (1979) simply considers that worker morale (labor productivity) is an increasing function of wages. Since then, however, researchers have addressed a range of factors that influence worker morale; in other words, they have assumed

²Using an overlapping generations model, Eggertsson and Mehrotra (2014) show that a permanent deleveraging shock gives rise to secular stagnation.
³Matsuzaki (2003), Johdo (2009), and Hashimoto (2011) also do not provide microeconomic foundations for adjustments of nominal wages and prices.
various types of effort functions. For example, Collard and de la Croix (2000) and Danthine and Kurmann (2004) develop dynamic general equilibrium models where worker morale depends on current and past real wages and the level of employment.\footnote{See de la Croix et al. (2009) and Vaona (2013a, 2015) for similar dynamic general equilibrium models.} Akerlof et al. (2000) and Campbell (2008) propose models where a rise in the unemployment rate and a rise in current wages compared with a reference level, including previous wages, encourage workers to provide greater effort.\footnote{See Campbell (2010) for a similar model.} Shafir et al. (1997) consider that because of money illusion, not only the level of current real wages but also the level of current nominal wages relative to previous nominal wages influences worker morale. Following these studies, we assume that an increase in the unemployment rate and an increase in current nominal wages as against previous nominal wages boost worker morale. In other words, worker effort is given as an increasing function of the unemployment rate and of current nominal wages over previous nominal wages.

The notion that an increase in the unemployment rate boosts worker morale is not only theoretically adopted by many studies, including those cited above, but is also empirically supported (e.g., Blinder and Choi, 1990; Agell and Lundborg, 1995, 2003). At the same time, following Shafir et al. (1997), we consider money illusion to be the reason why an increase in current nominal wages relative to previous nominal wages induces workers to provide greater effort. In this setting, workers use previous nominal wages as a reference to judge whether their employers treat them fairly. This setting is also supported by empirical studies. For instance, Kahneman et al. (1986)
and Blinder and Choi (1990) find that money illusion affects people’s judgment of fairness. Similarly, Shafir et al. (1997) conclude that money illusion influences such judgment and, consequently, worker morale. Bewley (1999) and Kawaguchi and Ohtake (2007) find that a reduction in nominal wages harms worker morale. Furthermore, the effects of money illusion may be persistent. Fehr and Tyran (2007) show that the effects of money illusion on equilibrium selection are long-lasting, and they (2007, p. 263) state: “Thus, the argument that the impact of money illusion on aggregate outcomes will eventually vanish through learning, can be seriously misleading.” Recently, money illusion is studied in the macroeconomic context. Vaona (2013b) analyzes the effects of money illusion on a long-run Phillips curve in a New Keynesian model with staggered wages, while Miao and Xie (2013) examine its effects on long-run economic growth in an endogenous growth model.

In Section 3, we show that the firm’s profit maximization subject to the above-mentioned effort function adopted in the present model gives rise to a Phillips curve, as in Akerlof et al. (2000) and Campbell (2008). In Section 4, we treat the case without Ono’s (1994, 2001) assumption of the insatiable desire for money as a benchmark. In this case, an aggregate demand deficiency does not arise but the nominal wage stickiness characterized by the Phillips curve generates unemployment in a steady state. We investigate the effect of an employment subsidy and show that an increase in the subsidy reduces this unemployment.

Employment subsidies are adopted as a way to preserve employment or

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6See also Campbell (2010) and Vaona (2013a, 2015) for Phillips curves in efficiency wage models.
reduce unemployment. In fact, the government of Japan has massively increased such subsidies since the Lehman shock (Fukushima, 2012, pp. 138–139). The amount rapidly increased from about 7 billion yen in fiscal year 2008 to about 653 billion yen in fiscal year 2009, and then it decreased but was still about 113 billion yen in fiscal year 2012. There are studies that find positive effects of wage and hiring subsidies on employment (e.g., Jaenichen and Stephan, 2011). However, the effectiveness of such subsidies is called into question because of deadweight, substitution, and displacement effects (Layard et al., 2005, pp. 476–478). As mentioned in Boockmann et al. (2012, pp. 737–738), due to these effects, such subsidies may not be effective in a macroeconomic sense even when they succeed in increasing the employment of a targeted group. Similarly, Martin and Grubb (2001, p. 31) state: “At the same time, most evaluations which focus on firm behaviour show that subsidies to private-sector employment have both large dead-weight and substitution effects. As a result, most such schemes yield small net employment gains, particularly in the short term when aggregate demand and vacancies are fixed.”

Therefore, we examine the effect of the employment subsidy in a macroeconomic framework where aggregate demand deficiency creates unemployment. However, in this regard, we deal with not short-run slumps but such

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7In a static partial equilibrium framework, Fukushima (2012) analyzes the effects of employment subsidies.
9In addition, Martin and Grubb (2001, p. 31) state: “For instance, evaluations of wage subsidies in Australia, Belgium, Ireland and the Netherlands have suggested combined dead-weight and substitution effects amounting to around 90 per cent, implying that for every 100 jobs subsidised by these schemes only ten were net gains in employment.”
long-run stagnation as that which Japan has experienced, by adopting Ono’s (1994, 2001) assumption of the insatiable desire for money. In Section 5, we present a steady state where unemployment is generated not only by the above-mentioned nominal wage stickiness but also by an aggregate demand deficiency, and show that a generous employment subsidy may worsen the demand deficiency and unemployment in this steady state. In Section 6, we summarize the results and discuss the effectiveness of the employment subsidy.

At the conclusion of the introduction, we mention differences from the most related papers. Ono and Ishida (2014) also analyze long-run stagnation with an aggregate demand deficiency in a model with microeconomically founded nominal wage stickiness, but their microeconomic foundation differs from the present efficiency wage setting.\textsuperscript{10} In addition, we treat persistent unemployment attributable to the efficiency wage in Section 4, whereas neither Ono (1994, 2001) nor Ono and Ishida (2014) consider this unemployment. Most importantly, the main aim of the present paper is to investigate the effects of the employment subsidy on aggregate demand and unemployment, but neither Ono (1994, 2001) nor Ono and Ishida (2014) examine.\textsuperscript{11}

\textsuperscript{10}We express labor productivity as a continuous function of the unemployment rate and the nominal wage change, whereas Ono and Ishida (2014) simply consider that it is always a positive constant if the nominal wage offered to workers is higher than a fair level and is zero if the wage is lower than the fair level.

2 Model

Following Collard and de la Croix (2000), Danthine and Kurmann (2004), de la Croix et al. (2009), and Vaona (2013a, 2015), we develop a dynamic general equilibrium model. In particular, as in de la Croix et al. (2009) and Vaona (2013a, 2015), we introduce the notion of a fair wage into a MIUF model. However, in contrast with them, we assume that worker morale hinges not on real wages but on nominal wages.\textsuperscript{12}

2.1 Household Sector

There is a continuum of identical households, the size of which is unity. Each household consists of a continuum of identical workers, the size of which is also unity. Hence the aggregate population size equals unity.

A typical household seeks to maximize its lifetime utility:

$$\max_{c_t, m_t, e_t} \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t [u(c_t) + v(m_t) - n_t x(e_t)] ,$$

where $\rho (> 0)$ is the subjective discount rate, $u(c_t)$ is the utility of consumption $c_t$, $v(m_t)$ is the utility of real money holdings $m_t$, $n_t$ is the number or proportion of employed workers, and $-n_t x(e_t)$ is the disutility of effort $e_t$ that employed workers provide. Note that involuntary unemployment $1-n_t$ arises although all workers inelastically supply their one-unit labor endowment.

The household determines consumption $c_t$ and money holdings $m_t$ and distributes them equally among the workers belonging to the household. Moreover, the employed workers, whose size is $n_t$, provide the same effort.

\textsuperscript{12}Collard and de la Croix (2000, p. 172) suggest an extension where nominal wages affect worker morale.
and derive the same disutility because identical firms pay the same wage. Therefore, we can analyze unemployment in a representative agent framework without considering the awkward problem that the workers are heterogeneous ex post, i.e., employed or unemployed (see in detail Danthine and Kurmann (2004), Vaona (2013a), and the literature cited therein).

As usual, we assume that \( u(c_t) \) and \( v(m_t) \) satisfy

\[
\begin{align*}
    u'(c_t) &> 0, \quad u''(c_t) < 0, \quad u'(0) = \infty, \quad u'(\infty) = 0; \\
    v'(m_t) &> 0, \quad v''(m_t) < 0, \quad v'(0) = \infty, \quad v'(\infty) = 0.
\end{align*}
\]

(1)

However, in Section 5, following Ono (1994, 2001), we analyze the case where the last property \( v'(\infty) = 0 \) is violated. Following Akerlof (1982), Collard and de la Croix (2000), Danthine and Kurmann (2004), de la Croix et al. (2009), and Vaona (2013a, 2015), for the disutility of effort to be a quadratic function, we assume that

\[
x(e_t) = (e_t - \bar{e}_t)^2, \quad (2)
\]

where \( \bar{e}_t \) is the norm of effort. However, in contrast with these studies, the norm \( \bar{e}_t \) depends not on real wages but on nominal wages and is given by

\[
\bar{e}_t = e \left( \frac{W_t}{W_{t-1}^s}, 1 - n_t^n \right),
\]

where \( W_t \) is the nominal wage received by a worker in period \( t \), \( W_{t-1}^s \) is the social average of nominal wages in period \( t - 1 \), and \( n_t^n \) is the aggregate amount of employment, all of which the household takes as given. \( \bar{e}_t \) satisfies

\[
\frac{\partial \bar{e}_t}{\partial (W_t/W_{t-1}^s)} > 0, \quad \frac{\partial^2 \bar{e}_t}{\partial (W_t/W_{t-1}^s)^2} < 0; \quad \frac{\partial \bar{e}_t}{\partial (1 - n_t^n)} > 0.
\]

(3)

Note that \( 1 - n_t^n \) denotes the economy-wide unemployment rate because the size of the aggregate population is unity.
The household faces the following budget constraint:

\[
\frac{M_{t+1} - M_t}{P_t} = w_t n_t - c_t - \tau_t,
\]

where \( M_t \) is nominal money holdings, \( P_t \) is a commodity price, \( w_t (\equiv W_t/P_t) \) is a real wage, and \( \tau_t \) is a lump-sum tax. Taking (2) and \( m_t \equiv M_t/P_t \) into account, we obtain the first-order optimality conditions for the utility-maximization problem with respect to \( c_t, m_{t+1}, \) and \( e_t \):\(^{13}\)

\[
u'(c_t) = \lambda_t, \tag{4}
\]

\[
\frac{\nu'(m_{t+1}) + \lambda_{t+1}}{1 + \rho} = \lambda_t (1 + \pi_{t+1}), \tag{5}
\]

\[
e_t = \tilde{e}_t = e \left( \frac{W_t}{W_{t-1}} \right)^a \left( 1 - n_a^a \right), \tag{6}
\]

where \( \lambda_t \) is the Lagrange multiplier and \( \pi_{t+1} (\equiv (P_{t+1} - P_t)/P_t) \) is the rate of change in the price, and the transversality condition is

\[
\lim_{t \to \infty} \frac{\lambda_t (1 + \pi_{t+1}) m_{t+1}}{(1 + \rho)^t} = 0. \tag{7}
\]

From (4) and (5), we derive

\[
u'(c_t) = \frac{\nu'(m_{t+1})}{(1 + \rho)(1 + \pi_{t+1})} + \frac{\nu'(c_{t+1})}{(1 + \rho)(1 + \pi_{t+1})}, \tag{8}
\]

where the left-hand side (LHS) and the right-hand side (RHS) denote the marginal disutility of and the total marginal utility of giving up consumption by one unit (i.e., saving money by \( P_t \) yen) in period \( t \), respectively. \( P_t \) yen marginally saved in period \( t \) (one unit in real terms) changes to \( P_t/P_{t+1} (=\)
\[ \frac{1}{(1 + \pi_{t+1})} \] in real terms in period \( t + 1 \). It yields the utility of money (the first term) and the utility of consumption (the second term). Equation (8) implies that an increase in the rate of change in the price, \( \pi_{t+1} \), reduces the total marginal utility of saving money because it decreases the future purchasing power of money, in other words, increases the cost of holding money.

From (6), worker morale depends on the nominal wages.\(^{14}\) Taking (3) into account, we have

\[
\frac{\partial e_t}{\partial (W_t/W_{t-1})} = e_1 > 0, \quad \frac{\partial^2 e_t}{\partial (W_t/W_{t-1})^2} = e_{11} < 0; \quad \frac{\partial e_t}{\partial (1 - n_t^e)} = e_2 > 0. \quad (9)
\]

Following Akerlof (1982) and Akerlof and Yellen (1990), we discuss the implications of \( e_1 > 0 \) and \( e_2 > 0 \) in (9). If a firm pays a higher current nominal wage compared with the previous nominal wage, which serves as a reference for a worker to judge whether the firm is treating him or her fairly, then the worker provides greater effort in return. The worse the labor market condition becomes (i.e., the higher the unemployment rate \( 1 - n_t^e \)), the more the worker appreciates being hired by the firm and paid the wage, namely, the more valuable the gift from the firm to the worker. Thus, an increase in unemployment causes the worker to provide greater effort.

### 2.2 Firm Sector

The firm sector is composed of a continuum of identical firms, the size of which we normalize to unity. A typical firm produces a commodity according

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\(^{14}\)Shafir et al. (1997, Section 3) assume that worker morale depends both on the level of real wages and on the ratio of current nominal wages to previous nominal wages.
to the following linear technology:

\[ y_t = e_t n_t^d, \]  

(10)

where \(y_t\) is the production of the commodity, the effort \(e_t\), given by (6), is labor productivity, and \(n_t^d\) is labor input. The firm sets \(n_t^d\) and \(W_t\) to maximize its profits:

\[ P_t e \left( W_t/W_{t-1}^s, 1 - n_t^a \right) n_t^d - W_t n_t^d + P_t z n_t^d, \]

where \(z\) denotes an employment subsidy in real terms and the firm takes \(P_t\), \(W_{t-1}^s\), \(n_t^a\), and \(z\) as given.\(^{15}\) This profit maximization leads to

\[ e \left( W_t/W_{t-1}^s, 1 - n_t^a \right) + z = \frac{W_t}{P_t}, \]  

(11)

\[ P_t e_1 \left( W_t/W_{t-1}^s, 1 - n_t^a \right) = 1. \]  

(12)

From (11), we take an increase in \(z\) as a rise in the marginal productivity of labor. Naturally, we can regard it as a decrease in the marginal cost of labor by arranging (11) as follows:

\[ e \left( W_t/W_{t-1}^s, 1 - n_t^a \right) = \frac{W_t}{P_t} - z. \]

By eliminating \(P_t\) from (11) and (12), we obtain a modified Solow condition:

\[ \frac{(W_t/W_{t-1}^s) e_1 \left( W_t/W_{t-1}^s, 1 - n_t^a \right)}{e \left( W_t/W_{t-1}^s, 1 - n_t^a \right) + z} = 1. \]  

(13)

2.3 Government

The budget equation of the government is

\[ \frac{M_{t+1} - M_t}{P_t} + \tau_t = g + z n_t^d, \]

\(^{15}\)The commodity market is perfectly competitive.
where $g$ is government purchases. The nominal money stock changes at a constant rate $\mu (> -\rho/(1 + \rho))$:

$$\frac{M_{t+1} - M_t}{M_t} = \mu,$$

which yields the law of motion for real money balances as follows:

$$\frac{m_{t+1}}{m_t} = \frac{1 + \mu}{1 + \pi_{t+1}}.$$  \hspace{1cm} (14)

## 3 Phillips Curve

Since households and firms are identical and the sizes of both are unity, we obtain

$$W_{t-1} = W_{t-1}, \quad n_t^d = n_t^a = n_t.$$  \hspace{1cm} (15)

From (13) and (15), we find

$$\frac{(W_t/W_{t-1}) e_1 (W_t/W_{t-1}, 1 - n_t)}{e (W_t/W_{t-1}, 1 - n_t) + z} = 1,$$  \hspace{1cm} (16)

which gives $W_t/W_{t-1}$ as a function of $1 - n_t$ and $z$:

$$\frac{W_t}{W_{t-1}} = \psi(1 - n_t; z).$$  \hspace{1cm} (17)

Following Campbell (2008) and Vaona (2015), we assume that\(^{16}\)

$$\frac{\partial^2 e_t}{\partial (W_t/W_{t-1}) \partial (1 - n_t)} \equiv e_{12} \leq 0.$$

\(^{16}\)Campbell (2008, p. 1391) makes a similar assumption and argues for the validity of the assumption.
Then, differentiating (16) and taking (9) into account, we derive\(^{17}\)

\[
\frac{\partial(W_t/W_{t-1})}{\partial(1-n_t)} \equiv \psi_1 = \frac{e_2 - (W_t/W_{t-1})e_{12}}{(W_t/W_{t-1})e_{11}} < 0, \tag{18}
\]

\[
\frac{\partial(W_t/W_{t-1})}{\partial z} = \psi_2 = \frac{1}{(W_t/W_{t-1})e_{11}} < 0. \tag{19}
\]

Equation (18) implies the existence of a Phillips curve: a negative relationship between the rate of change in the nominal wage \((W_t - W_{t-1})/W_{t-1}\) and the unemployment rate \(1 - n_t\). By subtracting one from both sides of (17), we indeed obtain this Phillips curve as follows:

\[
\frac{W_t - W_{t-1}}{W_{t-1}} = \psi(1 - n_t; z) - 1,
\]

where its slope equals \(\psi_1\) of (18):

\[
\frac{\partial((W_t - W_{t-1})/W_{t-1})}{\partial(1-n_t)} = \psi_1 < 0.
\]

This Phillips curve, depicted in Figure 2, implies the following effect of unem-

\(^{17}\)Totally differentiating (16), we obtain

\[
\left[ \frac{e_1}{e + z} + \frac{(W_t/W_{t-1})e_{11}}{e + z} - \frac{(W_t/W_{t-1})e_{12}^2}{(e + z)^2} \right] d(W_t/W_{t-1})
+ \left[ \frac{(W_t/W_{t-1})e_{12}}{e + z} - \frac{(W_t/W_{t-1})e_{11}e_{12}}{(e + z)^2} \right] d(1 - n_t) - \frac{(W_t/W_{t-1})e_{11}}{(e + z)^2} dz = 0.
\]

Substituting (16) into this equation yields

\[
\left[ \frac{e_1}{e + z} + \frac{(W_t/W_{t-1})e_{11}}{e + z} - \frac{e_1}{e + z} \right] d(W_t/W_{t-1})
+ \left[ \frac{(W_t/W_{t-1})e_{12}}{e + z} - \frac{e_2}{e + z} \right] d(1 - n_t) - \frac{dz}{e + z} = 0.
\]

Therefore we have

\[
d(W_t/W_{t-1}) = \frac{e_2 - (W_t/W_{t-1})e_{12}}{(W_t/W_{t-1})e_{11}} \cdot d(1 - n_t) + \frac{1}{(W_t/W_{t-1})e_{11}} \cdot dz,
\]

which implies that (18) holds if \(dz = 0\) and that (19) holds if \(d(1 - n_t) = 0\).
ployment on firm behavior. An increase in unemployment extracts greater effort from workers, giving firms less incentive to raise the current nominal wage compared with the previous nominal wage. Meanwhile, from (19), an increase in $z$ shifts the Phillips curve downward (see Figure 2). This shift implies the following effect of the employment subsidy on firm behavior. Since a subsidy increase works like a rise in labor productivity, it becomes less important for firms to extract effort from workers and firms become reluctant to raise the current nominal wage compared with the previous nominal wage.

Using (6), (10), (15), and (17), we obtain the commodity market equilibrium as follows:

$$c_t + g = y_t = e(\psi(1 - n_t; z), 1 - n_t) n_t,$$

where it is naturally assumed that an increase in employment $n_t$ leads to an increase in production $y_t$:

$$\frac{dy_t}{dn_t} = e - e_1 \psi_1 n_t - e_2 n_t > 0.$$  

From (11), (15), and (17), the rate of change in the price, $\pi_t$, is given as a function of the unemployment rates $1 - n_t$ and $1 - n_{t-1}$:

$$\pi_t = \psi(1 - n_t; z) \cdot \frac{e(\psi(1 - n_{t-1}; z), 1 - n_{t-1}) + z}{e(\psi(1 - n_t; z), 1 - n_t) + z} - 1.$$  

## 4 Benchmark Case

In this section, we analyze the steady state where an aggregate demand deficiency does not occur but the nominal wage stickiness characterized by

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18 Figure 2 illustrates the case where $\psi(0; z) - 1$ is positive and $\psi(1; z) - 1$ is negative. In this regard, however, both can be positive or negative, depending on the form of $\psi(\cdot)$, i.e., the effort function.
the Phillips curve generates unemployment. From (8), (14), (20), and (22), we obtain

\[ u'(c^*) = \frac{v'(m^*) + u'(c^*)}{(1 + \rho)(1 + \pi^*)}, \]  \hspace{1cm} (23)

\[ \pi^* = \mu, \]  \hspace{1cm} (24)

\[ c^* + g = y^* = e(\psi(1 - n^*; z), 1 - n^*) n^*, \]  \hspace{1cm} (25)

\[ \pi^* = \psi(1 - n^*; z) - 1, \]  \hspace{1cm} (26)

where the asterisk is attached to endogenous variables in this steady state.

Equation (26) shows that the price as well as the nominal wage obeys the Phillips curve relationship.

4.1 Existence of Steady State

From (24) and (26), we have

\[ \mu = \psi(1 - n^*; z) - 1. \]  \hspace{1cm} (27)

From (27), if the money growth rate \( \mu \) satisfies

\[ \psi(0; z) - 1 > \mu > \psi(1; z) - 1, \]

then \( n^* \) is determined so as to satisfy \( 1 > n^* > 0 \); that is, unemployment (or the unemployment rate) is\(^{19} \)

\[ 1 - n^* \ (> 0). \]

With the determination of \( n^* \), from (25), we obtain \( y^* \) and then \( c^* \ (= y^* - g) \).

Lastly, from (23) and (24), \( m^* \) is determined such that it satisfies

\[ u'(c^*) = \frac{v'(m^*) + u'(c^*)}{(1 + \rho)(1 + \mu)} \]  \hspace{1cm} (28)

under the properties of the marginal utility of money in (1).

\(^{19}\)Full employment \( (n^* = 1) \) is reached only if \( \mu = \psi(0; z) - 1 \).
4.2 Effects of Fiscal and Monetary Expansions

To understand the properties of this steady state, we examine the effects of fiscal and monetary expansions. From (18), (21), (25), and (27), an increase in government purchases $g$ has no effect on employment $n^*$ and completely crowds out private consumption $c^*$, whereas an increase in the money growth rate $\mu$ boosts them:

$$\frac{dn^*}{dg} = 0, \quad \frac{dc^*}{dg} = -1 < 0; \quad \frac{dn^*}{d\mu} > 0, \quad \frac{dc^*}{d\mu} = \frac{dy^*}{dn^*} \cdot \frac{dn^*}{d\mu} > 0.$$

The result of government purchases implies that unemployment in this steady state is not Keynesian but arises because of the efficiency wage. Moreover, in contrast to Keynesian economics, the monetary expansion affects employment and consumption not through the demand side but through the supply side as follows. An increase in the money growth rate raises the rate of change in the price $\pi^*$ and, hence, that in the nominal wage, which enhances labor productivity $e$. This rise in productivity boosts production both directly and indirectly by motivating firms to employ more labor—and this increase in production leads to an increase in consumption. Vaona (2013a) obtains a similar effect of monetary expansion in a model with a price Phillips curve different from (26).\(^{20}\)

4.3 Effect of Employment Subsidy

We next examine the effect of the employment subsidy. Differentiating (27) and taking (18) and (19) into account, we find that the subsidy reduces

\(^{20}\)In a setting where individuals use the price at time $t+i$ to assess the real value of the nominal wage at time $t+i-1$, Vaona (2013a) also derives a Phillips curve relationship between inflation and unemployment.
unemployment:

**Proposition 1.** In the steady state with only unemployment attributable to the efficiency wage, an increase in the employment subsidy $z$ improves unemployment:

$$\frac{dn^*}{dz} = \frac{\psi_2}{\psi_1} > 0.$$ 

Since the identity of (27) is the modified Solow condition, derived from the first-order conditions for the profit-maximization problem, in this steady state:

$$\frac{(1 + \mu)e_1(1 + \mu, 1 - n^*)}{e(1 + \mu, 1 - n^*) + z} = 1,$$

this result is simply produced as follows. An increase in the subsidy works like a reduction in the marginal cost of labor; therefore the firm’s demand for labor is stimulated and unemployment declines.

## 5 Aggregate Demand Deficiency

In this section, we analyze the steady state where an aggregate demand deficiency in addition to the above-mentioned nominal wage stickiness creates unemployment. For this purpose, we abandon the assumption that $v'(\infty) = 0$ in (1) and adopt Ono’s (1994, 2001) assumption that the marginal utility of money reaches a positive lower bound $\beta$ even when real money holdings increase to infinity:

$$\lim_{m \to \infty} v'(m) = \beta (> 0).$$

This assumption has a significant advantage that enables us to easily analyze aggregate demand deficiency and Keynesian unemployment even in a
dynamic optimization framework. Given this assumption, we do not require the conventional Keynesian consumption function, which lacks a microeconomic foundation. See Murota and Ono (2015, p. 598) for a discussion on the validity of this assumption.

We first show that if the insatiable desire for money is strong (\(\beta\) is high):

\[ u'(c^*) < \frac{\beta + u'(c^*)}{(1 + \rho)(1 + \mu)}, \]  

(29)

then aggregate demand becomes insufficient and unemployment becomes higher than \(1 - n^*\). When (29) is true, the steady state of Section 4 does not exist because there is no value of \(m^*\) that satisfies (28). In this case, from (8), (14), (20), and (22), the following steady state exists:

\[ u'(c) = \frac{\beta + u'(c)}{(1 + \rho)(1 + \mu)}, \]  

(30)

\[ \lim_{t \to \infty} \frac{m_{t+1}}{m_t} = \frac{1 + \mu}{1 + \pi} > 1, \]  

(31)

\[ c + g = e(\psi(1 - n; z), 1 - n) n, \]  

(32)

\[ \pi = \psi(1 - n; z) - 1. \]  

(33)

Equation (33) shows that the price Phillips curve also appears.\(^{21}\)

5.1 Existence of Steady State

Let us examine the existence of this steady state. From (32) and (33), \(n\) and \(\pi\) are expressed as functions of \(c, z,\) and \(g,\) respectively:

\[ n = n(c; z, g), \quad \pi = \pi(c; z, g) = \psi(1 - n(c; z, g); z) - 1. \]  

(34)

\(^{21}\)It is well known that the Phillips curve relationship stably exists in Japan (see, e.g., Ono, 2010, Figure 2.7; Nishizaki et al., 2014, Figure 2).
They satisfy
\[ n(c^*; z, g) = n^*, \quad \pi(c^*; z, g) = \pi^* = \mu, \quad (35) \]

where \( c^*, n^*, \) and \( \pi^* \) are the values given in the steady state of Section 4. In addition, they satisfy
\[ \frac{\partial n}{\partial c} = \frac{1}{e - e_1\psi_1 n - e_2 n} > 0, \quad \frac{\partial \pi}{\partial c} = \frac{-\psi_1}{e - e_1\psi_1 n - e_2 n} > 0, \quad (36) \]

where the inequalities hold under (18) and (21).

Following Ono (1994, 2001), we investigate the conditions for the unique existence of the steady state. From (30) and (34), we obtain
\[ (f(c) \equiv) u'(c) - \frac{\beta + u'(c)}{(1 + \rho)[1 + \pi(c; z, g)]} = 0. \quad (37) \]

If (29) is true, taking the second property of (35) into account, we find
\[ f(c^*) = u'(c^*) - \frac{\beta + u'(c^*)}{(1 + \rho)[1 + \pi(c^*; z, g)]} < 0. \]

Since \( \beta/u'(0) = 0 \) (\( u'(0) = \infty \) from (1)), \( f(0) \) is
\[ f(0) = u'(0) \left[ 1 - \frac{[\beta/u'(0)] + 1}{(1 + \rho)[1 + \pi(0; z, g)]} \right] = u'(0) \left[ 1 - \frac{1}{(1 + \rho)[1 + \pi(0; z, g)]} \right], \]

which implies that if \( \pi(0; z, g) > -\rho/(1 + \rho) \):
\[ (1 + \rho)[1 + \pi(0; z, g)] > 1, \quad (38) \]
then
\[ f(0) > 0. \]

Therefore, if (29) and (38) are true, the value(s) of \( c \) that satisfies (37) lies
between 0 and \(c^*\). Furthermore, if \(f'(c) < 0\):\(^{22}\)

\[
f'(c) = u''(c) \left[ 1 - \frac{1}{(1 + \rho)(1 + \pi)} \right] + \frac{\beta + u'(c)}{(1 + \rho)(1 + \pi)^2} \frac{\partial \pi}{\partial c} < 0, \tag{39}
\]

we get the unique value of \(c\). In sum, if the unique value satisfying (37) is denoted by \(\tilde{c}\), we obtain

\[
0 < \tilde{c} < c^* = (y^* - g).
\]

From (34), the values of \(n\) and \(\pi\) in this steady state, denoted by \(\hat{n}\) and \(\hat{\pi}\), also uniquely exist as follows:

\[
\hat{n} = n(\tilde{c}; z, g), \quad \hat{\pi} = \pi(\tilde{c}; z, g). \tag{40}
\]

We briefly discuss how the consumption deficiency \((\tilde{c} < c^*)\) occurs.\(^ {23}\) If \(c = c^*\) and (29) is true, the total marginal utility of saving money (giving up consumption) exceeds the marginal disutility of doing so even when \(m = \infty\), which implies that the household desires to decrease consumption and save more money because \(c^*\) is too much for the household. Therefore, consumption is reduced to \(\hat{c} (< c^*)\) so that (37) holds (the total marginal utility equals the marginal disutility).

This consumption deficiency persistently aggravates unemployment. Taking \(\tilde{c} < c^*\) into account, from the first equations of (35), (36), and (40), we

\(^{22}\)Because of the second property of (36): \(\partial \pi / \partial c > 0\) and (38), the expression in the square brackets of (39) is always positive:

\[
1 - \frac{1}{(1 + \rho)(1 + \pi)} > 0 \quad \text{for } \forall c.
\]

Hence, in (39), the first term is negative while the second term is positive, which implies that \(f'(c) < 0\) is possible.

indeed find that \( \hat{n} < n^* \), i.e.,

\[
1 - \hat{n} > 1 - n^*.
\]

In contrast to Ono (1994, 2001) and Ono and Ishida (2014), unemployment in this steady state, \( 1 - \hat{n} \), is the sum of unemployment attributable to the efficiency wage, \( 1 - n^* \), and unemployment attributable to the consumption deficiency, \( n^* - \hat{n} \). Moreover, the consumption deficiency depresses the rate of change in the price. Given \( \hat{c} < c^* \), from the second equations of (35), (36), and (40), we obtain

\[
\hat{\pi} < \pi^* = \mu, \tag{41}
\]

which implies that real money balances continue to expand, as shown by (31).\(^{24}\) Thus, if (29), (38), and (39) are true, the steady state characterized by (30)–(33) uniquely exists.

### 5.2 Effects of Fiscal and Monetary Expansions

We show further Keynesian properties of this steady state. The effects of fiscal and monetary expansions are consistent with those obtained in the Keynesian liquidity trap, such as the case where the IS curve intersects with the horizontal part of the LM curve in the IS-LM analysis. Differentiating

\[^{24}\text{From (4), (7), and (14), for the transversality condition to be satisfied:}
\]

\[
\lim_{t \to \infty} \frac{\lambda_t(1 + \pi_{t+1})m_{t+1}}{(1 + \rho)^t} = u'(\hat{c})(1 + \mu) \lim_{t \to \infty} \frac{m_t}{(1 + \rho)^t} = 0,
\]

real money balances must expand at a rate less than \( \rho \). Hence, as in Ono and Ishida (2014), \( \mu \) must satisfy not only (29) but also

\[
\frac{1 + \mu}{1 + \hat{\pi}} < 1 + \rho.
\]
and (40) and taking (36) and (39) into account, we find that an increase in government purchases $g$ increases consumption $\tilde{c}$ and employment $\tilde{n}$ whereas an increase in the money growth rate $\mu$ has no effect on either:

$$\frac{d\tilde{c}}{dg} = -\frac{\beta + u'(\tilde{c})}{f'(\tilde{c})(1 + \rho)(1 + \pi)^2} \cdot \frac{\partial \tilde{n}}{\partial g} > 0,$$

$$\frac{d\tilde{n}}{dg} = \frac{\partial \tilde{n}}{\partial \tilde{c}} \cdot \frac{d\tilde{c}}{dg} + \frac{\partial \tilde{n}}{\partial g} > 0;$$

$$\frac{d\tilde{c}}{d\mu} = 0, \quad \frac{d\tilde{n}}{d\mu} = 0,$$

where $\partial \tilde{n}/\partial g > 0$ and $\partial \tilde{c}/\partial g > 0.$

An increase in $g$ boosts employment $\tilde{n}$ through two channels. It directly increases aggregate demand and creates employment ($\partial \tilde{n}/\partial g > 0$). At the same time, it raises the rate of change in the price ($\partial \tilde{c}/\partial g > 0$), which reduces the total marginal utility of saving money (the RHS of (30)). Therefore, consumption is stimulated and further employment is created ($((\partial \tilde{n}/\partial \tilde{c}) \cdot (d\tilde{c}/dg) > 0$). By contrast, an increase in $\mu$ is ineffective because the Pigou effect is not working. Even if real money holdings increase, the marginal utility of money remains at $\beta$ and consequently consumption is not stimulated. Ono and Ishida (2014) obtain the same results of fiscal and monetary expansions in a model with nominal wage stickiness different from that of the present model.

From (33), the monetary expansion is also ineffective for the rate of change in the price $\tilde{\pi}$:

$$\frac{d\tilde{\pi}}{d\mu} = -\psi_1 \frac{d\tilde{n}}{d\mu} = 0.$$

---

$^{25}$Differentiating (32) and (33) and using (18) and (21), we have

$$\frac{\partial \tilde{n}}{\partial g} = \frac{1}{e - e_1 \psi_1 n - e_2 n} > 0, \quad \frac{\partial \tilde{\pi}}{\partial g} = \frac{-\psi_1}{e - e_1 \psi_1 n - e_2 n} > 0.$$

$^{26}$This increase in private consumption implies that the multiplier of government purchases on output is larger than one (see in detail Murota and Ono, 2015).
However, as shown by (41), \( \tilde{\pi} \) is subject to the restriction that it does not exceed \( \mu \). If \( \mu \) is negative, \( \tilde{\pi} \) takes a negative value. Meanwhile, if \( \mu \) is positive, \( \tilde{\pi} \) may be positive or negative. The latter implies that stagflation can arise or that deflationary stagnation can arise even though money expands—which of these appears depends on the shape of the Phillips curve. This result is different from Ono (1994, 2001) and Ono and Ishida (2014), in whose models only deflationary stagnation occurs. The theoretical result that an increase in \( \mu \) is not effective either for stopping deflation or for stimulating aggregate demand may be consistent with the Japanese experience that monetary expansions were not very effective in long-run stagnation since the 1990s (see, e.g., Ugai, 2007 for a discussion on this ineffectiveness).

Note that the causal relationship between the price change rate and the unemployment rate differs between Section 4 and Section 5 although the Phillips curve relationship between them exists in both sections. A change in the unemployment rate affects the price change rate in Section 5, whereas the opposite causal relationship appears in Section 4. In this way, the causation of the Phillips curve depends crucially on the presence of aggregate demand deficiency.

### 5.3 Effect of Employment Subsidy

We finally examine the effect of the employment subsidy \( z \) in this steady state. An increase in \( z \) affects employment \( \tilde{n} \) through two channels. From (32), it has a direct effect on \( \tilde{n} \). At the same time, from (32) and (37), it has an indirect effect on \( \tilde{n} \) through consumption \( \tilde{c} \). These effects are shown by
differentiating the first equation of (40) as follows:

\[
\frac{d\tilde{n}}{dz} = \frac{\partial \tilde{n}}{\partial \tilde{c}} \cdot \frac{d\tilde{c}}{dz} + \frac{\partial \tilde{n}}{\partial z} = \frac{1}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}} \cdot \frac{d\tilde{c}}{dz} + \frac{-e_1\psi_2\tilde{n}}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}},
\]

(42)

where \(\partial \tilde{n}/\partial \tilde{c}\) is given by the first equation of (36) and \(\partial \tilde{n}/\partial z\) is derived by differentiating (32).\(^{27}\) In (42), the second term \(\partial \tilde{n}/\partial z\) denotes the direct effect. It is positive from (9), (19), and (21) and implies the following. An increase in the subsidy depresses the rate of change in the nominal wage, as shown by (19). Since this lowers labor productivity, more labor is needed to produce a given amount, as implied by (32).

In (42), the first term \((\partial \tilde{n}/\partial \tilde{c}) \cdot (d\tilde{c}/dz)\) denotes the indirect effect through consumption (aggregate demand). To identify its sign, we have to investigate the sign of \(d\tilde{c}/dz\) because \(\partial \tilde{n}/\partial \tilde{c}\) is already known to be positive. Totally differentiating (37), we find that an increase in the subsidy has an impact on consumption by affecting the rate of change in the price as follows:

\[
\frac{d\tilde{c}}{dz} = -\frac{\beta + u'(\tilde{c})}{f'((1 + \rho)(1 + \tilde{\pi})^2)} \cdot \frac{\partial \tilde{\pi}}{\partial z},
\]

(43)

where the sign of \(d\tilde{c}/dz\) depends on that of \(\partial \tilde{\pi}/\partial z\). Partially differentiating the second property of (34) and using \(\partial \tilde{n}/\partial z\) derived by differentiating (32), we obtain

\[
\frac{\partial \tilde{\pi}}{\partial z} = -\psi_1 \frac{\partial \tilde{n}}{\partial z} + \psi_2 = \frac{\psi_2(e - e_2\tilde{n})}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}},
\]

(44)

where the expression in the parentheses, \(e - e_2\tilde{n}\), can be positive or negative, namely, the sign of \(\partial \tilde{\pi}/\partial z\) is ambiguous. This ambiguity is because of the following opposing effects. An increase in \(\tilde{n}\) caused by an increase in \(z\)

\(^{27}\)Equation (42) is also obtained by totally differentiating (32).
positively affects $\tilde{\pi}$ along the Phillips curve ($-\psi_1(\partial\tilde{n}/\partial z) > 0$), whereas an increase in $z$ negatively affects it by shifting the Phillips curve downward ($\psi_2 < 0$). If the total effect on $\tilde{\pi}$ is negative: $\partial\tilde{\pi}/\partial z < 0$ in (44), then this decline in $\tilde{\pi}$ encourages households to save more and consume less: $d\tilde{c}/dz < 0$ in (43), which reduces employment. That is, in this case, the indirect effect in (42) is negative: $(\partial\tilde{n}/\partial\tilde{c}) \cdot (d\tilde{c}/dz) < 0$.

From (42), if this negative indirect effect dominates the positive direct effect, then a subsidy increase leads to a decrease in employment: $d\tilde{n}/dz < 0$. Otherwise, if the negative indirect effect is dominated by the positive direct effect or if the indirect effect as well as the direct effect is positive, it expands employment: $d\tilde{n}/dz > 0$.\textsuperscript{28} That is, the sign of $d\tilde{n}/dz$ is ambiguous.\textsuperscript{29} We summarize these results in the following proposition.

**Proposition 2.** In the steady state with both unemployment attributable to the efficiency wage and unemployment attributable to the aggregate demand deficiency, an increase in the employment subsidy $z$ may improve or worsen unemployment.

This result of the employment subsidy is in sharp contrast with that shown in Section 4. In the present section, a subsidy increase affects unemployment through the two channels, making the effect of the subsidy ambiguous.\textsuperscript{30} This is fundamentally because aggregate demand determines output

\textsuperscript{28}The indirect effect is positive in the case where $\partial\tilde{\pi}/\partial z > 0$.

\textsuperscript{29}To be precise, from the second equation of (36), (39), (42), (43), and (44), $d\tilde{n}/dz$ is given by

$$
\frac{d\tilde{n}}{dz} = \frac{-\psi_2}{f'(\bar{c})(\bar{c} - \psi_1\tilde{n} - e_2\tilde{n})} \left[ \frac{\beta + u'(\bar{c})}{(1 + \rho)(1 + \tilde{\pi})^2} + e_1\tilde{n}u''(\bar{c}) \left( 1 - \frac{1}{(1 + \rho)(1 + \tilde{\pi})} \right) \right],
$$

where the expression in the square brackets may be positive or negative.

\textsuperscript{30}Neumark and Grijalva (2013, 2014) empirically examine the effects of various types
and the price change rate is not pinned down by the money growth rate.

6 Conclusion

We develop a MIUF model where worker effort depends on the unemployment rate and the nominal wage change and show that the firm’s profit maximization subject to this effort function gives rise to a Phillips curve. We then analyze two steady states with and without an aggregate demand deficiency and find that the presence of the demand deficiency crucially influences the effects of an employment subsidy as well as those of fiscal and monetary expansions.

In the steady state without the demand deficiency, only unemployment attributable to the efficiency wage occurs. In this steady state, a monetary expansion and a generous employment subsidy reduce unemployment whereas a fiscal expansion has no effect on it. However, in the steady state where the demand deficiency arises and aggravates unemployment, the effects of these policies are quite different. The fiscal expansion reduces unemployment but the monetary expansion has no effect on it. The effect of the employment subsidy is ambiguous; namely, the subsidy may serve to aggravate unemployment by worsening the demand deficiency. Thus, when an economy is in long-run stagnation where aggregate demand is insufficient and unemployment is serious, such as the Japanese economy since the 1990s, creating employment through government purchases is more reliable and effective in reducing unemployment than promoting hiring through employment subsi-
dies.
References


Source: IMF World Economic Outlook Database (April 2015)

Figure 1: Japan’s output gap (as a percentage of potential GDP)
Figure 2: A Phillips curve and the effect of an increase in $z$