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Abstract

Using a dynamic efficiency wage model where a Phillips curve relationship arises because worker morale depends on the unemployment rate and the change in nominal wages, we analyze both structural and Keynesian unemployment and the effects of an employment subsidy on the two types of unemployment. We show that the presence of an aggregate demand deficiency crucially influences the impact of the employment subsidy.

Keywords: Aggregate demand, Efficiency wage, Employment subsidy, Phillips curve, Unemployment

JEL Classification Codes: E12, E24

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1 Introduction

In a typical early contribution to efficiency wage theory, Solow (1979) simply considers that worker morale (labor productivity) is an increasing function of wages. Since this study, however, many researchers have dealt with a range of factors that influence worker morale. In other words, they have assumed various types of effort functions. For example, Agell and Lundborg (1992) assume that an increase in economy-wide unemployment causes workers to provide greater effort. Several other studies regard past wages as a factor that also influences worker morale. Collard and de la Croix (2000) and Danthine and Kurmann (2004) develop dynamic general equilibrium models where worker morale depends on current and past real wages and the level of employment.[1] Akerlof et al. (2000) and Campbell (2008) propose models where a rise in the unemployment rate and a rise in current wages against some reference level, including previous wages, encourage workers to provide greater effort.[2] Shafir et al. (1997) consider that because of money illusion, not only the level of current real wages but also the level of current nominal wages against previous nominal wages influences worker morale.

Following these studies, we assume that an increase in the unemployment rate and an increase in current nominal wages against previous nominal wages boost worker morale, namely, a worker's effort is an increasing function of the unemployment rate and of current nominal wages over previous nominal wages. We then introduce this effort function into a money-in-the-utility-

[1] See Danthine and Donaldson (1990), de la Croix et al. (2009), and Vaona (2013a, 2014) for similar dynamic general equilibrium models.

[2] See also Campbell (2010).

function (MIUF) model. The idea that an increase in the unemployment rate boosts worker morale is not only theoretically adopted by many studies, including those cited above, but is also empirically supported (e.g., Blinder and Choi, 1990; Agell and Lundborg, 1995, 2003). At the same time, following Shafir et al. (1997), we consider that money illusion is the reason why an increase in current nominal wages against previous nominal wages induces workers to provide greater effort. In this setting, workers use previous nominal wages as a reference to judge whether their employers treat them fairly. This setting is also supported by empirical studies. For instance, Kahneman et al. (1986) and Blinder and Choi (1990) find that money illusion affects people's judgment of fairness. Similarly, Shafir et al. (1997) conclude that money illusion influences it, and consequently, worker morale. Bewley (1999) and Kawaguchi and Ohtake (2007) find that a cut in nominal compensation harms worker morale. The neuroscience literature also supports the presence of money illusion (Weber et al., 2009). Furthermore, the effects of money illusion may be persistent. Fehr and Tyran (2007) show that the effects of money illusion on equilibrium selection are long-lasting, and they (2007, p. 263) state: "Thus, the argument that the impact of money illusion on aggregate outcomes will eventually vanish through learning, can be seriously misleading." Recently, money illusion is studied in the macroeconomic context. Vaona (2013b) analyzes the effects of money illusion on a long-run Phillips curve in a New Keynesian model with staggered wages, while Miao and Xie (2013) examine its effects on long-run economic growth in an endogenous growth model.

In the present model, as in Akerlof et al. (2000) and Campbell (2008), the firm's profit maximization, subject to the effort function that includes the current and past wages and the unemployment rate, gives rise to a Phillips curve.[3] The nominal wage stickiness represented by this Phillips curve leads to persistent unemployment in a steady state. Unemployment in this steady state is not Keynesian despite the nominal wage stickiness, but it is structural unemployment generated by the efficiency wage setting. We examine the effect of an employment subsidy on this structural unemployment.

Employment subsidies, which are one kind of active labor market policy, are expected to serve as a way for reducing unemployment. In fact, there are studies that find positive effects of wage and hiring subsidies on employment (e.g., Jaenichen and Stephan, 2011). Meanwhile, the effectiveness of such subsidies is called into question because of deadweight, substitution, and displacement effects (Layard et al., 2005, pp. 476–478). As mentioned in Boockmann et al. (2012, pp. 737–738), such subsidies may not be macroeconomically effective even when they succeed in increasing the employment of some targeted group. Martin and Grubb (2001, p. 31) state: "At the same time, most evaluations which focus on firm behaviour show that subsidies to private-sector employment have both large dead-weight and substitution effects. As a result, most such schemes yield small net employment gains, particularly in the short term when aggregate demand and vacancies are fixed."[4]

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[3]See also Campbell (2010) and Vaona (2013a, 2014) for Phillips curves in efficiency wage models.

[4]In addition, Martin and Grubb (2001, p. 31) state: "For instance, evaluations of wage subsidies in Australia, Belgium, Ireland and the Netherlands have suggested combined dead-weight and substitution effects amounting to around 90 per cent, implying that for every 100 jobs subsidised by these schemes only ten were net gains in employment."

Therefore, we also examine the effect of the employment subsidy in the macroeconomic framework where an aggregate demand deficiency creates involuntary unemployment. Following Ono (1994, 2001) and Ono and Ishida (2014), we present a steady state with this Keynesian property. In this steady state, Keynesian unemployment in addition to the abovementioned structural unemployment arises.[5] We show that the effects of the employment subsidy depend crucially on whether the aggregate demand deficiency is present.

The remainder of this paper is organized as follows. Section 2 details the structure of the model. Section 3 derives the Phillips curve. Section 4 analyzes the steady state with only structural unemployment, and Section 5 investigates the steady state with both structural and Keynesian unemployment. Section 6 concludes.

2 The Model

Following Collard and de la Croix (2000), Danthine and Kurmann (2004), de la Croix et al. (2009), and Vaona (2013a, 2014), we construct a dynamic general equilibrium model. In particular, as in de la Croix et al. (2009) and Vaona (2013a, 2014), we introduce the idea of a fair wage into a MIUF model. However, there is a key difference between the present model and their models.[6] In the present model, worker morale hinges not upon real wages but upon nominal wages.[7] Concretely, as in Shafir et al. (1997), it simply depends on the ratio of current nominal wages to previous nominal

[5] Ono (1994, 2001) and Ono and Ishida (2014) do not consider structural unemployment.

[6] They do not analyze employment subsidies but mainly investigate the business cycle implications of fair wages and the effects of monetary shocks.

[7] Collard and de la Croix (2000) suggest an extension where nominal wages affect worker morale.

wages, which gives rise to a Phillips curve.[8]

2.1 The Household Sector

There is a continuum of identical households, the size of which is unity. Each household consists of a continuum of identical workers, the size of which is also unity. Therefore, the aggregate population size equals unity.

The lifetime utility of a typical household is

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}\left[u(c_t)+v(m_t)-n_t\chi(e_t)\right],$$

where ρ (> 0) is the subjective discount rate, $u(c_t)$ is the utility of consumption c_t, $v(m_t)$ is the utility of real money holdings m_t, n_t is the number or proportion of employed workers, and $\chi(e_t)$ is the disutility of effort e_t that an employed worker provides.[9] As usual, we assume that

$$\begin{aligned} &u'(c_t)>0,\quad u''(c_t)<0,\quad u'(0)=\infty,\quad u'(\infty)=0;\\ &v'(m_t)>0,\quad v''(m_t)<0,\quad v'(0)=\infty,\quad v'(\infty)=0. \end{aligned} \tag{1}$$

Following Akerlof (1982), Collard and de la Croix (2000), Danthine and Kurmann (2004), Campbell (2006), de la Croix et al. (2009), and Vaona (2013a, 2014), we assume that the disutility of effort is given by a quadratic function:

$$\chi(e_t)=(e_t-\bar{e}_t)^2, \tag{2}$$

[8]In the setting that individuals use the price at time $t+i$ to assess the real value of the nominal wage at time $t+i-1$, which is unlike the present setting, Vaona (2013a) also derives a Phillips curve relationship between inflation and unemployment.

[9]We assume that the household determines consumption c_t and money holdings m_t and distributes them equally among the workers belonging to the household. Moreover, employed workers, whose size is n_t, provide the same effort and derive the same disutility from effort, because identical firms pay the same wage. Therefore, we can analyze unemployment in a representative agent framework without considering the awkward problem that the workers are heterogeneous ex post, i.e., employed or unemployed (see in detail Danthine and Kurmann (2004), Vaona (2013a), and references cited therein).

where $\bar{e}_t$ is the norm of effort. However, the norm $\bar{e}_t$ depends not on real wages but on nominal wages, and it is given by

$$\bar{e}_t = e\left(W_t/W_{t-1}^s, 1 - n_t^a\right),$$

where W_t is the nominal wage received by a worker in period t, W_{t-1}^s is the social average of nominal wages in period $t-1$, and n_t^a is the aggregate amount of employment, all of which the household takes as given. It satisfies

$$\frac{\partial \bar{e}_t}{\partial (W_t/W_{t-1}^s)} > 0, \quad \frac{\partial^2 \bar{e}_t}{\partial (W_t/W_{t-1}^s)^2} < 0; \quad \frac{\partial \bar{e}_t}{\partial (1 - n_t^a)} > 0. \tag{3}$$

Note that $1 - n_t^a$ denotes the economy-wide unemployment rate because the size of the aggregate population is unity.

The household faces the following budget constraint:

$$\frac{M_{t+1} - M_t}{P_t} = w_t n_t - c_t - \tau_t,$$

where M_t is nominal money holdings, P_t is the commodity price, w_t ($\equiv W_t/P_t$) is the real wage, and τ_t is a lump-sum tax. Although all workers inelastically supply their one-unit labor endowment, unemployment arises. As a consequence, the number of employed workers is n_t (≤ 1) and the labor income of the household is $w_t n_t$.

The household maximizes its lifetime utility subject to the budget constraint. Taking (2) and $m_t \equiv M_t/P_t$ into account, we obtain the first-order optimality conditions with respect to c_t, m_{t+1}, and e_t:

$$u'(c_t) = \lambda_t, \tag{4}$$

$$\frac{v'(m_{t+1}) + \lambda_{t+1}}{1 + \rho} = \lambda_t(1 + \pi_{t+1}), \tag{5}$$

$$e_t = \bar{e}_t = e\left(W_t/W^s_{t-1}, 1 - n^a_t\right), \tag{6}$$

where λ_t is the Lagrange multiplier and π_{t+1} ($\equiv (P_{t+1} - P_t)/P_t$) is the rate of change in the price, and the transversality condition is

$$\lim_{t\to\infty} \frac{\lambda_t(1+\pi_{t+1})m_{t+1}}{(1+\rho)^t} = 0. \tag{7}$$

From (4) and (5), we derive

$$(1+\rho)(1+\pi_{t+1})\frac{u'(c_t)}{u'(c_{t+1})} - 1 = \frac{v'(m_{t+1})}{u'(c_{t+1})}, \tag{8}$$

where the left-hand side (LHS) denotes the marginal benefit of spending money for consumption and the right-hand side (RHS) denotes the marginal benefit of saving money. This equation implies that an increase in the rate of change in the price, π_{t+1}, motivates the household to save less and consume more because it decreases the future purchasing power of money, or equivalently, it increases the cost of holding money.

From (3) and (6), we have

$$\frac{\partial e_t}{\partial(W_t/W^s_{t-1})} \equiv e_1 > 0, \quad \frac{\partial^2 e_t}{\partial(W_t/W^s_{t-1})^2} \equiv e_{11} < 0; \quad \frac{\partial e_t}{\partial(1-n^a_t)} \equiv e_2 > 0. \tag{9}$$

Following Akerlof (1982) and Akerlof and Yellen (1990), we discuss the implication of (9). If a firm pays a higher current nominal wage against the previous nominal wage, which serves as a reference for a worker to judge whether the firm is treating him/her fairly, then the worker provides greater effort in return. The worse the labor market condition becomes (i.e., the higher the unemployment rate $1 - n^a_t$), the more the worker appreciates being hired by the firm and paid the wage, in other words, the more valuable the gift from the firm to the worker. Thus, an increase in unemployment causes the worker to provide greater effort.

2.2 The Firm Sector

The firm sector is composed of a continuum of identical firms, the size of which we normalize to unity. A typical firm produces a commodity according to the following linear technology:

$$y_t = e_t n_t, \tag{10}$$

where y_t is the production of the commodity, the effort e_t, given by (6), is labor productivity, and n_t is labor input. The firm sets n_t and W_t to maximize profits:

$$P_t e\left(W_t/W_{t-1}^s, 1-n_t^a\right) n_t - W_t n_t + P_t z n_t,$$

where z denotes an employment subsidy in real terms and the firm takes P_t, W_{t-1}^s, n_t^a, and z as given. This profit maximization yields

$$e\left(W_t/W_{t-1}^s, 1-n_t^a\right) + z = \frac{W_t}{P_t}, \tag{11}$$

$$\frac{P_t e_1\left(W_t/W_{t-1}^s, 1-n_t^a\right)}{W_{t-1}^s} = 1. \tag{12}$$

From (11), we take an increase in z as a rise in the marginal productivity of labor. Naturally, we can regard it as a decrease in the marginal cost of labor by arranging (11) as follows:

$$e\left(W_t/W_{t-1}^s, 1-n_t^a\right) = \frac{W_t}{P_t} - z.$$

By eliminating P_t from (11) and (12), we obtain a modified Solow condition:

$$\frac{\left(W_t/W_{t-1}^s\right) e_1\left(W_t/W_{t-1}^s, 1-n_t^a\right)}{e\left(W_t/W_{t-1}^s, 1-n_t^a\right) + z} = 1. \tag{13}$$

2.3 The Government

The budget equation of the government is

$$\frac{M_{t+1} - M_t}{P_t} + \tau_t = g + zn_t,$$

where g is government purchases. The nominal money supply grows at a constant rate μ ($> -\rho/(1+\rho)$):

$$\frac{M_{t+1} - M_t}{M_t} = \mu,$$

which yields the law of motion for real money balances as follows:

$$\frac{m_{t+1}}{m_t} = \frac{1+\mu}{1+\pi_{t+1}}. \tag{14}$$

3 The Dynamics

Because households and firms are identical and the sizes of both are unity, we obtain

$$W^s_{t-1} = W_{t-1}, \quad n^a_t = n_t. \tag{15}$$

From (13) and (15), we find

$$\frac{(W_t/W_{t-1})\, e_1\, (W_t/W_{t-1}, 1-n_t)}{e\, (W_t/W_{t-1}, 1-n_t) + z} = 1, \tag{16}$$

which gives W_t/W_{t-1} as a function of $1-n_t$ and z:

$$\frac{W_t}{W_{t-1}} = \psi(1-n_t; z). \tag{17}$$

Following Campbell (2008), we assume that[10]

$$\frac{\partial^2 e_t}{\partial(W_t/W_{t-1})\partial(1-n_t)} \equiv e_{12} \leq 0.$$

[10]Campbell (2008) makes a similar assumption and argues for the validity of the assumption.

Then, differentiating (16) and taking (9) into account, we derive[11]

$$\frac{\partial(W_t/W_{t-1})}{\partial(1-n_t)} \equiv \psi_1 = \frac{e_2 - (W_t/W_{t-1})e_{12}}{(W_t/W_{t-1})e_{11}} < 0, \tag{18}$$

$$\frac{\partial(W_t/W_{t-1})}{\partial z} \equiv \psi_2 = \frac{1}{(W_t/W_{t-1})e_{11}} < 0. \tag{19}$$

Equation (18) implies the existence of a Phillips curve: a negative relationship between the rate of change in the nominal wage $(W_t - W_{t-1})/W_{t-1}$ and the unemployment rate $1 - n_t$. By subtracting one from both sides of (17), we obtain this Phillips curve as follows:

$$\frac{W_t - W_{t-1}}{W_{t-1}} = \psi(1 - n_t; z) - 1,$$

where its slope equals ψ_1 of (18):

$$\frac{\partial((W_t - W_{t-1})/W_{t-1})}{\partial(1-n_t)} = \psi_1 < 0.$$

This is depicted in Figure 1, which illustrates the case where $\psi(0; z) - 1$ is positive and $\psi(1; z) - 1$ is negative. Note that both can be positive or negative, depending on the form of $\psi(\cdot)$, i.e., the effort function. This Phillips curve implies the following effect of unemployment on firm behavior. An increase in unemployment extracts greater effort from workers, so that firms have less incentive to raise the current nominal wage against the previous nominal wage. Meanwhile, from (19), an increase in z shifts the Phillips

[11] If $z = 0$, differentiating (16) and using percentage changes in the wages, dW_t/W_t ($\equiv \hat{W}_t$) and dW_{t-1}/W_{t-1} ($\equiv \hat{W}_{t-1}$), we obtain the following expression:

$$\hat{W}_t = \left[\frac{e^{-1}e_1e_2 - e_{12}}{e_{11}ee_1^{-1}}\right] d(1-n_t) + \hat{W}_{t-1},$$

which is essentially the same as the case of Campbell (2008, Section 6) where $\omega = 0$ is substituted into Equation (17a).

curve downward (see Figure 1). This shift implies the following effect of the employment subsidy on firm behavior. Because a subsidy increase works like a rise in the labor productivity, it becomes less important for firms to extract effort from workers and the firms become reluctant to raise the current nominal wage against the previous nominal wage.

Using (6), (10), (15), and (17), we obtain the commodity market equilibrium as follows:

$$c_t + g = y_t = e\left(\psi(1-n_t; z), 1-n_t\right) n_t, \tag{20}$$

where it is naturally assumed that an increase in employment n_t leads to an increase in production y_t:

$$\frac{dy_t}{dn_t} = e - e_1\psi_1 n_t - e_2 n_t > 0. \tag{21}$$

From (11), (15), and (17), the rate of change in the price, π_t, is given as a function of the unemployment rates, $1-n_t$ and $1-n_{t-1}$:

$$\pi_t = \psi(1-n_t; z) \cdot \frac{e\left(\psi(1-n_{t-1}; z), 1-n_{t-1}\right) + z}{e\left(\psi(1-n_t; z), 1-n_t\right) + z} - 1. \tag{22}$$

4 Structural Unemployment

In this section, we analyze a steady state where the nominal wage stickiness represented by the Phillips curve generates persistent unemployment. From (8), (14), (20), and (22), we obtain

$$(1+\rho)(1+\pi^*) - 1 = \frac{v'(m^*)}{u'(c^*)}, \tag{23}$$

$$\pi^* = \mu, \tag{24}$$

$$c^* + g = y^* = e\left(\psi(1-n^*; z), 1-n^*\right) n^*, \tag{25}$$

$$\pi^* = \psi(1-n^*; z) - 1, \tag{26}$$

where the asterisk is attached to endogenous variables in this steady state. From (26), the price as well as the nominal wage obeys the Phillips curve relationship.

4.1 The Existence of the Steady State

From (24) and (26), we have

$$\mu = \psi(1-n^*; z) - 1. \tag{27}$$

From (27), if the money growth rate μ satisfies

$$\psi(0; z) - 1 \geq \mu > \psi(1; z) - 1,$$

then n^* is determined so as to satisfy $1 \geq n^* > 0$. That is, unemployment (or the unemployment rate) is

$$1 - n^* \ (\geq 0),$$

and full employment ($n^* = 1$) is reached only if $\mu = \psi(0; z) - 1$. Once n^* is determined, from (25), we obtain y^* and then c^* ($= y^* - g$). Last, from (23), (24), and (25), m^* is determined so as to satisfy

$$(1+\rho)(1+\mu) - 1 = \frac{v'(m^*)}{u'(y^* - g)}. \tag{28}$$

4.2 The Effects of Fiscal and Monetary Expansions

To understand the properties of this steady state better, we examine the effects of fiscal and monetary expansions. From (18), (21), (25), and (27), an increase in government purchases g has no effect on employment n^* and completely crowds out private consumption c^*, whereas an increase in the money growth rate μ boosts them:

$$\frac{dn^*}{dg} = 0, \quad \frac{dc^*}{dg} = -1 < 0; \quad \frac{dn^*}{d\mu} = -\frac{1}{\psi_1} > 0, \quad \frac{dc^*}{d\mu} = \frac{dy^*}{dn^*} \cdot \frac{dn^*}{d\mu} > 0.$$

The effects of government purchases are the same as those obtained in New Classical models. Therefore, unemployment in this steady state is not Keynesian but rather structural unemployment caused by the efficiency wage setting, although the adjustment of the nominal wage is sticky. Moreover, in contrast to Keynesian economics, the monetary expansion affects employment and consumption not through the demand side but through the supply side as follows. An increase in the money growth rate raises the rate of change in the price π^* and hence that in the nominal wage, which enhances the labor productivity e. This rise in productivity motivates firms to employ more labor. Consequently, production expands, which leads to an increase in consumption. This effect of the monetary expansion is similar to that of Vaona (2013a), who derives a price Phillips curve different from (26).

4.3 The Effect of an Employment Subsidy

We next examine the effect of the employment subsidy. Differentiating (27) and taking (18) and (19) into account, we find that as expected, the subsidy works as a way for reducing unemployment:

Proposition 1. *In the steady state with only structural unemployment, an increase in the employment subsidy improves unemployment:*

$$\frac{dn}{dz} = \frac{\psi_2}{\psi_1} > 0.$$

Since the identity of (27) is the modified Solow condition, derived from the first-order conditions for the profit-maximization problem, in this steady state:

$$\frac{(1+\mu)e_1(1+\mu, 1-n^*)}{e(1+\mu, 1-n^*)+z} = 1,$$

this result simply arises as follows. An increase in the subsidy works like a reduction in the marginal cost of labor, and therefore the firm's demand for labor is stimulated and unemployment declines.

5 Structural and Keynesian Unemployment

In this section, we analyze a steady state where not only the above structural unemployment occurs but also a deficiency of aggregate demand creates involuntary unemployment. For this purpose, we abandon the assumption that $v'(\infty) = 0$ in (1). Instead, following Ono (1994, 2001), we assume that the marginal utility of money has a positive lower bound β as follows:[12]

$$\lim_{m \to \infty} v'(m) = \beta \ (> 0), \tag{29}$$

and show that if this insatiable liquidity preference is strong, then aggregate demand becomes insufficient and unemployment becomes higher than $1 - n^*$.[13] The assumption (29) has the great advantage that enables us to analyze easily the aggregate demand deficiency and Keynesian unemployment even in a framework where households dynamically optimize their lifetime utility.

[12]Recently, many studies have adopted this assumption in various models (e.g., Matsuzaki, 2003; Johdo, 2009; Hashimoto, 2011). In contrast to the present paper, Ono (1994, 2001) and these studies assume an exogenous adjustment process for nominal wages, which lacks a microeconomic foundation.

[13]Financial crises, such as the collapse of banking systems, may account for the strong liquidity preference, but this is beyond the scope of this paper.

Because of this assumption, we do not require the conventional Keynesian consumption function, which lacks a microeconomic foundation.

When a MIUF model is adopted,[14] it is almost always assumed that the marginal utility of money eventually decreases to zero as money holdings increase.[15] However, using both parametric and nonparametric methods, Ono et al. (2004) empirically find that the assumption (29) is better supported than the usual assumption that it eventually reaches zero. Theoretically, Murota and Ono (2011) show that it remains positive if money is a status symbol, and Murota and Ono (2012) show that it reaches a positive lower bound when nominal interest rates are zero in a model that incorporates both money and deposits into a utility function. Quoting Keynes, Marx, and Simmel, Ono (1994) argues for the validity of the assumption (29). In addition, Camerer et al. (2005) mention the possibility that the utility of money has little association with consumption. If this is true, it may be possible that the marginal utility of money, in contrast to that of consumption, does not decline to zero.

We further assume that β in (29) is high enough to satisfy

$$(1+\rho)(1+\mu) - 1 < \frac{\beta}{u'(y^* - g)}. \tag{30}$$

Then, the steady state of Section 4 does not exist because there is no value

[14]The dominant view of money in contemporary economics is that people do not derive utility directly from money. Therefore, incorporating money into a utility function is usually criticized and a cash-in-advance model is preferred to a MIUF model. However, Camerer et al. (2004, 2005) argue that money may directly provide utility on the ground of neuroscientific evidence that money and various reinforcers, i.e., attractive faces, funny cartoons, cultural objects such as sports cars, and drugs, activate the same dopaminergic reward circuitry of the brain.

[15]Devoe et al. (2013) present evidence that may conflict with the assumption of the decreasing marginal utility of money. They find that individuals who earn more money from labor view money as more important.

of m^* that satisfies (28). Instead, from (8), (14), (20), and (22), the following steady state exists:

$$(1+\rho)(1+\pi) - 1 = \frac{\beta}{u'(c)}, \tag{31}$$

$$\lim_{t\to\infty} \frac{m_{t+1}}{m_t} = \frac{1+\mu}{1+\pi} > 1, \tag{32}$$

$$c + g = e\left(\psi(1-n;z), 1-n\right) n, \tag{33}$$

$$\pi = \psi(1-n;z) - 1, \tag{34}$$

where Keynesian unemployment persistently occurs and real money balances continue to increase.

5.1 The Existence of the Steady State

Let us prove the existence of this steady state. From (33) and (34), n and π are expressed as functions of c, z, and g:

$$n = n(c; z, g), \quad \pi = \pi(c; z, g) = \psi(1 - n(c; z, g); z) - 1. \tag{35}$$

They satisfy

$$n(c^*; z, g) = n^*, \quad \pi(c^*; z, g) = \pi^* = \mu, \tag{36}$$

where c^*, n^*, and π^* are the values given in the steady state of Section 4. In addition, they satisfy

$$\frac{\partial n}{\partial c} = \frac{1}{e - e_1\psi_1 n - e_2 n} > 0, \quad \frac{\partial \pi}{\partial c} = \frac{-\psi_1}{e - e_1\psi_1 n - e_2 n} > 0, \tag{37}$$

where the inequalities hold under (18) and (21).

From (31) and (35), we obtain

$$(1+\rho)[1 + \pi(c; z, g)] - 1 = \frac{\beta}{u'(c)}. \tag{38}$$

If (30) is true when $c = y^* - g$ $(= c^*)$ and if the following inequality is true when $c = 0$:

$$(1+\rho)[1+\pi(0; z, g)] - 1 > 0,$$

then the value(s) of c satisfying (38) lies between 0 and c^*. Furthermore, if the slope of the LHS is smaller than that of the RHS at the value of c satisfying (38):

$$(1+\rho)\frac{\partial \pi}{\partial c} < -\frac{\beta u''}{(u')^2}, \tag{39}$$

then the value of c is uniquely determined (see Figure 2).[16] We denote the unique value by $\tilde{c}$ and then from (35) we obtain the values of n and π in this steady state as follows:

$$\tilde{n} = n(\tilde{c}; z, g), \quad \tilde{\pi} = \pi(\tilde{c}; z, g). \tag{40}$$

We briefly discuss how this deficiency of consumption occurs.[17] Under (30), the marginal benefit of money exceeds that of consumption even when $m = \infty$, which implies that c^* $(= y^* - g)$ is too much for the household. Therefore, the household desires to save more money even by decreasing consumption. As a consequence, consumption is reduced to $\tilde{c}$ $(< c^*)$ so that (38) holds (both marginal benefits equal). This deficiency of consumption persistently aggravates unemployment. Taking $\tilde{c} < c^*$ into account, from the first equations of (36), (37), and (40), we indeed find that $\tilde{n} < n^*$, i.e.,

$$1 - \tilde{n} > 1 - n^*.$$

In contrast to Ono (1994, 2001) and Ono and Ishida (2014), unemployment in this steady state, $1-\tilde{n}$, is the sum of structural unemployment, $1-n^*$, and

[16]See Ono (1994, 2001) for the detailed proof of the existence of this type of steady state.

[17]See Ono (1994, 2001) for the detailed mechanism.

Keynesian unemployment caused by the deficiency of consumption, $n^* - \tilde{n}$. Moreover, this deficiency of consumption depresses the rate of change in the price. Given $\tilde{c} < c^*$, from the second equations of (36), (37), and (40), we obtain

$$\tilde{\pi} < \pi^* = \mu, \tag{41}$$

which implies that real money balances continue to expand, as shown by (32).[18]

5.2 The Effects of Fiscal and Monetary Expansions

We show further Keynesian properties of this steady state. The effects of fiscal and monetary expansions are consistent with those obtained in the Keynesian liquidity trap, such as the case where the IS curve intersects with the horizontal part of the LM curve in IS–LM analysis. In fact, differentiating (38) and (40) and taking (37) and (39) into account, we find that an increase in government purchases g increases consumption $\tilde{c}$ and employment $\tilde{n}$, whereas an increase in the money growth rate μ has no effect on either:

$$\frac{d\tilde{c}}{dg} = \frac{(1+\rho)\partial\tilde{\pi}/\partial g}{-[\beta u''/(u')^2] - (1+\rho)\partial\tilde{\pi}/\partial\tilde{c}} > 0, \quad \frac{d\tilde{n}}{dg} = \frac{\partial\tilde{n}}{\partial\tilde{c}} \cdot \frac{d\tilde{c}}{dg} + \frac{\partial\tilde{n}}{\partial g} > 0;$$
$$\frac{d\tilde{c}}{d\mu} = 0, \quad \frac{d\tilde{n}}{d\mu} = 0,$$

[18]However, from (4), (7), and (14), for the transversality condition to be satisfied:

$$\lim_{t\to\infty} \frac{\lambda_t(1+\pi_{t+1})m_{t+1}}{(1+\rho)^t} = u'(\tilde{c})(1+\mu) \lim_{t\to\infty} \frac{m_t}{(1+\rho)^t} = 0,$$

real money balances must expand at a rate less than ρ. Hence, as in Ono and Ishida (2014), μ must satisfy not only (30) but also

$$\frac{1+\mu}{1+\tilde{\pi}} < 1+\rho.$$

where $\partial\tilde{n}/\partial g > 0$ and $\partial\tilde{\pi}/\partial g > 0$.[19]

An increase in g boosts employment $\tilde{n}$ through two channels. It directly increases aggregate demand and creates employment ($\partial\tilde{n}/\partial g > 0$). At the same time, it raises the rate of change in the price ($\partial\tilde{\pi}/\partial g > 0$). Since this increases the cost of holding money, consumption is stimulated and further employment is created ($(\partial\tilde{n}/\partial\tilde{c})\cdot(d\tilde{c}/dg) > 0$). Meanwhile, an increase in μ is ineffective because the Pigou effect is not working. That is, even if real money holdings increase, the marginal utility of money remains at β and therefore consumption is not stimulated. These results of the fiscal and monetary expansions are the same as those in Ono and Ishida (2014). However, the nominal wage stickiness of their model differs from that of the present model. Moreover, they neither consider structural unemployment nor investigate the effects of an employment subsidy.

From (34), the monetary expansion is also ineffective for the rate of change in the price $\tilde{\pi}$:

$$\frac{d\tilde{\pi}}{d\mu} = -\psi_1 \frac{d\tilde{n}}{d\mu} = 0.$$

However, as shown by (41), $\tilde{\pi}$ is subject to the restriction that it does not exceed μ. Therefore, if μ is negative, $\tilde{\pi}$ takes a negative value. Meanwhile, if μ is positive, $\tilde{\pi}$ may be positive or negative. The latter implies that stagflation can arise or that deflationary stagnation can arise even though money expands—which one of these occurs depends on the form of the Phillips curve. This is different from Ono (1994, 2001) and Ono and Ishida (2014),

[19] Differentiating (33) and (34) and using (18) and (21), we have

$$\frac{\partial\tilde{n}}{\partial g} = \frac{1}{e - e_1\psi_1 n - e_2 n} > 0, \quad \frac{\partial\tilde{\pi}}{\partial g} = \frac{-\psi_1}{e - e_1\psi_1 n - e_2 n} > 0.$$

in whose models only deflationary stagnation occurs.

The results obtained in the present section are consistent with the phenomena observed in Japan's long-lasting stagnation since the 1990s, now called the Lost Two Decades. In Japan, the output gap was negative during most of this period (Nishizaki et al., 2014, Figure 4), and the unemployment rate increased and remained high (Ono, 2010, Figure 2.1).[20] At the same time, deflation continued and monetary expansions were not effective either for stopping deflation or for stimulating aggregate demand (see, e.g., Ugai, 2007).

5.3 The Effect of an Employment Subsidy

We finally examine the effect of the employment subsidy z in this steady state. An increase in z affects employment $\tilde{n}$ through two channels. From (33), it has a direct impact on $\tilde{n}$. At the same time, from (33) and (38), it has an indirect impact on $\tilde{n}$ through consumption $\tilde{c}$.

Differentiating the first equation of (40) yields

$$\frac{d\tilde{n}}{dz} = \frac{\partial \tilde{n}}{\partial \tilde{c}} \cdot \frac{d\tilde{c}}{dz} + \frac{\partial \tilde{n}}{\partial z} = \underbrace{\frac{1}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}}}_{\text{positive}} \cdot \frac{d\tilde{c}}{dz} + \underbrace{\frac{-e_1\psi_2\tilde{n}}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}}}_{\text{positive}}, \tag{42}$$

where $\partial\tilde{n}/\partial\tilde{c}$ is given by the first equation of (37) and $\partial\tilde{n}/\partial z$ is derived by differentiating (33). In (42), the first term, $(\partial\tilde{n}/\partial\tilde{c}) \cdot (d\tilde{c}/dz)$, is the indirect effect through consumption (aggregate demand) and the second term, $\partial\tilde{n}/\partial z$, is the direct effect, which is positive from (9), (19), and (21). The direct effect implies the following. As shown by (19), an increase in the subsidy depresses

[20] Ono (2010, Figure 2.7) and Nishizaki et al. (2014, Figure 2) show the existence of Japan's Phillips curves.

the rate of change in the nominal wage. Since this lowers labor productivity, more labor is needed to produce a given amount of the commodity. Note that aggregate demand determines output in this steady state.

Let us explore the sign of the indirect effect. Differentiating (38), we find that an increase in the subsidy has an impact on consumption by affecting the rate of change in the price as follows:

$$\frac{d\tilde{c}}{dz} = \underbrace{\frac{1+\rho}{-[\beta u''/(u')^2] - (1+\rho)\partial\tilde{\pi}/\partial\tilde{c}}}_{\text{positive}} \cdot \frac{\partial\tilde{\pi}}{\partial z},$$

where from (39) the denominator is positive. Differentiating (34) and using $\partial\tilde{n}/\partial z$ in (42), we obtain

$$\frac{\partial\tilde{\pi}}{\partial z} = -\psi_1 \frac{\partial\tilde{n}}{\partial z} + \psi_2 = \frac{\psi_2(e - e_2\tilde{n})}{e - e_1\psi_1\tilde{n} - e_2\tilde{n}},$$

where the expression in parentheses, $e - e_2\tilde{n}$, can be positive or negative and therefore the sign of $\partial\tilde{\pi}/\partial z$ is ambiguous. This is because of the following opposing effects. An increase in $\tilde{n}$ caused by an increase in z positively affects $\tilde{\pi}$ along the Phillips curve ($-\psi_1(\partial\tilde{n}/\partial z) > 0$), whereas an increase in z negatively affects it by shifting the Phillips curve downward ($\psi_2 < 0$). If the total effect on $\tilde{\pi}$ is negative: $\partial\tilde{\pi}/\partial z < 0$, then this decline in $\tilde{\pi}$ encourages households to save more and consume less: $d\tilde{c}/dz < 0$, which reduces employment. That is, in this case, the indirect effect in (42) is negative: $(\partial\tilde{n}/\partial\tilde{c}) \cdot (d\tilde{c}/dz) < 0$.

From (42), if this negative indirect effect dominates the positive direct effect, then a subsidy increase leads to a decrease in employment: $d\tilde{n}/dz < 0$. Otherwise, if the negative indirect effect is dominated by the positive direct effect or if the indirect effect as well as the direct effect is positive, it expands employment: $d\tilde{n}/dz > 0$. Note that the indirect effect is positive when $\partial\tilde{\pi}/\partial z > 0$. Thus, the sign of $d\tilde{n}/dz$ is ambiguous.[21] We summarize these results in the following.

Proposition 2. *In the steady state with both structural and Keynesian unemployment, an increase in the employment subsidy z may improve or worsen unemployment.*

This effect of the employment subsidy drastically differs from that in Section 4. A subsidy increase affects unemployment through the two channels different from that in Section 4, so that the effect of the subsidy is ambiguous.[22] This is essentially because aggregate demand determines output, as in Keynesian economics, and because the rate of change in the price is not pinned down by the money growth rate but governed by the demand-supply gap.

6 Concluding Remarks

We develop a MIUF model where a worker's effort depends on the unemployment rate and the change in the nominal wage, and show that the firm's profit maximization subject to this effort function gives rise to a Phillips curve. We then analyze two steady states with and without an aggregate

[21] To be precise, $d\tilde{n}/dz$ is given by

$$\frac{d\tilde{n}}{dz} = \frac{-e_1\psi_2\tilde{n}\cdot\partial\tilde{n}/\partial\tilde{c}}{(-\beta u''/(u')^2) - (1+\rho)\partial\tilde{\pi}/\partial\tilde{c}}\left[-\frac{\beta u''}{(u')^2} - \frac{1+\rho}{e_1\tilde{n}}\right],$$

where the expression in the square brackets may be positive or negative.

[22] Neumark and Grijalva (2013, 2014) examine the effects of various types of hiring credits enacted during and after the Great Recession in the USA and find mixed evidence of the effects: many types of hiring credits did not promote employment although some types succeeded.

demand deficiency, and find that the presence of the aggregate demand deficiency crucially influences the effects of an employment subsidy as well as those of fiscal and monetary expansions.

In the steady state without the aggregate demand deficiency, only structural unemployment occurs, and a monetary expansion and a generous employment subsidy reduce unemployment but a fiscal expansion has no effect on unemployment. In the steady state with the aggregate demand deficiency, Keynesian unemployment in addition to structural unemployment arises. In this case, the fiscal expansion reduces unemployment but the monetary expansion has no effect on unemployment. The effect of the employment subsidy is also in contrast to the above case, and the effect is ambiguous. In other words, the employment subsidy may not be a way for improving unemployment but may instead serve to aggravate the existing unemployment by worsening the deficiency of aggregate demand. Therefore, we conclude that when aggregate demand is insufficient and Keynesian unemployment arises, creating employment by government purchases is more effective and helpful for reducing unemployment than promoting hiring through employment subsidies.

References

[1] Agell, J., Lundborg, P. (1992): 'Fair wages, involuntary unemployment and tax policies in the simple general equilibrium model', Journal of Public Economics, 47, pp. 299–320.

[2] Agell, J., Lundborg, P. (1995): 'Theories of pay and unemployment: Survey evidence from Swedish manufacturing firms', Scandinavian Journal of Economics, 97, pp. 295–307.

[3] Agell, J., Lundborg, P. (2003): 'Survey evidence on wage rigidity and unemployment: Sweden in the 1990s', Scandinavian Journal of Economics, 105, pp. 15–29.

[4] Akerlof, G. A. (1982): 'Labor contracts as partial gift exchange', Quarterly Journal of Economics, 97, pp. 543–569.

[5] Akerlof, G. A., Dickens, W. T., Perry, G. L. (2000): 'Near-rational wage and price setting and the long-run Phillips curve', Brookings Papers on Economic Activity, 1, pp. 1–44.

[6] Akerlof, G. A., Yellen, J. L. (1990): 'The fair wage-effort hypothesis and unemployment', Quarterly Journal of Economics, 105, pp. 255–283.

[7] Bewley, T. F. (1999): Why Wages Don't Fall During a Recession, Harvard University Press, Cambridge, Mass., and London, England.

[8] Blinder, A. S., Choi, D. H. (1990): 'A shred of evidence on theories of wage stickiness', Quarterly Journal of Economics, 105, pp. 1003–1015.

[9] Boockmann, B., Zwick, T., Ammermüller, A., Maier, M. (2012): 'Do hiring subsidies reduce unemployment among older workers? Evidence from natural experiments', Journal of the European Economic Association, 10, pp. 735–764.

[10] Camerer, C. F., Loewenstein, G., Prelec, D. (2004): 'Neuroeconomics: Why economics needs brains', Scandinavian Journal of Economics, 106, pp. 555–579.

[11] Camerer, C. F., Loewenstein, G., Prelec, D. (2005): 'Neuroeconomics: How neuroscience can inform economics', Journal of Economic Literature, 43, pp. 9–64.

[12] Campbell, C. M. (2006): 'A model of the determinants of effort', Economic Modelling, 23, pp. 215–237.

[13] Campbell, C. M. (2008): 'An efficiency wage approach to reconciling the wage curve and the Phillips curve', Labour Economics, 15, pp. 1388–1415.

[14] Campbell, C. M. (2010): 'Deriving the wage–wage and price–price Phillips curves from a model with efficiency wages and imperfect information', Economics Letters, 107, pp. 242–245.

[15] Collard, F., de la Croix, D. (2000): 'Gift exchange and the business cycle: The fair wage strikes back', Review of Economic Dynamics, 3, pp. 166–193.

[16] Danthine, J. P., Donaldson, J. B. (1990): 'Efficiency wages and the business cycle puzzle', European Economic Review, 34, pp. 1275–1301.

[17] Danthine, J. P., Kurmann, A. (2004): 'Fair wages in a New Keynesian model of the business cycle', Review of Economic Dynamics, 7, pp. 107–142.

[18] de la Croix, D., de Walque, G., Wouters, R. (2009): 'A note on inflation persistence in a fair wage model of the business cycle', Macroeconomic Dynamics, 13, pp. 673–684.

[19] Devoe, S. E., Pfeffer, J., Lee, B. Y. (2013): 'When does money make money more important? Survey and experimental evidence', Industrial and Labor Relations Review, 66, pp. 1078–1096.

[20] Fehr, E., Tyran, J.-R. (2007): 'Money illusion and coordination failure', Games and Economic Behavior, 58, pp. 246–268.

[21] Hashimoto, K. (2011): 'International outsourcing, the real exchange rate and effective demand', Metroeconomica, 62, pp. 305–327.

[22] Jaenichen, U., Stephan, G. (2011): 'The effectiveness of targeted wage subsidies for hard-to-place workers', Applied Economics, 43, pp. 1209–1225.

[23] Johdo, W. (2009): 'Habit persistence and stagnation', Economic Modelling, 26, pp. 1110–1114.

[24] Kahneman, D., Knetsch, J. L., Thaler, R. (1986): 'Fairness as a constraint on profit seeking: Entitlements in the market', American Economic Review, 76, pp. 728–741.

[25] Kawaguchi, D., Ohtake, F. (2007): 'Testing the morale theory of nominal wage rigidity', Industrial and Labor Relations Review, 61, pp. 59–74.

[26] Layard, R., Nickell, S., Jackman, R. (2005): Unemployment: Macroeconomic Performance and the Labour Market, the Second Edition, Oxford University Press, New York.

[27] Martin, J. P., Grubb, D. (2001): 'What works and for whom: A review of OECD countries' experiences with active labour market policies', Swedish Economic Policy Review, 8, pp. 9–56.

[28] Matsuzaki, D. (2003): 'The effects of a consumption tax on effective demand under stagnation', Japanese Economic Review, 54, pp. 101–118.

[29] Miao, J., Xie, D. (2013): 'Economic growth under money illusion', Journal of Economic Dynamics and Control, 37, pp. 84–103.

[30] Murota, R., Ono, Y. (2011): 'Growth, stagnation and status preference', Metroeconomica, 62, pp. 122–149.

[31] Murota, R., Ono, Y. (2012): 'Zero nominal interest rates, unemployment, excess reserves and deflation in a liquidity trap', Metroeconomica, 63, pp. 335–357.

[32] Neumark, D., Grijalva, D. (2013): 'The employment effects of state hiring credits during and after the Great Recession', NBER Working Paper 18928.

[33] Neumark, D., Grijalva, D. (2014): 'State hiring credits and recent job growth', FRBSF Economic Letter 2014-05.

[34] Nishizaki, K., Sekine, T., Ueno, Y. (2014): 'Chronic deflation in Japan', Asian Economic Policy Review, 9, pp. 20–39.

[35] Ono, Y. (1994): Money, Interest, and Stagnation, Clarendon Press, Oxford.

[36] Ono, Y. (2001): 'A reinterpretation of Chapter 17 of Keynes's *General Theory:* Effective demand shortage under dynamic optimization', International Economic Review, 42, pp. 207–236.

[37] Ono, Y. (2010): 'Japan's long-run stagnation and economic policies', in Bateman, B. W., Hirai, T., Marcuzzo, M. C. (ed.): The Return to Keynes, Belknap Press, Cambridge, Mass., and London, England.

[38] Ono, Y., Ishida, J. (2014): 'On persistent demand shortages: A behavioural approach', Japanese Economic Review, 65, pp. 42–69.

[39] Ono, Y., Ogawa, K., Yoshida, A. (2004): 'The liquidity trap and persistent unemployment with dynamic optimizing agents: Empirical evidence', Japanese Economic Review, 55, pp. 355–371.

[40] Shafir, E., Diamond, P., Tversky, A. (1997): 'Money illusion', Quarterly Journal of Economics, 112, pp. 341–374.

[41] Solow, R. M. (1979): 'Another possible source of wage stickiness', Journal of Macroeconomics, 1, pp. 79–82.

[42] Ugai, H. (2007): 'Effects of the Quantitative easing policy: A survey of empirical analyses', Monetary and Economic Studies, 25, pp. 1–48.

[43] Vaona, A. (2013a): 'The most beautiful variations on fair wages and the Phillips curve', Journal of Money, Credit and Banking, 45, pp. 1069–1084.

[44] Vaona, A. (2013b): 'Money illusion and the long-run Phillips curve in staggered wage-setting models', Research in Economics, 67, pp. 88–99.

[45] Vaona, A. (2014): 'The price-price Phillips curve in small open economies and monetary unions: Theory and empirics', International Economics and Economic Policy, Forthcoming.

[46] Weber, B., Rangel, A., Wibral, M., Falk, A. (2009): 'The medial prefrontal cortex exhibits money illusion', Proceedings of the National Academy of Sciences of the United States of America, 106, pp. 5025–5028.

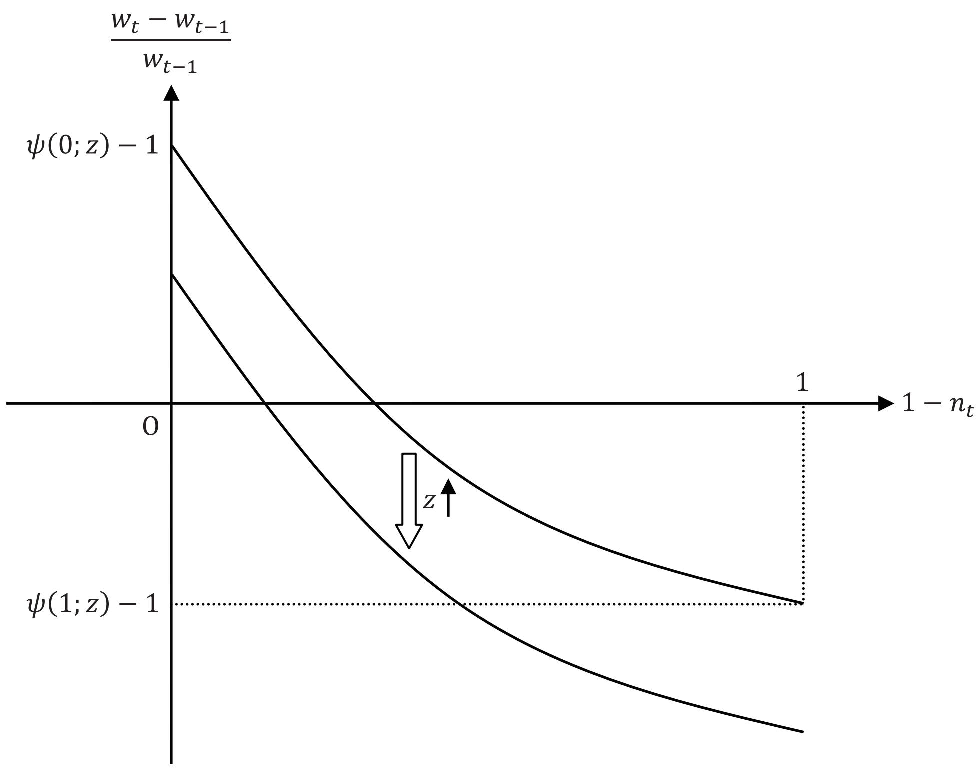

Figure 1: A Phillips curve and the effect of an increase in z

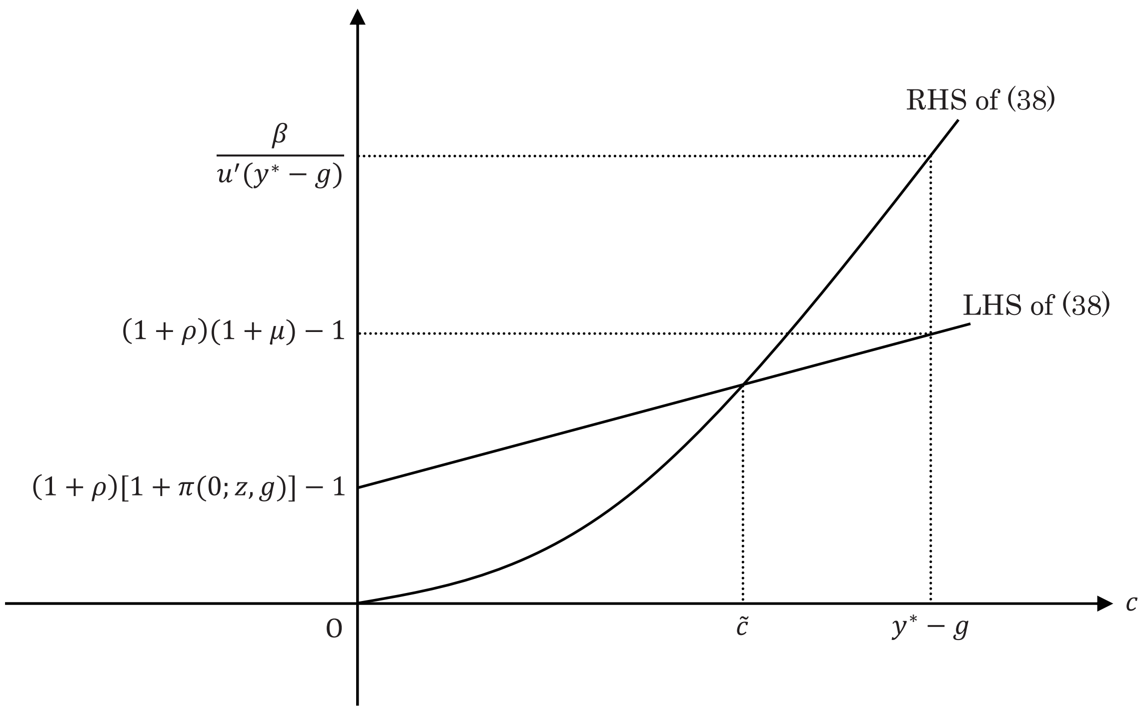

Figure 2: The existence of a unique value of c that satisfies (38)