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Optimal Fiscal Policy in an Endogenous Growth Model with Public Capital as Common Property

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Abstract

This study discerns the optimal fiscal policy in a dynamic general equilibrium model featuring constant-returns-to-scale production technology and public capital as common property. Results indicate that free-access common property engenders distributive distortion of factor income in a competitive market. Alongside debt management to adjust the initial disparity between private and public capital, factor-income taxes are required for correcting distributive distortion of factor income. A consumption tax/subsidy is also needed to adjust initial conditions for satisfying the government's intertemporal balanced budget.

Keywords: Optimal fiscal policy; Public capital; Common property
JEL classification: H21; H54; H60

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1 Introduction

The productivity effect of public capital has been widely asserted during the three decades since Aschauer's (1989) pioneering empirical study. Many empirical studies support a positive effect of public capital on total output.[1] Arrow and Kurz (1970) presented an infuential theoretical study of public investment, including optimal fiscal policy, by incorporating public capital accumulation into a neoclassical growth model.

Barro (1990) developed an endogenous growth model with government goods as a production factor. Alternatively, Futagami et al. (1993) and Glomm and Ravikumar (1994) presented endogenous growth models with public capital as a production factor. Turnovsky (1997), Gómez (2004), and Tamai (2008) studied optimal fiscal policy in an endogenous growth model featuring public capital.[2]

These previous studies assume that the benefits of public capital return directly to households as labor income or dividends and ensuing studies seldom regard them as distortions in the distribution of factor income. However, it is natural that public capital assets such as streets, roads, and freeways become unpaid factors in the sense of Meade (1952). Moreover, public capital may include human capital by public education or public health expenditures. Therefore, freely accessible public capital will engender distortions in the distribution of factor income.

Feehan and Batina (2007) examined the efficiency of decentralized equilibrium with public inputs as free-access common property and derived the optimal factor-income tax using a static model.[3] Torregrosa (2012) provided a welfare analysis of long-run equilibrium using an overlapping generations model with free-access public capital. These studies incorporate free-access public capital into models and clalify the distortionary effects of factor-income distribution through the presence of unpaid factors.

This paper studies the optimal fiscal policy in a dynamic general equilibrium model with constant-returns-to-scale production function and free-access public capital. The presence of free-access public capital under a linear homogenous production function brings about a distributive distortion of factor income because the benefit of public capital is distributed between the rewards for marketable inputs. Then, each factor price exceeds the marginal productivity of its factor. These distortionary effects increase the possibility of the over-accumulation of private capital and the overprovision of other production factors.

We show that government must use instruments such as factor-income taxes to adjust factor prices to optimal levels, issue public bonds to adjust initial capital to optimal levels, and impose additional instruments to clear government's intertemporal budget constraints. Thus, this study adds insights about the policy instruments needed to attain the first-best equilibrium for optimal public invetment and taxation of factor

[1] Gramlich (1994) surveyed of early empirical studies in this literature. See also Pereira and Andraz (2013) and Bom and Ligthart (2014) for surveys encompassing more recent research.

[2] Agénor (2009) studied optimal fiscal policy, including optimal allocation rules for investment and maintenance expenditure. Misch et al. (2013) investigated the relationship between growth-maximization and welfare-maximization in the two endogenous growth models of Barro (1990) and Futagami et al. (1993).

[3] Matsumoto and Feehan (2010) also examined the analysis of unpaid factors (public inputs) in a tax comeptition model. They assumed that public input is rationed to firms in proportion to their capital inputs.

income.

This paper proceeds as follows. Section 2 explains our mathematical model and solves the social planner's optimization problem. Section 3 explains the behavior of economic agents in a decentralized economy and characterizes decentralized equilibrium. Section 4 examines the optimal fiscal policy that sets the decentralized growth path onto the optimal growth path. Section 5 extends the basic model by incorporating an elastic labor supply in the model. Finally, Section 6 concludes.

2 Social optimum

Consider a closed dynamic economy with a single good that can be consumed or used for capital accumulation at any time.[4] Time is continuous and indexed by t. A single good is produced using two capital inputs (private capital k_p and public capital k_g) and labor input l. The production function $F(k_p, k_g, l)$ is concave, constant-returns-to-scale, and twice continuously differentiable. We assume the labor endowment is constant over time and normalized to unity ($l = 1$). Defined the function f as $f(k_p, k_g) \equiv F(k_p, k_g, 1)$. It is also assumed to be $f(0, \cdot) = f(\cdot, 0) = 0$, $f(\infty, \cdot) = f(\cdot, \infty) = +\infty$, $f_i \equiv \partial f/\partial k_i = \partial F/\partial k_i \equiv F_i > 0$, $f_{ii} \equiv \partial^2 f/\partial k_i^2 = \partial^2 F/\partial k_i^2 \equiv F_{ii} < 0$, and $f_{ij} \equiv \partial^2 f/(\partial k_j \partial k_i) = \partial^2 F/(\partial k_j \partial k_i) \equiv F_{ij}$.

Private and public capital are formed by net investments in each. Specifically, we assume $\dot{k}_p = i_p - \delta_p k_p$ and $\dot{k}_g = i_g - \delta_g k_g$ where $\dot{k}_i = dk_i/dt$, i_i is the investment in k_i, and δ_i is the depreciation rate of k_i ($i = p, g$). Total capital stock in the economy is defined as

$$k \equiv k_p + k_g. \tag{1}$$

In the economy, total output y should be allocated to investment and consumption: $y = F(k_p, k_g, l) = c + i_p + i_g$. Using (1), the dynamic equations of capital accumulation and the resource constraint, the dynamic equation of total capital accumulation becomes

$$\dot{k} = f(k_p, k_g) - \delta_p k_p - \delta_g k_g - c. \tag{2}$$

We now consider the first-best allocation of total output between investment and consumption. The planner for this economy aims for

$$\max_{c, k_p, k_g} \int_0^\infty u(c) e^{-\rho t} dt,$$

subject to (1), (2), and $k(0)$. In the equation above, ρ stands for the rate of time preference and $u(c)$ denotes the instantaneous utility function, which is defined over instantaneous consumption and satisfies $u'(c) > 0$ and $u''(c) < 0$.

[4] Our model is based on Arrow and Kurz (1970, Ch 4). We allow differences between depreciation rates.

Optimality conditions are

$$u'(c) = p, \tag{3a}$$
$$p(f_p - \delta_p) = q, \tag{3b}$$
$$p(f_g - \delta_g) = q, \tag{3c}$$
$$\rho p - q = \dot{p}, \tag{3d}$$
$$\lim_{t\to\infty} e^{-\rho t} pk = 0. \tag{3e}$$

Equation (3a) leads to $c = c(p)$ where $c'(p) = dc/dp = 1/u''(c) < 0$. From equations (3b) and (3c), net returns on private and public capital are equalized to the common rate (e.g., Arrow and Kurz 1970; Turnovsky 1997). Equation (3e) is the transversality condition. Equations (1)-(3d) give the optimal consumption rate of interest as

$$f_p - \delta_p = f_g - \delta_g = \rho - \frac{d \log u'(c)}{dt} \equiv r_c^*. \tag{4}$$

Equation (4) leads to the Keynes-Ramsey rule in the social optimum. It also establishes a relationship between private and public capital, such that $k_g = k_g(k_p)$ or $k_p = k_p(k_g)$ where

$$\frac{dk_g}{dk_p} = \frac{f_{pp} - f_{pg}}{f_{gg} - f_{pg}}. \tag{5a}$$

Taking into account $k_p + k_g = k$, these equations provide $k_p = k_p(k)$ and $k_g = k_g(k)$ where

$$\frac{dk_p}{dk} = \frac{f_{gg} - f_{pg}}{f_{pp} + f_{gg} - 2f_{pg}}, \tag{5b}$$
$$\frac{dk_g}{dk} = \frac{f_{pp} - f_{pg}}{f_{pp} + f_{gg} - 2f_{pg}}. \tag{5c}$$

Following Allow and Kurz (1970), we impose the *dominant diagonal* assumption:

$$f_{pp} - f_{pg} < 0 \text{ and } f_{gg} - f_{gp} < 0.$$

For instance, $F_{pg} \geq 0$ is sufficient for satisfying the dominant diagonal assumption.[5] However, some empirical studies find that private and public capital are substitutes (e.g. Nadiri and Mamuneas 1994; Vijverberg and Vijverberg 2007). On the ground of empirical evidence, we do not exclude the case where $F_{pg} < 0$.

Using (5b), (5c), and the dominant diagonal assumption, we get

$$\frac{df_p}{dk} = f_{pp}\frac{dk_p}{dk} + f_{pg}\frac{dk_g}{dk} = \frac{f_{pp}f_{gg} - f_{pg}^2}{f_{pp} + f_{gg} - 2f_{pg}} < 0, \tag{6}$$

where the numerator of equation (6) is positive by the strict concavity of f. Equation (6) shows that an increase in total capital stock reduces the marginal productivity of private capital. By equation (4), equation (6) also implies $df_g/dk < 0$.

[5] For example, Lynde and Richmond (1992), Seitz (1994), and Morrison and Schwartz (1996) found that private and public capital are complements.

Equations (1)-(3d) are reduced to

$$\dot{p} = (\rho + \delta_p - f_p)p, \tag{7a}$$
$$\dot{k} = f(k_p, k_g) - \delta_p k_p - \delta_g k_g - c(p). \tag{7b}$$

Note that $k_p = k_p(k)$ and $k_g = k_g(k)$ hold. Hereafter, we use a superscripted $*$ to indicate the optimal values of endogenous variables that satisfy equations (1)-(3e) or, equivalently, equations (3e), (7a), and (7b). Regarding the existence, uniqueness, and stability of optimal stationary equilibrium, we obtain the following proposition:

Proposition 1. *There exists a unique optimal stationary equilibrium, and the optimal growth path to stationary equilibrium is uniquely determined if* $\lim_{k\to\infty} f_p < \rho + \delta_p < \lim_{k\to 0} f_p$.

Proof. See Appendix A. □

By the dominant diagonal assumption, the marginal productivity of private capital is decreasing in k. Therefore, the optimal consumption rate of interest is also a decreasing function with respect to k. The sufficient condition in Proposition 1 is to establish the single crossing between the optimal consumption rate of interest and the discount rate in the r_c-k plane; we impose a weak boundary assumption for the marginal productivity of private capital compared with the Inada condition.

The production function that satisfies the Inada condition–for instance, the Cobb-Douglas function is sufficient to establish the existence, uniqueness, and stability of stationary equilibrium. However, the Inada condition might be strong assumption when focusing on the specific production function such as the CES production function.

3 Decentralized economy

3.1 Economic agents and their behaviors

Households. The representative household provides labor and lends financial resources. It receives wages for labor and interest as a return on financial assets. Financial assets are composed of private capital k_p and a goverment bond b, which are perfect substitutes:

$$a \equiv k_p + b. \tag{8}$$

Note that the rate of return on the govermnet bond equals that on private capital. The post-tax budget constraint of the prepresentative household is

$$\dot{a} = (1 - \tau_r)ra + (1 - \tau_w)w - (1 + \tau_c)c, \tag{9}$$

where $\dot{a} \equiv da/dt$, r is the rate of return on financial assets, w is the wage rate, and τ_i is the tax rate on i ($i = r, w, c$).

The representative household allocates its iincome between private consumption and savings to maximize lifetime utility.[6] The optimization problem of the household

[6]We assume no difference between social and private objective functions.

is formulated as

$$\max_{c} \int_0^{\infty} u(c) e^{-\rho t} dt,$$

subject to (8), (9), $a(0)$, and $\{r(t), w(t), \tau_r(t), \tau_w(t), \tau_c(t) | t \in R_{++}\}$.

Solving the optimization problem of the representative household, we obtain the following conditions:

$$u'(c) = v\lambda, \tag{10a}$$

$$[\rho - (1 - \tau_r) r]\lambda = \dot{\lambda}, \tag{10b}$$

$$\lim_{t \to \infty} e^{-\rho t} \lambda a = 0, \tag{10c}$$

where $v \equiv 1 + \tau_c$ and λ are the post-tax price of consumption goods and the shadow price of private capital, respectively. Equations (10a) and (10b) lead to the private consumption rate of interest as

$$(1 - \tau_r) r - \frac{\dot{v}}{v} = \rho - \frac{d \log u'(c)}{dt} \equiv r_c. \tag{11}$$

Furthermore, equation (10c) is the transversality condition.

Firms. The presence of an unpaid factor brings about a windfall profit. In the competitive market, the rent, as long as it, exists continues new entries. Output should be completely distributed to labor and capital income. Consequently, factor prices will satisfy

$$r = F_p + x \frac{F_g k_g}{k_p} - \delta_p, \tag{12a}$$

$$w = F_l + (1 - x) \frac{F_g k_g}{l}, \tag{12b}$$

where $x \in [0, 1]$. We will show later that x might depend on k_p and k_g. The factor prices of private capital and labor are marked up by the presence of the unpaid factor; each factor price exceeds the marginal productivity of its factor.

Equations (12a) and (12b) are justified by Feehan and Batina (2007), who set $x \equiv r k_p / y$.[7] This setup is consistent with firms' cost-minimization behavior and factor-income distribution. Regarding the cost-minimization problem in which the objective is $(r + \delta_p) k_p + wl$ and the constraint is $y = F(k, g, l)$, the first-order condition is

$$\frac{r + \delta_p}{w} = \frac{F_p}{F_l}. \tag{13}$$

Using equation (13) and $y = (r + \delta_p) k_p + wl$, we obtain

$$r = \frac{F}{F - F_g k_g} F_p - \delta_p, \tag{14a}$$

$$w = \frac{F}{F - F_g k_g} F_l, \tag{14b}$$

[7] Alternatively, it can be justified by a rent-sharing model such as Blanchflower et al. (1996).

where $x = F_p k_p/(F - F_g k_g)$. Therefore, we have

$$\frac{\partial r}{\partial k_p} = -\frac{\left[(1-\eta_g)\theta_{pp} + \eta_p\eta_g - \theta_{pg}k_g\right] f_p}{(1-\eta_g)^2 k_p}.$$

Government. Government imposes taxes on the net return on private capital, labor income, private consumption. It allocates tax revenue and public borrowing to pay government bond interest and to invest public capital. Furthermore, the rate of return on governmnet bond equals that on private capital. Therefore, the government's budget constraint is given as

$$\dot{b} = (1-\tau_r)rb + \dot{k}_g + \delta_g k_g - \tau_r r k_p - \tau_w w - \tau_c c. \tag{15}$$

The government aims to steer the decentralized equilibrium path to the optimal growth path using policy instruments such as public investment, taxes, and public borrowing.

3.2 Dynamic properties of decentralized equilibrium

The decentralized dynamic general equilibrium is characterized by equations (8)-(10c), (14a), (14b), and (15). Different pricing of the production factor, financing of both public investment and interest payment on public debt and combinations of policy instruments engender different dynamic equilibria. To characterize the difference between social optimum and a decentralized economy and to clarify the effects of public capital as an unpaid factor, we focus on a balanced budget funded by a lump-sum tax and featuring a fixed stock of public capital. Specifically, we assume $b = \dot{b} = \dot{k}_g = 0$, $k_g = k_g^*$, $\tau_r = \tau_c = 0$, and $\tau_w = \delta_g k_g^*/w$.[8] Therefore, equations (8)-(10b), (14a), (14b), and (15) are reduced to

$$\dot{\lambda} = \left[\rho - r(k_p, k_g^*)\right]\lambda, \tag{16a}$$

$$\dot{k}_p = f(k_p, k_g^*) - \delta_p k_p - \delta_g k_g^* - c(\lambda), \tag{16b}$$

where $c'(\lambda) = 1/u''(c) < 0$.

Regarding the dynamic properties of decentralized equilibrium that satisfy (10c), (16a), and (16b), we establish the following proposition:

Proposition 2. *Suppose that the stock of public capital is fixed at k_g^*. There exists a unique stationary equilibrium, and the equilibrium growth path is determined as a unique stable trajectory to the stationary equilibrium.*

Proof. See Appendix B. □

This result resembles the result for Proposition 1. The net return on private capital decreases in private capital stock. Private capital deepening raises the shadow price of private capital. Therefore, the speed of capital accumulation is slowed by capital deepening.

[8] Sinc labor is inelastically supplied, government can use the tax on labor income as the lump-sum tax.

We now consider the effect of the unpaid factor on the long-run stock of private capital. In optimal stationary equilibrium, we have $f_p(k_p^*, k_g^*) - \delta_p = \rho$. On the other hand, we obtain $r(k_p^\dagger, k_g^*) = \rho$ in the decentralized stationary equilibrium. These equations and (12a) lead to

$$f_p^* - f_p^\dagger = x^\dagger \frac{f_g^\dagger k_g^*}{k_p^\dagger} > 0 \Leftrightarrow f_p(k_p^*, k_g^*) > f_p(k_p^\dagger, k_g^*),$$

where $f_p^\dagger \equiv f_p(k_p^\dagger, k_g^*)$. The concavity of f with respect to k_p, i.e., $f_{pp} < 0$, shows that $k_p^\dagger$ exceeds k_p^*. Thus, it shows $k^* < k^\dagger \equiv k_p^\dagger + k_g^*$. This result is summarized as follows:

Proposition 3. *For a fixed level of public capital stock that equals the socially optimum level, an over-accumulation of private capital occurs in the stationary decentralized equilibrium. Therefore, total capital stock is also over-accumulated through the presence of public capital as unpaid factor.*

Equivalently,

Corollary 1. *In the decentralized economy, the marginal productivity of private capital, which diverges from the rate of return on financial asset, is less than the social optimal rate of return on private capital, which is same as the optimal marginal productivity of private capital.*

The Keynes-Ramsey rule (11) implies that it is optimal for households to reduce consumption in the short-run if the rate of return on financial assets exceeds the rate of time preference. Momentary patience encourages investment in private capital and allows future consumption. When public capital is present as an unpaid factor, the rate of return on financial assets exceeds the marginal productivity of private capital for identical levels of k_p and k_g because the return on public capital as an unpaid factor is divided between return on capital and return on labor. Therefore, more private capital is accumulated in the decentralized economy than in the centrally planned economy.

4 Optimal fiscal policy

This section considers the optimal fiscal policy that sets the decentralized growth path on the optimal growth path. It is necessary that optimality conditions (10a) and (10b) coincide with the socially optimum conditions (3a)-(3d). Optimal fiscal policy also needs identical conditions of initial capital stock, and the government's budget constraints should be consistent with the No-Ponzi game (NPG) condition.

The first necessary condition corresponds to $r_c = r_c^*$. In other words, the consumption rates of interest in two different economies should be equalized at a common rate. Using (4) and (11), we obtain

$$(1 - \tau_r)r - \frac{\dot{v}}{v} = f_p - \delta_p = f_g - \delta_g. \tag{17}$$

The middle and right terms of (17) imply that the net rates of return on private and public capital should be equalized in the optimum. Government provides public capital linked to private capital accumulation. Specifically, public investment is ruled by $k_g = k_g(k_p)$, where (5a).

The left and middle terms of (17) determine the optimal tax rate on capital income and the dynamics of the consumption tax rate. To interpret this relation, we rewrite equation (17) as $(1-\tau_r)r - (f_p - \delta_p) = \dot{v}/v$. Rewriting shows that the difference between the post-tax net rate of return on financial assets and the net rate of return on private capital equal the growth rate of the post-tax price of consumption goods. The net rate of return on financial assets takes a different value from the net rate of return on private capital because there are differences in prices of consumption goods and disparities between factor prices and marginal productivity of input given the presence of the unpaid factor. Therefore, the optimal tax rate on capital income should be used to actualize a parity in rates of return.

However, the instruments discussed above cannot attain the socially optimum growth path. Initial stocks of private and public capital in the decentralized economy might diverge from the initial capital allocation under the social optimum. Regarding this point, the second necessary condition requires further instruments to fill the gap in initial capital allocation. In the decentralized economy, a representative household appropriates initial wealth as a financial resource for private capital as a material asset and a national bond. By adjusting public borrowing or lending, the government can set the initial private capital –which is ruled by $k_p^*(0) = k_p(k_g^*(0))$– to its optimal level.[9] Thus, the government's budget constraint be an NPG condition, and shows the initial condition of the national bond, which is tied to tax instruments.

These results are formally summarized as follows:

Proposition 4. *The government can steer the equilibrium growth path of the decentralized economy onto the optimal growth path if and only if*

$$-\frac{\dot{v}}{v} = [f_p - \delta_p]\,\tau_r + (1-\tau_r)\frac{f_g k_g}{k_p}x, \tag{18a}$$

$$f_p - \delta_p = f_g - \delta_g, \tag{18b}$$

$$b(0) = [v(0)-1]k(0) + k_g(0) + v(0)\Pi(0), \tag{18c}$$

$$\Pi(0) \equiv \int_0^\infty \left[(r - r_c)k_p + (1-z)w - \delta_g k_g\right] e^{-R_c(t)} dt, \tag{18d}$$

where $R_c(t) \equiv \int_0^t r_c(s)ds$ *and* $z \equiv (1-\tau_w)/v$.

Proof. Equation (18a) is derived from (17) with (12a). Equation (18b) is also derived from (17). Equation (18c) is obtained as follows: Let $\tilde{a} \equiv a/v$ and $\tilde{w} \equiv (1-\tau_w)w/v$. Using these notations and (9), we obtain

$$\dot{\tilde{a}} = \left[(1-\tau_r)r - \frac{\dot{v}}{v}\right]\tilde{a} + \tilde{w} - c.$$

[9] Gómez (2004) studied the optimal fiscal policy in an endogenous growth model with public capital by incorporating irreversible investment.

Multiplying both sides of this equation by $R_c(t)$ and integrating it with respect to t from 0 to ∞, we reach

$$\int_0^\infty e^{-R_c(t)}\dot{\tilde{a}}dt = \int_0^\infty e^{-R_c(t)} r_c \tilde{a} dt + \int_0^\infty e^{-R_c(t)} \tilde{w} dt - \int_0^\infty e^{-R_c(t)} c dt.$$

Applying the integral by part and $\lim_{t\to\infty} \tilde{a}e^{-R_c(t)} = 0$ into the equation above, we obtain

$$\int_0^\infty e^{-R_c(t)} c dt = \int_0^\infty e^{-R_c(t)} \tilde{w} dt + \tilde{a}(0). \tag{19a}$$

On the other hand, equations (9), (12a), (12b) and (15) lead to $\dot{k} = f(k_p, k_g) - \delta_p k_p - \delta_g k_g - c = rk_p + w - \delta_g k_g - c$, which is equivalent to (2). Multiplying both sides of this equation by $R_c(t) \equiv \int_0^t r_c(s)ds$ and integrating it with respect to t from 0 to ∞, we get

$$\begin{aligned}\int_0^\infty e^{-R_c(t)} \dot{k} dt = \int_0^\infty e^{-R_c(t)} r k_p dt + \int_0^\infty e^{-R_c(t)} w dt \\ - \delta_g \int_0^\infty e^{-R_c(t)} k_g dt - \int_0^\infty e^{-R_c(t)} c dt.\end{aligned}$$

Using the integral by part and $\lim_{t\to\infty} ke^{-R_c(t)}$, we obtain

$$\begin{aligned}\int_0^\infty e^{-R_c(t)} c dt = \int_0^\infty e^{-R_c(t)} (r - r_c) k_p dt \\ + \int_0^\infty e^{-R_c(t)} w dt - \delta_g \int_0^\infty e^{-R_c(t)} k_g dt + k(0).\end{aligned} \tag{19b}$$

Equations (19a) and (19b) provide

$$a(0) = v(0) \left[\int_0^\infty \{(r - r_c)k_p + (w - \tilde{w}) - \delta_g k_g\} e^{-R_c(t)} dt + k(0) \right].$$

This equation and (8) lead to

$$\begin{aligned}b(0) &= v(0) \left[\int_0^\infty \{(r - r_c)k_p + (1 - z)w - \delta_g k_g\} e^{-R_c(t)} dt + k(0) \right] - k_p(0) \\ &= [v(0) - 1]k(0) + k_g(0) + v(0)\Pi(0).\end{aligned}$$

□

We explained most of the economic meaning of Proposition 4 is explained previously. The remainder of this section presents further notes about Proposition 4 and provides examples illustrating specific rule for distribution of factor income and production functions. First, we consider the long-run optimal tax rate. In the long-run, the growth rate of the post-tax prices of consumption goods should be asymptotically zero ($\dot{v}/v = 0$) because $\dot{v}/v \neq 0$ means the divergence of the consumption tax rate. For a given initial condition, the tax on labor income and the consumption tax should be determined by equations (18c) and (18d). Therefore, the optimal tax rate on capital income is obtained as $\tau_r^* = [r - (f_p - \delta_p)]/r$. Taking into account (12a), it is optimal to impose a non-negative capital income tax $0 \le \tau_r^* < 1$:

Corollary 2. *The optimal tax rate on capital income is*

$$\begin{aligned} \tau_r^* &= \frac{(\rho + \delta_g) x^* k_g^*}{\rho k_p^* + (\rho + \delta_g) x^* k_g^*} \qquad (20) \\ &= \frac{(\rho + \delta_p)\eta_g}{\rho + \delta_p \eta_g} \ge \eta_g \text{ if (14a) and (14b) hold.} \end{aligned}$$

For the given parameters and capital stocks, equation (20) implies that the optimal tax rate on capital income is increasing in the distribution rate of rent allocated to private capital. Therefore, if $x^* = 0$, the optimal tax rate on capital income is 0 in existing studies on optimal fiscal policy that includes the second best tax policy (e.g., Arrow and Kurz 1970; Judd 1985, 1999; Chamley 1986). However, the presence of public capital as common property requires levying a charge for public capital on private capital. The distributive manner of the unpaid factor is quantitatively important for a positive optimal tax rate on capital income. Therefore, the optimal tax rate on capital income can be greater or less than the elasticity of the output with respect to public capital η_g.

Let $\delta_p = 0$; note that $\tau_r^* = \eta_g$ if $\delta_p = 0$. When production technology is given by the Cobb-Douglas function, the optimal tax rate on capital income depends only on the intensity of public capital and is independent of other parameters. However, when the elasticity of factor substitution is not equal to unity, the optimal tax rate depends on all parameters of the model.

5 Endogenous labor supply

5.1 Social optimum

In previous sections, we assumed that labor is inelastically supplied. Therefore, the labor income tax and the time-invariant consumption tax do not themselves engender distortion in economic allocatios. If the government can use either of these two taxes, the optimal growth path is feasible for the decentralized economy. However, as is well known, the labor income tax distorts the labor supply, and a time-invariant consumption tax affects the labor supply through consumption and labor choice. Thus, we examine an extension of our basic model by incorporating endogenous labor.

By writing the labor supply explicitly, the resource constraint is given as

$$\dot{k} = F(k_p, k_g, l) - \delta_p k_p - \delta_g k_g - c. \qquad (21)$$

The time endowment is normalized to unity, and it can be allocated to labor supply and leisure. The household benefits from leisure and consumption. Assume $U(c, 1-l)$ is strictly concave, homogenous of degree $1-\sigma$ (σ is a positive constant) and twice continuously differentiable.[10] The social planner's optimization problem is

$$\max_{c,l,k_p,k_g} \int_0^\infty U(c, 1-l)e^{-\rho t}dt,$$

subject to (1), (2), and $k(0)$.

Solving the social planner's optimization problem, we obtain (3b)-(3e) and

$$U_c = p, \tag{22a}$$

$$U_l = pF_l = p\left[f - f_p\hat{k}_p - f_g\hat{k}_g\right], \tag{22b}$$

where $\hat{k}_i \equiv k_i/l$ denotes the capital per labor ($i = p, g$). Note that $f = f(\hat{k}_p, \hat{k}_g)$, $f_p = \partial f(\hat{k}_p, \hat{k}_g)/\partial \hat{k}_p$ and $f_g = \partial f(\hat{k}_p, \hat{k}_g)/\partial \hat{k}_g$ here. Equations (3b)-(3d), (22a), and (22b) lead to

$$\frac{U_l}{U_c} = F_l = f - f_p\hat{k}_p - f_g\hat{k}_g, \tag{23a}$$

$$f_p - \delta_p = f_g - \delta_g = \rho - \frac{d\log U_c(c, 1-l)}{dt} \equiv r_c^*. \tag{23b}$$

Equation (23a) requires that the marginal rate of substitution (MRS) eqauls the relative price. Equation (23b) corresponds to equation (4).

From equations (1), (3b), (3c), (22a), (22b), and the definition of $\hat{k}_i \equiv k_i/l$, we obtain $c = c(p, k)$, $l = l(p, k)$, $\hat{k}_p = \hat{k}_p(p, k)$ and $\hat{k}_g = \hat{k}_g(p, k)$.[11] Thus, the dynamics of the centrally planned economy are obtained from

$$\dot{p} = [\rho + \delta_p - f_p(\hat{k}_p, \hat{k}_g)]p, \tag{24a}$$

$$\dot{k} = [f(\hat{k}_p, \hat{k}_g) - \delta_p\hat{k}_p - \delta_g\hat{k}_g]l(p, k) - c(p, k). \tag{24b}$$

Regarding the existence, uniqueness, and stability of stationary equilibrium, we establish the following proposition:

Proposition 5. *Suppose the instantaneous utility function is homogenous of degree* $1-\sigma$*. There exists a unique optimal stationary equilibrium and a unique stable trajectory to optimal equilibrium if* $\lim_{\hat{k}\to 0} f_p > \rho + \delta_p$ *and* $\lim_{\hat{k}\to +\infty} f_p < \rho + \delta_p$ *where* $\hat{k} \equiv k/l$.

Proof. Appendix C. □

The interpretation of this proposition resembles that of Proposition 1. Note that we impose homogeneity on the instantaneous utility function. Without this assumption, we will impose another assumption about the properties of utility function to prove the stability of the stationary equilibrium.

[10] Let be $U_i > 0$ and $U_{ii} < 0$ ($i = c, l$). Under that assumption, we obtain $U_{cc}U_{ll} - U_{cl}^2 > 0$.

[11] Appendix C presents properties of these functions.

5.2 Decentralized economy and optimal fiscal policy[Kokokara]

Several equations in Section 3 should be replaced with new equations. First, for endogenous labor supply, the budget constraint of household is given as

$$\dot{a} = (1-\tau_r)ra + (1-\tau_w)wl - (1+\tau_c)c. \tag{25}$$

The optimization problem of households in Section 3 is rewritten as

$$\max_{c,l} \int_0^\infty U(c, 1-l)e^{-\rho t}dt,$$

subject to (25), $a(0)$, and $\{r(t), w(t), \tau_r(t), \tau_w(t), \tau_c(t) | t \in R_{++}\}$.

The optimality conditions for the household's decision-making are equations (10b), (10c) and the following equations:

$$U_c = v\lambda, \tag{26a}$$

$$U_l = (1-\tau_w)w\lambda. \tag{26b}$$

Equations (10b), (26a) and (26b) are simplified to

$$\frac{U_l}{U_c} = \frac{(1-\tau_w)w}{v} = zw, \tag{27a}$$

$$(1-\tau_r)r - \frac{\dot{v}}{v} = \rho - \frac{d \log U_c(c, 1-l)}{dt} \equiv r_c. \tag{27b}$$

Equation (27a) asserts that the marginal rate of substitution must equal the post-tax relative price. The price of leisure relative to consumption will exceed that in the case of marginal pricing of inputs for the given k_p and k_g. Therefore, leisure time will be brief and the amount of consumption will be large compared with the marginal pricing of inputs. Furthermore, equation (27b) corresponds to equation (11).

The dynamics of a decentralized economy are obtained from equations (10b) and (25)-(26b). The choice of the financial source of fiscal policy might affect the dynamics of the economy through public capital accumulation. However, sustainable fiscal policy can attain the socially optimum equilibrium that duplicates the uniquely stable growth path toward stationary optimal equilibrium (Proposition 5). Therefore, it is sufficient for analyzing the effect of the unpaid factor to characterize the long-run property of decentralized equilibrium with only lump-sum tax financing.

Assume the government sets k_g to satisfy $\hat{k}_g = \hat{k}_g^*$ in the long-run. A comparison of equations (23b) and (27b) shows that the social optimum rate of return on private capital exceeds the rate of return on private capital in the decentralized economy for a given $\hat{k}_g^*$ in the same way as that derived in Proposition 3. The disparity in the rate of return leads to inefficient intertemporal allocation between consumption and saving; there is an over-accumulation of private capital per labor unit. The disparity of relative price brings about inefficient allocation between consumption and leisure. In particular, it will bring about more consumption demand and more labor supply in the decentralized economy compared with the social optimum.

Therefore, the remainder of this section considers the optimal fiscal policy under endogenous labor supply to overcome this type of inefficiency. To attain socially optimum equilibrium, it is necessary to solve the disparity in the initial allocation between private and public capital, the disparity in the rates of return on private and public capital, the disparity in the consumption rate of interest, the disparities in relative prices of goods and leisure. As shown in Section 4, the government must impose (18a)-(18c) to solve the first three disparities. Note that equations (18a)-(18c) depend on capital stock per labor unit. Therefore, some notations accompanying these equations are modified later.

The disparity in relative prices must be solved using the tax on labor income and the consumption tax. Equations (23a) and (27a) yield

$$\frac{U_l}{U_c} = F_l = z\left[F_l + (1-x)\frac{F_g k_g}{l}\right] \Leftrightarrow \frac{1-z}{1-x} = \frac{F_g k_g}{F_l l} = \frac{\eta_g}{\eta_l}. \tag{28}$$

Recall that z is the index related to the taxation of labor income and consumption. Equation (28) implies that the ratio of tax index to the distribution rate of rent allocated to labor should equal the ratio of elasticity of output with respect to public capital to the elasticity of output with respect to labor. The foregoing discussion is summarized as the following proposition:

Proposition 6. *With endogenous labor supply, the government can steer the equilibrium growth path of the decentralized economy onto the optimal growth path if and only if* (18a)-(18c),

$$z = \frac{f - f_p\hat{k}_p - f_g\hat{k}_g}{f - f_p\hat{k}_p - x f_g\hat{k}_g} = \frac{\eta_l}{\eta_l + (1-x)\eta_g}, \tag{29a}$$

$$\Pi(0) \equiv \int_0^\infty \left[(r - r_c)k_p + (1-z)wl - \delta_g k_g\right] e^{-R_c(t)} dt. \tag{29b}$$

Proof. Equations (18a) and (18b) hold by replacing variables per capita with variables per labor. Thus, the term with the wage rate is replaced by the term with the wage rate multiplied by labor unit in the present value of tax revenue (i.e., equation (29b)). Solving equation (28) with respect to z, we obtain equation (29a). □

From Proposition 7, we immediately obtain the optimal tax rate on capital income, labor income, and consumption.

Corollary 3. *The optimal tax rates on private capital income, labor income and consumption are*

$$\tau_r^* = \frac{(\rho+\delta_g)x^*\hat{k}_g^*}{\rho\hat{k}_p^* + (\rho+\delta_g)x^*\hat{k}_g^*} \in [0,1), \tag{30a}$$

$$\begin{aligned}\tau_w^* &= \frac{(1-x^*)\eta_g - \eta_l\tau_c^*}{\eta_l + (1-x^*)\eta_g} \\ &= \eta_g - (1-\eta_g)\tau_c^* \textit{ if } \text{(14a)} \textit{ and } \text{(14b)} \textit{ hold}.\end{aligned} \tag{30b}$$

Equation (30a) is explained using the same interpretation as that used in equation (20). Equation (30b) shows that the optimal tax rate on labor income should be negatively associated with the optimal tax rate on consumption for the given x, η_g and η_l. A tax on labor income reduces the price of leisure relative to consumption and raises the price of consumption relative to leisure. Therefore, the government must set lower (higher) tax rates on labor income with regard to higher (lower) tax rate on consumption.

For example, $\tau_w^* = -\tau_c^*$ holds when $x^* = 1$ (Tamai 2008). In other words, the optimal tax rate on labor income must be negative (positive) if the tax rate on consumption is positive (negative). If $0 < x^* < 1$, the reward to the unpaid factor is distributed as not only an add-on reward of private capital but also an add-on reward to labor. Consequently, the optimal tax rate on labor income will be positive if the tax rate on consumption is positive.

6 Conclusion

This study developed a dynamic general equilibrium model with free-access public capital and investigated socially optimal fiscal policy. It has shown that the optimal tax rates on capital and labor income are generally positive in the Piguovian sense. A distributive distortion of factor income exists because free-access public capital brings about unpaid rewards by a marginal principle under a constant-returns-to-scale production function. Therefore, the consumption tax/subsidy is important for attaining the first-best equilibrium as the instrument to adjust the intertemporal balanced budget. This study has shown that taxes on factor income, consumption, and public debt in conjunction with public investment are required to attain socially optimal equilibrium in a decentralized economy with public capital as common property.

Appendix

A. Proof of Proposition 1

Using the definition of stationary equilibrium $\dot{p} = \dot{k} = 0$ and equation (7a), we obtain $f_p - \delta_p = f_g - \delta_g = \rho$. If $\lim_{k\to 0} f_p > \rho + \delta_p$ and $\lim_{k\to +\infty} f_p < \rho + \delta_p$, by the continuity of f_p, there exists at least one value that satisfies $f_p = \rho + \delta_p$. Note that f_p is a monotone decreasing function with respect to k under a dominant diagonal assumption. Therefore, the solution of $f_p = \rho + \delta_p$ is uniquely determined. Let k^* be socially optimal stock of private capital. Equation (1) with $k^*(t) = k^*$ provides $k(t)^*_p = k^*_p$ and $k^*_g(t) = k^*_g$. Using these values and equation (7b), we obtain $c(p^*) = f(k^*_p, k^*_g) - \delta_p k^*_p - \delta_g k^*_g$.

We now consider the stability of the stationary equilibrium. Using equations (7b) and (7a), a linearized dynamic system around the stationary equilibrium is given as

$$\begin{pmatrix} \dot{p}(t) \\ \dot{k}(t) \end{pmatrix} = \begin{pmatrix} J^*_{11} & J^*_{12} \\ J^*_{21} & J^*_{22} \end{pmatrix} \begin{pmatrix} p(t) - p^* \\ k(t) - k^* \end{pmatrix},$$

where $J^*_{11} = 0$, $J^*_{21} = -c'(p^*) > 0$,

$$\begin{aligned} J^*_{12} &= -\left[f^*_{pp} \frac{dk_p}{dk}\bigg|_{\dot{p}=\dot{k}=0} + f^*_{pg} \frac{dk_g}{dk}\bigg|_{\dot{p}=\dot{k}=0} \right] p^* \\ &= -\frac{f^*_{pp}(f^*_{gg} - f^*_{pg}) + f^*_{pg}(f^*_{pp} - f^*_{pg})}{f^*_{pp} + f^*_{gg} - 2f^*_{pg}} p^* \\ &= -\frac{f^*_{pp} f^*_{gg} - (f^*_{pg})^2}{f^*_{pp} + f^*_{gg} - 2f^*_{pg}} p^* > 0, \\ J^*_{22} &= (f^*_p - \delta_p)\ \frac{dk_p}{dk}\bigg|_{\dot{p}=\dot{k}=0} + (f^*_g - \delta_g)\ \frac{dk_g}{dk}\bigg|_{\dot{p}=\dot{k}=0} = \rho. \end{aligned}$$

Consequently, we have $\mathrm{trace} J = J^*_{11} + J^*_{22} = J^*_{22} = \rho > 0$ and $\det J = -J^*_{12} J^*_{21} < 0$. These two values show that the coefficient matrix has one negative and one positive eigenvalue; the stationary point is a saddle point. Since there are one control and one state variable, the saddle-point stability provides a unique stable trajectory to the stationary equilibrium.

B. Proof of Proposition 2

According to the definition of stationary equilibrium, we have $\dot{\lambda} = \dot{k}_p = 0$. Equation (16a) with $\dot{\lambda} = 0$ leads to $r(k_p, k^*_g) = \rho$. Let be (14a). Then, $\partial r / \partial k_p < 0$ holds. If $\lim_{k_p\to\infty} r < \rho < \lim_{k_p \to 0} r$, there exists $k^\dagger_p$ such as $r(k^\dagger_p, k^*_g) = \rho$ by the continuity of r. Thus, the total capital stock is given as $k^\dagger = k^\dagger_p + k^*_g$. These results and (19b) with $\dot{k} = 0$ lead to $c(p^\dagger) = f(k^\dagger_p, k^*_g) - \delta_p k^\dagger_p - \delta_g k^*_g$.

The linearized dynamic system around the stationary equilibrium is

$$\begin{pmatrix} \dot{\lambda}(t) \\ \dot{k}_p(t) \end{pmatrix} = \begin{pmatrix} J^\dagger_{11} & J^\dagger_{12} \\ J^\dagger_{21} & J^\dagger_{22} \end{pmatrix} \begin{pmatrix} \lambda(t) - \lambda^\dagger \\ k_p(t) - k^\dagger_p \end{pmatrix},$$

where

$$J_{11}^{\dagger} = 0,\ J_{12}^{\dagger} = -\lambda^{\dagger}\frac{\partial r}{\partial k_p},\ J_{21}^{\dagger} = -c'(\lambda^{\dagger}) > 0,$$
$$J_{22}^{\dagger} = f_p^{\dagger} - \delta_p = \frac{(\rho+\delta_p)(f - f_g k_g^*)}{f} - \delta_p = (1-\eta_g)\rho - \eta_g\delta_p.$$

Therefore, we have

$$\mathrm{trace}J = J_{11}^{\dagger} + J_{22}^{\dagger} = J_{22}^{\dagger} = (1-\eta_g)\rho - \eta_g\delta_p,$$
$$\det J = -J_{12}^{\dagger}J_{21}^{\dagger} = -c'(\lambda^{\dagger})\lambda^{\dagger}\frac{\partial r}{\partial k_p}.$$

If $\partial r/\partial k_p < 0$ (i.e. $[(\epsilon_p - 1)\eta_g + \theta_{pg}\eta_p]x < \eta_p\theta_{pp}$), $\mathrm{det}J < 0$ holds. Therefore, the stationary equilibrium is a saddle point, and only one stable trajectory exists.

C. Proof of Proposition 5

The total differentiation of equations (22a), (22b), (23b), and $k \equiv [\hat{k}_p + \hat{k}_g]l$ gives

$$A\begin{pmatrix} dc \\ dl \\ d\hat{k}_p \\ d\hat{k}_g \end{pmatrix} = \begin{pmatrix} dp \\ \omega dp \\ dk \\ 0 \end{pmatrix},$$

where $\omega \equiv f - f_p\hat{k}_p - f_g\hat{k}_g$ and

$$A \equiv \begin{pmatrix} U_{cc} & -U_{cl} & 0 & 0 \\ U_{cl} & -U_{ll} & p(f_{pp}\hat{k}_p + f_{pg}\hat{k}_g) & p(f_{gg}\hat{k}_g + f_{pg}\hat{k}_p) \\ 0 & \hat{k}_p + \hat{k}_g & l & l \\ 0 & 0 & f_{pp} - f_{pg} & -f_{gg} + f_{pg} \end{pmatrix}$$

Applying Cramer's rule into the above system, we obtain

$$\begin{aligned}
\frac{\partial c}{\partial p} &= \frac{(U_{ll}-\omega U_{cl})(f_{pp}+f_{gg}-2f_{pg})l+(\hat{k}_p+\hat{k}_g)^2(f_{pp}f_{gg}-f_{pg}^2)p}{\det A} < 0,\\
\frac{\partial c}{\partial k} &= -\frac{(\hat{k}_p+\hat{k}_g)(f_{pp}f_{gg}-f_{pg}^2)U_{cl}p}{\det A} > 0,\\
\frac{\partial l}{\partial p} &= -\frac{(\omega U_{cc}-U_{cl})(f_{pp}+f_{gg}-2f_{pg})l}{\det A} > 0,\\
\frac{\partial l}{\partial k} &= \frac{(\hat{k}_p+\hat{k}_g)(f_{pp}f_{gg}-f_{pg}^2)U_{cc}p}{\det A} > 0,\\
\frac{\partial \hat{k}_p}{\partial p} &= \frac{(\omega U_{cc}-U_{cl})(\hat{k}_p+\hat{k}_g)(f_{gg}-f_{pg})}{\det A} < 0,\\
\frac{\partial \hat{k}_p}{\partial k} &= \frac{(U_{cc}U_{ll}-U_{cl}^2)(f_{gg}-f_{pg})}{\det A} > 0,\\
\frac{\partial \hat{k}_g}{\partial p} &= \frac{(\omega U_{cc}-U_{cl})(\hat{k}_p+\hat{k}_g)(f_{pp}-f_{pg})}{\det A} < 0,\\
\frac{\partial \hat{k}_g}{\partial k} &= \frac{(U_{cc}U_{ll}-U_{cl}^2)(f_{pp}-f_{pg})}{\det A} > 0,
\end{aligned}$$

where

$$\det A = (U_{cc}U_{ll}-U_{cl}^2)(f_{pp}+f_{gg}-2f_{pg})l+(\hat{k}_p+\hat{k}_g)^2(f_{pp}f_{gg}-f_{pg}^2)U_{cc}p < 0.$$

In a stationary equilibrium, $\dot{p}=\dot{k}=0$. We obtain $f_p(\hat{k}_p,\hat{k}_g)=\rho+\delta_p$. In the same manner as Proposition 1, we have $\hat{k}_p+\hat{k}_g=\hat{k}$, where $\hat{k}\equiv k/l$, and $\hat{k}_i=\hat{k}_i(\hat{k})$ $(i=p,g)$, where $\hat{k}_i'>0$ under the dominant diagonal assumption (see (5b) and (5c)). Furthermore, $df_p/d\hat{k}<0$ holds (see (6)). If $\lim_{\hat{k}\to 0} f_p > \rho+\delta_p$ and $\lim_{\hat{k}\to +\infty} f_p < \rho+\delta_p$, there exists a unique value of $\hat{k}$ that satisfies $f_p(\hat{k}_p(\hat{k}),\hat{k}_g(\hat{k}))=\rho+\delta_p$ by the continuity of f_p. Using $\hat{k}_i=\hat{k}_i(\hat{k})$, $\hat{k}_i$ is also uniquely determined. Finally, $\hat{k}_i^*=\hat{k}_i(p,k)$ and $\dot{k}=0$ give the stationary values of p and k.

Linearizing the dynamic system around the stationary equilibrium, we obtain the linearized system similar to that of Appendix A, where

$$\begin{aligned}
J_{11}^* &= -\left[f_{pp}^*\frac{\partial \hat{k}_p}{\partial p}+f_{pg}^*\frac{\partial \hat{k}_g}{\partial p}\right]p^* < 0,\ J_{12}^* = -\left[f_{pp}^*\frac{\partial \hat{k}_p}{\partial k}+f_{pg}^*\frac{\partial \hat{k}_g}{\partial k}\right]p^* > 0,\\
J_{21}^* &= \left[(f_p^*-\delta_p)\frac{\partial \hat{k}_p}{\partial p}+(f_g^*-\delta_g)\frac{\partial \hat{k}_g}{\partial p}\right]l^* + \left[f^*-\delta_p\hat{k}_p^*-\delta_g\hat{k}_g^*\right]\frac{\partial l}{\partial p}-\frac{\partial c}{\partial p},\\
J_{22}^* &= \left[(f_p^*-\delta_p)\frac{\partial \hat{k}_p}{\partial k}+(f_g^*-\delta_g)\frac{\partial \hat{k}_g}{\partial k}\right]l^* + \left[f^*-\delta_p\hat{k}_p^*-\delta_g\hat{k}_g^*\right]\frac{\partial l}{\partial k}-\frac{\partial c}{\partial k}.
\end{aligned}$$

The determinant of the coefficient matrix is

$$\det J = (f_{pp}^* - f_{pg}^*)\left[\frac{\partial \hat{k}_p}{\partial k}\frac{\partial \hat{k}_g}{\partial p} - \frac{\partial \hat{k}_p}{\partial p}\frac{\partial \hat{k}_g}{\partial k}\right]\rho l^*$$
$$- \left[f_{pp}^*\frac{\partial \hat{k}_p}{\partial p} + f_{pg}^*\frac{\partial \hat{k}_g}{\partial p}\right]\left[\frac{1}{l^*}\frac{\partial l}{\partial k} - \frac{1}{c^*}\frac{\partial c}{\partial k}\right]c^*$$
$$+ \left[f_{pp}^*\frac{\partial \hat{k}_p}{\partial k} + f_{pg}^*\frac{\partial \hat{k}_g}{\partial k}\right]\left[\frac{1}{l^*}\frac{\partial l}{\partial p} - \frac{1}{c^*}\frac{\partial c}{\partial p}\right]c^*.$$

Note that the following equations hold:

$$\frac{\partial \hat{k}_p}{\partial k}\frac{\partial \hat{k}_g}{\partial p} - \frac{\partial \hat{k}_p}{\partial p}\frac{\partial \hat{k}_g}{\partial k} = 0,$$
$$f_{pp}\frac{\partial \hat{k}_p}{\partial p} + f_{pg}\frac{\partial \hat{k}_g}{\partial p} = \frac{(\omega U_{cc} - U_{cl})(\hat{k}_p + \hat{k}_g)(f_{pp}f_{gg} - f_{pg}^2)}{\det A} > 0,$$
$$f_{pp}\frac{\partial \hat{k}_p}{\partial k} + f_{pg}\frac{\partial \hat{k}_g}{\partial k} = \frac{(U_{cc}U_{ll} - U_{cl}^2)(f_{pp}f_{gg} - f_{pg}^2)}{\det A} < 0,$$
$$\frac{1}{l^*}\frac{\partial l}{\partial k} - \frac{1}{c^*}\frac{\partial c}{\partial k} = -\frac{(\hat{k}_p + \hat{k}_g)(f_{pp}f_{gg} - f_{pg}^2)\sigma U_c p}{cl \det A} > 0,$$
$$\frac{1}{l^*}\frac{\partial l}{\partial p} - \frac{1}{c^*}\frac{\partial c}{\partial p} = \frac{(f_{pp} + f_{gg} - 2f_{pg})(\omega U_c + U_l)\sigma + (\hat{k}_p + \hat{k}_g)^2(f_{pp}f_{gg} - f_{pg}^2)p}{c \det A} > 0,$$

$-\sigma U_c = U_{cc}c + U_{cl}l$ and $-\sigma U_l = U_{ll}l + U_{cl}c$. Finally, we have $\det J < 0$. Therefore, there exists a unique stable trajectory to the stationary equilibrium in the manner indicated in Appendix A.

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