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Optimal Commodity Taxes and Tariffs under a Revenue Constraint in an Open Economy

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Abstract

This paper analyzes optimal commodity taxes and tariffs under a revenue constraint in an open economy. In the paper, we consider the four alternatives of a small or large country adopting the destination or origin principle for commodity taxation. In each case, we provide the expressions for the optimal commodity taxes and tariffs, and then, using these expressions, we derive the optimal tax rules regarding the signs of the optimal commodity taxes and tariffs, and the relative optimal commodity tax and tariff rates for different goods. We find that the optimal commodity tax rules are the same across all four cases, while the optimal tariff rules are different.

Keywords: Commodity Tax, Tariff, Terms-of-Trade Effect, Destination Principle, Origin Principle

JEL Classification: F11, H21

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1 Introduction

The theory of optimal commodity taxation, which identifies the tax structure that minimizes inevitable tax-induced deadweight losses under a revenue constraint, constitutes a major field in economics and has thus far provided many results suggestive of actual tax policy (for useful surveys, see Sandmo 1976, Auerbach 1985, and Sorensen 2007).[1] Importantly, as the revenue problem is largely specific to each country, much of this existing body of work has developed in the context of a closed economy. However, the increasing linkages between countries through international trade urge the examination of optimal taxation in an open economy.

When extending the analysis of optimal taxation to an open economy, we need to allow for the imposition of tariffs. Although average tariff levels in many countries have fallen substantially under the General Agreement on Tariffs and Trade (GATT)–World Trade Organization (WTO) regime, nearly all countries continue to employ tariffs of some form. Even today, tariffs represent a primary source of government revenue in many developing countries (IMF 2002).[2] Incorporating tariffs in a revenue-constrained optimal tax problem is also important in an open economy from the perspective of theoretical analysis. This is because both commodity taxes and tariffs yield price distortions in consumption,[3] and hence the tariff directly affects the optimal commodity tax structure. In addition, the tariff also affects the optimal commodity taxes through terms-of-trade effects when a tax-imposing country has monopoly power in trade.

The aim of this paper is to analyze optimal commodity taxes and tariffs under a revenue constraint in an open economy. Although few studies have analyzed the revenue-constrained optimal tax mix of commodity taxes and tariffs in an open economy, Keen and Wildasin (2004) are a notable exception.[4] They examine Pareto-efficient international taxation, in which tariffs serve as devices for transferring tax revenue across countries. Such complete tax coordination across countries aimed at achieving Pareto-efficient allocation is, however, difficult in practice. For example, when a country has monopoly power in trade, it faces the optimal tariff problem in identifying the tariff structure that maximizes its own welfare through improvements in the terms of trade

[1] Ramsey (1927) initiated the theoretical analysis of optimal commodity taxation. The field developed rapidly following work by Diamond and Mirrlees (1971a, 1971b).

[2] In evidence, the International Monetary Fund (2002) finds that the revenue from import duties as a proportion of total tax revenue is more than 50% in many African countries. For example, in 2000 it was 53.5% in Madagascar, 54.7% in Swaziland, and 50.3% in Uganda, and outside Africa, it was 50.2% in the Bahamas (in 2001).

[3] When commodity taxes operate under the origin principle, the common tax base for the commodity taxes and tariffs is on the production side.

[4] Dixit (1985) examines revenue-constrained optimal commodity taxation in an open economy, while Hatta and Ogawa (2007) analyze the revenue-constrained optimal tariff problem.

given welfare losses in the other countries.[5] Broda, Limão, and Weinstein (2008) have recently shown empirically that the actual tariff structure in the United States is consistent with the optimal tariff rule for improving the terms-of-trade effects. Noting such actual behavior, we consider a tax-imposing country that arbitrarily acts only in its own interest in our analysis. This is in contrast to Keen and Wildasin (2004), in that the tax-imposing country herein seeks the improvement of its own terms of trade.

This paper treats four alternative cases: a small country adopting the destination principle for commodity taxation, a large country adopting the destination principle, a small country adopting the origin principle, and a large country adopting the origin principle. A small country faces constant world prices, while a large country can manipulate the terms of trade. Under destination-based (origin-based) commodity taxes, the country in which goods and services are consumed (produced) constitutes the tax jurisdiction, and so revenues accrue to that country.[6] This paper specifies the small-country case with the destination principle as the benchmark and compares the optimal tax structure across the four cases. In each case, we provide the expressions for the optimal commodity tax and tariff vectors. Then, from the expressions, we derive the optimal commodity tax and tariff rules regarding the sign of the optimal commodity taxes and tariffs, and the relative optimal commodity tax and tariff rates for different goods.

The expression for the optimal commodity tax vector provided in this paper consists of its own price-distortion effects and the tariff-induced price distortion effects. Of these, the latter arises because the commodity taxes and tariffs have a common tax base. One of the most interesting and important findings in this paper is that the expression for the optimal commodity tax vector is identical across the four cases. Thus, the optimal tax rules derived from this expression, such as the signs and the relative magnitudes of the optimal commodity tax rates, are also identical, irrespective of the country's type and commodity tax system. As another important result, this paper provides the generalized Corlett and Hague rule for the optimal commodity taxes,[7] which shows that the relative optimal commodity tax rates are determined by the cross-price elasticities of both compensated demand and supply.

The expression for the optimal tariff vector provided in this paper consists of a term

[5] This optimal tariff problem is one of the main issues in international trade theory, and the resultant literature is voluminous. See, for example, Kaldor (1940), Johnson (1953–54), and more recently, Bond (1990), Syropoulos (2002), and Ogawa (2007b, 2012) on the optimal tariffs.

[6] The origin principle for a commodity tax system is applied to services in much of the European Union. However, this will shift, in principle, to the destination principle after 2015.

[7] Corlett and Hague (1953) show that in a three-good economy with uniform commodity taxation, increasing the tax rate on the commodity that is less of a substitute for leisure, and decreasing the tax rate on the other commodity so as to keep the tax revenue constant, enhances welfare. This rule is derived in the context of an optimal commodity tax framework by Harberger (1964), Diamond and Mirrlees (1971), and Ogawa (2007a).

representing the price-distortion effects generated by a revenue constraint and a term representing the optimal tariff that improves the terms of trade. In contrast to the optimal commodity taxes, the optimal tariff vector takes different forms in all four cases. This implies that tariff policies should take the type of country and commodity taxation system into consideration. For example, in a small country, the optimal trade taxes are import subsidies and export taxes under the destination principle,[8] but import tariffs and export subsidies under the origin principle. It should be noted that these results readily imply that the optimal tariffs in a small country are nonzero even though commodity taxes are available. An intuition for this result is given in the text.

The remainder of the paper is structured as follows. Section 2 describes the model. Sections 3 and 4 respectively analyze optimal taxation in the small- and large-country cases under the destination principle. Section 5 considers the application of origin-based commodity taxes. Section 6 provides the conclusion.

2 The Model

For our analysis, we employ the framework of a general equilibrium model of international trade. There are $N+1$ tradable goods, which are indexed as $0, 1, \ldots, N$, where good 0 is the numeraire. Production factors, fixed in supply, are internationally immobile and fully employed in the production sectors. The markets for goods and factors are perfectly competitive. The government of the home country imposes commodity taxes on the consumption of the goods, which shows destination-based commodity taxes,[9] and tariffs on the net imports of the goods to collect revenue. A lump-sum tax and a profit tax are not available.[10]

Let us denote the commodity tax vector as $\mathbf{t}' \equiv (t_0, \mathbf{t}'_\mathbf{N})$,[11] the tariff vector as $\boldsymbol{\tau}' \equiv (\tau_0, \boldsymbol{\tau}'_\mathbf{N})$,[12] the consumer price vector as $\mathbf{q}' \equiv (q_0, \mathbf{q}'_\mathbf{N})$, the producer price vector as $\mathbf{p}' \equiv (p_0, \mathbf{p}'_\mathbf{N})$, and the world price vector as $\mathbf{w}' \equiv (w_0, \mathbf{w}'_\mathbf{N})$, where $\mathbf{t}_\mathbf{N}$, $\boldsymbol{\tau}_\mathbf{N}$, $\mathbf{q}_\mathbf{N}$, $\mathbf{p}_\mathbf{N}$, and $\mathbf{w}_\mathbf{N}$ represent the N-dimensional vector excluding the index 0 of the numeraire good. Then,

$$\mathbf{q} = \mathbf{t} + \mathbf{p}, \;\; \mathbf{p} = \boldsymbol{\tau} + \mathbf{w}. \tag{1}$$

Following Keen and Wildasin (2004), we assume that $t_0 = \tau_0 = 0$ and, without loss of generality, set $w_0 = 1$. Thus, $q_0 = p_0 = w_0 = 1$.

[8] Iran gives an import subsidy to gasoline, and Saudi Arabia gives an import subsidy to barley.

[9] In Section 5, we exchange this tax system for the origin principle.

[10] No tax or subsidy is imposed on production factors. Production factors, such as mines, oilfields, and cultivated land, are effectively not subject to commodity taxes. In Japan, the consumption tax is not imposed on land transactions.

[11] A prime on a vector or matrix denotes the transpose.

[12] Note that $\tau_i > 0$ is an import tariff (export subsidy) if good i is imported (exported), and $\tau_i < 0$ is an import subsidy (export tax) if good i is imported (exported).

In the home country, there is a representative consumer with a well-behaved utility function. The expenditure function is given by $e(\mathbf{q}, u, g)$, where u is the utility level and g is a publicly provided good. The demand vector is then given by $\mathbf{e_q}(\equiv \partial e/\partial \mathbf{q})$, whose elements are $e_i(\equiv \partial e/\partial q_i)$. Let $\mathbf{e_{qq}} \equiv \partial \mathbf{e_q}/\partial \mathbf{q}'$, whose elements are $e_{ij}(\equiv \partial e_i/\partial q_j)$, and $\mathbf{e_{q}}_u \equiv \partial \mathbf{e_q}/\partial u$, whose elements are $e_{iu}(\equiv \partial e_i/\partial u)$. Good i is a substitute for good j in consumption if $e_{ij} > 0$. The expenditure function has the following properties: (i) symmetry, $\mathbf{e_{qq}} = \mathbf{e}'_{\mathbf{qq}}$; (ii) homogeneity, $\mathbf{e_{qq}q} = \mathbf{0}_{N+1}$, where $\mathbf{0}_m$ denotes the m-dimensional vector of zeros; and (iii) negative semidefiniteness, $\mathbf{h}'\mathbf{e_{qq}h} = 0$ if $\mathbf{h} = \zeta\mathbf{q}$ for some scalar ζ and $\mathbf{h}' \equiv (h_0, h_1, \ldots, h_n)$, and $\mathbf{h}'\mathbf{e_{qq}h} < 0$ otherwise.[13] Let us denote $\mathbf{e_N} \equiv \partial e/\partial \mathbf{q_N}$, $\mathbf{e_{NN}} \equiv \partial \mathbf{e_N}/\partial \mathbf{q}'_{\mathbf{N}}$, $e_u \equiv \partial e/\partial u$, and $\mathbf{e_N}_u \equiv \partial \mathbf{e_N}/\partial u$.

Given convex technology, the behavior of the production sectors is characterized by the revenue function $r(\mathbf{p}, \mathbf{v})$,[14] where $\mathbf{v}$ is the factor endowment vector with fixed elements in supply. The supply vector is then given by $\mathbf{r_p}(\equiv \partial r/\partial \mathbf{p})$, whose elements are $r_i(\equiv \partial r/\partial p_i)$. Let $\mathbf{r_{pp}} \equiv \partial \mathbf{r_p}/\partial \mathbf{p}'$, whose elements are $r_{ij}(\equiv \partial r_i/\partial p_j)$. Good i is a substitute for good j in production if $r_{ij} < 0$. The revenue function has the following properties: (i) symmetry, $\mathbf{r_{pp}} = \mathbf{r}'_{\mathbf{pp}}$; (ii) homogeneity, $\mathbf{r_{pp}p} = \mathbf{0}_{N+1}$; and (iii) positive semidefiniteness, $\mathbf{k}'\mathbf{r_{pp}k} = 0$ if $\mathbf{k} = \xi\mathbf{p}$, where ξ is a scalar and $\mathbf{k}' \equiv (k_0, k_1, \ldots, k_n)$, and $\mathbf{k}'\mathbf{r_{pp}k} > 0$ otherwise. Let $\mathbf{r_N} \equiv \partial r/\partial \mathbf{p_N}$ and $\mathbf{r_{NN}} \equiv \partial \mathbf{r_N}/\partial \mathbf{p}'_{\mathbf{N}}$. Hereafter, $\mathbf{v}$ is not explicitly shown in $r(\cdot)$ because each element of $\mathbf{v}$ is fixed.

The budget constraint of the private sector is given by

$$e(\mathbf{q}, u, g) = r(\mathbf{p}), \tag{2}$$

in which the left-hand side (LHS) represents the expenditure of the consumer, and the right-hand side (RHS) represents the income that the consumer receives, which is equal to the factor payment plus pure profit. As the revenue function is defined under constant-returns-to-scale (CRS), which leads to zero profit, and under decreasing-returns-to-scale (DRS) production technology, which yields positive profit, condition (2) holds irrespective of whether there is pure profit or no profit.

The government spends tax revenue on the purchase of the numeraire good and then provides it to the consumer.[15] The amount of the public provision g is assumed to be fixed. Hereafter, g is not explicitly shown in $e(\cdot)$. The government's budget constraint is

$$\mathbf{t}'_{\mathbf{N}}\mathbf{e_N}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e_N}(\mathbf{q}, u) - \mathbf{r_N}(\mathbf{p})] \geq g. \tag{3}$$

[13] We assume that there is some substitutability between the numeraire and nonnumeraire goods. See Dixit and Norman (1980).

[14] The technology of this economy is described by the technology set $T \equiv \{(\mathbf{y}, -\mathbf{v}) : \mathbf{y} \in Y(\mathbf{v})\}$, where $\mathbf{y}$ is the net output vector. We assume that T is a nonempty and closed convex set. The revenue function is defined by $r(\mathbf{p}, \mathbf{v}) \equiv \max_{\mathbf{y}}\{\mathbf{p}'\mathbf{y} : \mathbf{y} \in Y(\mathbf{v})\}$. See Dixit and Norman (1980) and Woodland (1982) for details.

[15] Keen and Wildasin (2004) likewise adopt this form of public purchase.

Equations (1), (2), and (3) describe the economy.[16]

3 A Small Country

This section examines the structure of the optimal commodity taxes and tariffs in a small country facing constant world prices, adopting destination-based commodity taxes. We first provide the optimal commodity tax and tariff vectors in the following proposition.

Proposition 1. *The optimal commodity taxes and tariffs in a small country are given by*

$$\mathbf{t}'_{\mathbf{N}} = -\alpha(\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}} - \mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}}), \tag{4}$$

$$\boldsymbol{\tau}'_{\mathbf{N}} = -\alpha\mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}}, \tag{5}$$

where α is a positive scalar.

Proof. The welfare maximization problem for the home country is given by[17]

$$\max_{\mathbf{t}_{\mathbf{N}},\, \boldsymbol{\tau}_{\mathbf{N}},\, u} \quad u$$

$$\text{s.t.} \quad e(\mathbf{q}, u) - r(\mathbf{p}) = 0,$$

$$\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] = g.$$

The Lagrangian is

$$\Im^{sd} = u - \mu[e(\mathbf{q}, u) - r(\mathbf{p})] - \lambda\{\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] - g\},$$

where μ and λ are Lagrangian multipliers. The first-order conditions (FOCs) with respect to $\mathbf{t}_{\mathbf{N}}$, $\boldsymbol{\tau}_{\mathbf{N}}$, and u are, respectively,

$$-\mu\mathbf{e}'_{\mathbf{N}} - \lambda(\mathbf{e}'_{\mathbf{N}} + \mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} + \boldsymbol{\tau}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}) = \mathbf{0}'_{N}, \tag{6}$$

$$-\mu(\mathbf{e}'_{\mathbf{N}} - \mathbf{r}'_{\mathbf{N}}) - \lambda[\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} + \mathbf{e}'_{\mathbf{N}} - \mathbf{r}'_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}})] = \mathbf{0}'_{N}, \tag{7}$$

$$1 - \mu e_u - \lambda(\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}u} + \boldsymbol{\tau}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}u}) = 0. \tag{8}$$

Defining $\alpha \equiv (\mu + \lambda)/\lambda$ and subtracting (6) from (7), we obtain

$$\alpha\mathbf{r}'_{\mathbf{N}} = -\boldsymbol{\tau}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}}, \tag{9}$$

[16] Conditions (2) and (3) with equality yield the international trade balance: $\mathbf{w}'(\mathbf{e}_{\mathbf{q}} - \mathbf{r}_{\mathbf{p}} + \mathbf{g}) = 0$.

[17] This particular formulation of the maximization problem is also used by Mirrlees (1976), Munk (1978), Hatta and Ogawa (2007), and Ogawa (2012), among others.

which yields (5). From (6) and the definition of α, we have

$$\alpha \mathbf{e}'_{\mathbf{N}} = -(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})' \mathbf{e}_{\mathbf{NN}}. \tag{10}$$

This and (9) yield (4).[18]

Next, let us examine the sign of α. From (9) and (10), we have

$$\alpha \mathbf{e}'_{\mathbf{N}}(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}}) = -(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})' \mathbf{e}_{\mathbf{NN}}(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}}), \tag{11}$$

$$\alpha \mathbf{r}'_{\mathbf{N}} \boldsymbol{\tau}_{\mathbf{N}} = -\boldsymbol{\tau}'_{\mathbf{N}} \mathbf{r}_{\mathbf{NN}} \boldsymbol{\tau}_{\mathbf{N}}, \tag{12}$$

which yield

$$\alpha[\mathbf{t}'_{\mathbf{N}} \mathbf{e}_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})] = -(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})' \mathbf{e}_{\mathbf{NN}}(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}}) + \boldsymbol{\tau}'_{\mathbf{N}} \mathbf{r}_{\mathbf{NN}} \boldsymbol{\tau}_{\mathbf{N}}.$$

The RHS is positive from property (iii) of the expenditure and revenue functions. The expression in square brackets on the LHS is the tax revenue. The sign of α is therefore the same as that of the revenue. ■

An increase in tax increases revenue, while it also extends the price distortions. Conditions (9) and (10) indicate that these effects balance across goods at the optimum in consumption and production, respectively. This optimal tax implication is related to the Ramsey–Samuelson optimal tax rule. Slight variations in (9) and (10) respectively yield

$$\alpha \mathbf{e}_{\mathbf{N}} = -\mathbf{e}_{\mathbf{NN}}(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}}),$$

$$\alpha \mathbf{r}_{\mathbf{N}} = -\mathbf{r}_{\mathbf{NN}} \boldsymbol{\tau}_{\mathbf{N}},$$

which shows that *at the optimum, (i) the percentage changes in demand that would result from the commodity tax and tariff changes are the same for all goods, (ii) the percentage changes in supply that would result from the tariff changes are also the same for all goods,*

[18] From (8), we have

$$\mu = \frac{1}{e_u} - \lambda \left[\frac{(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})' \mathbf{e}_{\mathbf{N}u}}{e_u} \right],$$

which shows that μ is the social marginal utility of income. See Diamond (1975, Equation 6, p. 338). The above equation can be rewritten as

$$-\alpha = -\frac{1}{\lambda e_u} + \frac{(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})' \mathbf{e}_{\mathbf{N}u}}{e_u} - 1,$$

which corresponds to Equation (13) in Mirrlees (1976, p. 332). As indicated in Mirrlees (1976), $-\alpha$ is the difference in a term proportional to the marginal utility of income, and the income derivative of household income net of taxes. Note that λ in our analysis differs from λ in Diamond (1975) and Mirrlees (1976) because λ in our analysis is the Lagrangian multiplier associated with the government's budget constraint, whereas λ in Diamond (1975) and Mirrlees (1976) is associated with the resource constraint.

and (iii) the percentage changes in demand and supply are equal. This is the generalized Ramsey–Samuelson optimal tax rule, as applied to a small open economy.[19]

Noting that $\mathbf{e}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}^{-1}$ and $-\mathbf{r}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}}^{-1}$ relate to the price distortions in consumption and production, respectively, we discuss the optimal commodity taxes and tariffs provided in Proposition 1 from the point of view of the tax base. The optimal tariffs are set allowing for price distortions only on the production, which is not the tax base for the commodity taxes, as shown in (5). Consequently, the optimal commodity taxes have to be set allowing for the optimal tariff structure, because consumption is the common tax base for the commodity taxes and tariffs. This is readily confirmed by noticing from (5) that $-\alpha\mathbf{r}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}}^{-1}$ in (4) represents the optimal tariffs. Thus, although the commodity taxes do not yield price distortions in production, the substitution terms of supply appear in the optimal commodity taxes in (4).

Ramsey (1927) and Munk (1978) also provide the optimal commodity tax formula depending on the substitution effects of both compensated demand and supply in a closed economy with untaxed pure profit. An untaxed profit ensures the supply side impacts upon the optimal commodity taxes in their models. In contrast to their models, in our model the tariffs ensure the supply side influences the optimal commodity taxes, as discussed earlier. Therefore, our optimal commodity tax vector depending on the supply side is intact irrespective of whether there is pure profit under DRS or no profit under CRS. If tariffs are unavailable in our model, the optimal commodity taxes are given by $\mathbf{t}'_{\mathbf{N}} = -\alpha\mathbf{e}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}^{-1}$,[20] which is a standard Ramsey tax rule in a closed economy with zero profit or with 100% profit tax. This standard optimal tax formula arises in our model even if there is untaxed profit, as long as tariffs are unavailable. This is also contrasts with Munk (1978).

We next examine the signs of the optimal commodity taxes and tariffs. If all goods are substitutes in consumption, $\mathbf{e}_{\mathbf{NN}}^{-1} < \mathbf{0}_{NN}$, where $\mathbf{0}_{NN}$ denotes the $N \times N$ matrix of zeros, and if in production, $\mathbf{r}_{\mathbf{NN}}^{-1} > \mathbf{0}_{NN}$.[21] Using these and $\alpha > 0$, it follows from (4) and (5) that $\mathbf{t}_{\mathbf{N}} > \mathbf{0}_N$ and $\boldsymbol{\tau}_{\mathbf{N}} < \mathbf{0}_N$, respectively. From (4) and (5), we have $\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}} = -\alpha\mathbf{e}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}^{-1} > \mathbf{0}_N$. These are summarized by the following corollary.

Corollary 1(a). *The optimal tax structure is such that (i)* $\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}} > \mathbf{0}_N$ *if all goods are substitutes for each other in consumption, (ii)* $\boldsymbol{\tau}_{\mathbf{N}} < \mathbf{0}_N$ *if all goods are substitutes for each other in production, and (iii)* $\mathbf{t}_{\mathbf{N}} > \mathbf{0}_N$ *if all goods are substitutes for each other in both consumption and production.*

[19] Samuelson's (1951) formula, given by $\alpha\mathbf{e}_{\mathbf{N}} = -\mathbf{e}_{\mathbf{NN}}\mathbf{t}_{\mathbf{N}}$, was derived in a closed economy.

[20] When tariffs are unavailable, from (6), we have $-\mu\mathbf{e}'_{\mathbf{N}} - \lambda(\mathbf{e}'_{\mathbf{N}} + \mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}) = \mathbf{0}'_N$, which yields $\mathbf{t}'_{\mathbf{N}} = -\alpha\mathbf{e}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}^{-1}$.

[21] See Hatta (1977) for $\mathbf{e}_{\mathbf{NN}}^{-1} < \mathbf{0}_{NN}$ and $\mathbf{r}_{\mathbf{NN}}^{-1} > \mathbf{0}_{NN}$ under the substitution condition.

Corollary 1(a) means that taxes are imposed on both consumption and production at the optimum, because an import subsidy and an export tax, represented by $\tau_i < 0$, are equivalent to a production tax in the production side,[22] and $t_i + \tau_i > 0$ means the imposition of a tax on consumption. When $\boldsymbol{\tau}_{\mathbf{N}} > \mathbf{0}_N$, the required revenue must be collected from the taxes on consumption alone, because $\tau_i > 0$ is equivalent to a consumption tax with a production subsidy.[23] Therefore, when $\boldsymbol{\tau}_{\mathbf{N}} < \mathbf{0}_N$, the economy has a larger tax base, which naturally leads to less tax distortion than when $\boldsymbol{\tau}_{\mathbf{N}} > \mathbf{0}_N$.

We can specify the signs of tax revenue from the consumption and production sides without the substitution condition. From (11), (12), and property (iii) of the expenditure and revenue functions, we obtain $(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}})'\mathbf{e}_{\mathbf{N}} > 0$ and $\boldsymbol{\tau}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}} < 0$. The former means that the revenue from taxation on consumption is positive. The latter means that the revenue from taxation on production is positive, because $\tau_i < 0$ is equivalent to a production tax in the production side.

The optimal tariffs in a small country with a revenue constraint are nonzero even though commodity taxes are available, as shown by $\boldsymbol{\tau}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}} < 0$. This result is in contrast to Dixit (1985), who shows that the tariffs are not required. Dixit (1985, Sect. 3.2) considers the case where the government manipulates consumer, but not producer, prices, with taxes and directly controls output supply and factor demand. Thus, in his model, the production side is not included in the tax base. As commodity taxes are imposed on nontradable as well as tradable commodities, a commodity tax has a larger tax base in the consumption side than a tariff. His model setting therefore leads to the straightforward conclusion that tariffs are not required. That is, "no tariff" in Dixit (1985) is not the result, but rather the assumption.[24] The result that the optimal tariffs are nonzero is suggested by Hatzipanayotou et al. (1994) and Keen and Ligthart (2002), who show that a small reduction in tariffs, accompanied by an increase in commodity taxes so as to keep consumer prices constant, enhances welfare and revenue. Their result implies that if initial tariffs are initially zero, a negative tariff enhances welfare and revenue. This shows that having no tariffs does not maximize welfare even in a small country; that is, the use of both tariffs and commodity taxes can achieve higher welfare than the case where only commodity taxes are used.

Next, we examine the relative optimal commodity tax and tariff rates for different goods. When the taxed goods are price independent, $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, we obtain the inverse elasticity rules. Let us define $\delta_i \equiv (t_i + \tau_i)/q_i$, $\phi_i \equiv \tau_i/p_i$,

[22] An import subsidy and an export subsidy (i.e., $\tau_i < 0$) make domestic consumer and producer prices lower than the world price. Thus, it is equivalent to a production tax cum consumption subsidy.

[23] An export subsidy and an import tariff make domestic consumer and producer prices higher than the world price. Thus, it is equivalent to a consumption tax cum production subsidy.

[24] Dixit (1985) excludes the production side from the tax base, even in the large country case. Therefore, Dixit (1985) does not analyze the optimal tax mix of commodity taxes and tariffs under a revenue constraint.

$\gamma_i \equiv t_i/q_i$, $\eta_{ij} \equiv q_j e_{ij}/e_i$, and $\sigma_{ij} \equiv p_j r_{ij}/r_i$. All tax rates defined here are in terms of the tax-inclusive price, and δ_i denotes the total tax rate on the consumption of good i.

Corollary 1(b). *The following optimal tax rules hold: (i) when $e_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, $\delta_i \lesseqgtr \delta_j$ if $-\eta_{ii} \gtreqless -\eta_{jj}$; (ii) when $r_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, $-\phi_i \lesseqgtr -\phi_j$ if $\sigma_{ii} \gtreqless \sigma_{jj}$; and (iii) when $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, $\gamma_i < \gamma_j$ if $-\eta_{ii} \geq -\eta_{jj}$ and $\sigma_{ii} \geq \sigma_{jj}$ with at least one strict inequality.*

Proof. The proof of (i): When $e_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, (10) is reduced to $\delta_i = \alpha/(-\eta_{ii})$ for $i = 1, \ldots, n$, which proves (i).

The proof of (ii): When $r_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, (5) is reduced to $-\phi_i = \alpha/\sigma_{ii}$ for $i = 1, \ldots, n$, which proves (ii).

The proof of (iii): When $e_{ij} = r_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, from (4) we have

$$\gamma_i = \alpha \left[-\frac{1}{\eta_{ii}} + (1 - \gamma_i) \frac{1}{\sigma_{ii}} \right], \qquad i = 1, \ldots, n,$$

where we have used $p_i/q_i = 1 - \gamma_i$. Solving this equation with respect to γ_i yields

$$\gamma_i = \left(-\frac{1}{\eta_{ii}} + \frac{1}{\sigma_{ii}} \right) \Big/ \left(\frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right), \qquad i = 1, \ldots, n, \tag{13}$$

which yields[25]

$$(\gamma_i - \gamma_j)\, \Theta = \left(\frac{1}{-\eta_{ii}} - \frac{1}{-\eta_{jj}} \right) \left(\frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right) + \left(\frac{1}{\sigma_{ii}} - \frac{1}{\sigma_{jj}} \right) \left(\frac{1}{\alpha} + \frac{1}{\eta_{ii}} \right), \tag{14}$$

where

$$\Theta \equiv \left(\frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right) \left(\frac{1}{\alpha} + \frac{1}{\sigma_{jj}} \right).$$

As $\alpha > 0$ and $\sigma_{ii} > 0$ for $i = 1, \ldots, n$, we have $\Theta > 0$. From (13), it follows that

$$\frac{1}{\alpha} + \frac{1}{\eta_{ii}} = (1 - \gamma_i) \left(\frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right) > 0, \qquad i = 1, \ldots, n, \tag{15}$$

where the inequality follows from $1 - \gamma_i = p_i/q_i > 0$, $\alpha > 0$, and $\sigma_{ii} > 0$. Using $\Theta > 0$ and (15), (14) proves (iii). ■

Corollary 1(b)-(iii) is the well-known inverse elasticity rule, first derived by Ramsey (1927) in a closed economy, and (ii) is an application of the inverse elasticity rule to the optimal tariff rates.

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[25] From (13) we have

$$(\gamma_i - \gamma_j)\, \Theta = \left(-\frac{1}{\eta_{ii}} + \frac{1}{\sigma_{ii}} \right) \left(\frac{1}{\alpha} + \frac{1}{\sigma_{jj}} \right) - \left(-\frac{1}{\eta_{jj}} + \frac{1}{\sigma_{jj}} \right) \left(\frac{1}{\alpha} + \frac{1}{\sigma_{ii}} \right).$$

Adding $1/\eta_{ii}\sigma_{ii} - 1/\eta_{ii}\sigma_{ii} = 0$ to the RHS in this equation and manipulating, we obtain (14).

The assumption that there are no cross-substitution effects between the taxed goods is, however, somewhat strong, and with it, we may neglect an important relation between the cross-substitution effects and the optimal tax structure. Next, we consider the case where there are cross-substitution effects between goods. To avoid undue analytical complexity, we consider a three-good model. The following corollary provides the Corlett–Hague rule applied to a small open economy.

Corollary 1(c). *The optimal tax structure in a three-good case is such that: (i)* $\delta_1 \gtreqless \delta_2$ *if* $\eta_{20} \gtreqless \eta_{10}$*, (ii)* $-\phi_1 \gtreqless -\phi_2$ *if* $-\sigma_{20} \gtreqless -\sigma_{10}$*, and (iii)* $\gamma_i > \gamma_j$ *if* $\eta_{j0} \geq \eta_{i0}$ *and* $-\sigma_{j0} \geq -\sigma_{i0}$ *with at least one strict inequality.*

Proof. The proof of (i): In a three-good case, (10) is reduced to $\delta_i = (-\eta_{jj} + \eta_{ij})\Psi$ for $i, j = 1, 2$ and $i \neq j$, where $\Psi \equiv \alpha/(\eta_{11}\eta_{22} - \eta_{12}\eta_{21})$. By using $\eta_{i0} + \eta_{ii} + \eta_{ij} = 0$ for $i, j = 1, 2$ and $i \neq j$,[26] this can be rewritten as

$$\delta_1 = (\eta_{20} + \eta_{12} + \eta_{21})\Psi, \quad \delta_2 = (\eta_{10} + \eta_{12} + \eta_{21})\Psi. \tag{16}$$

Note that $\Psi > 0$ because $\eta_{11}\eta_{22} - \eta_{12}\eta_{21} = (e_{11}e_{22} - e_{12}e_{21})(q_1 q_2/e_1 e_2) > 0$ from property (iii) of the expenditure function. From (16), we immediately have

$$\delta_1 - \delta_2 = (\eta_{20} - \eta_{10})\Psi, \tag{17}$$

which, together with $\Psi > 0$, proves (i).

The proof of (ii): In a three-good case, it follows from (5) that $\phi_i = (-\sigma_{jj} + \sigma_{ij})\Lambda$, where $\Lambda \equiv \alpha/(\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21})$. By using $\sigma_{i0} + \sigma_{ii} + \sigma_{ij} = 0$ for $i, j = 1, 2$ and $i \neq j$,[27] this can be rewritten as

$$-\phi_1 = -(\sigma_{20} + \sigma_{12} + \sigma_{21})\Lambda, \quad \phi_2 = -(\sigma_{10} + \sigma_{12} + \sigma_{21})\Lambda. \tag{18}$$

Note that $\Lambda > 0$ because $\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21} = (r_{11}r_{22} - r_{12}r_{21})(p_1 p_2/r_1 r_2) > 0$ from property (iii) of the revenue function. From (18), we immediately have

$$(-\phi_1) - (-\phi_2) = [(-\sigma_{20}) - (-\sigma_{10})]\Lambda, \tag{19}$$

which, together with $\Lambda > 0$, proves (ii).

The proof of (iii): From the definitions of δ_i, ϕ_i, and γ_i, we obtain $\delta_i = (1 - \phi_i)\gamma_i + \phi_i$.[28] Using this, (17), and (19), we obtain

$$(\gamma_1 - \gamma_2)(1 - \phi_1) = (\eta_{20} - \eta_{10})\Psi + [(-\sigma_{20}) - (-\sigma_{10})](1 - \gamma_2)\Lambda. \tag{20}$$

[26] This follows from property (ii) of the expenditure function.

[27] This follows from property (ii) of the revenue function.

[28] From the definitions of γ_i and ϕ_i, we obtain $\delta_i = t_i/q_i + (p_i/q_i)(\tau_i/p_i)$. From this and $p_i/q_i = 1 - \gamma_i$, we have $\delta_i = (1 - \phi_i)\gamma_i + \phi_i$.

As $1-\phi_1 = w_1/p_1 > 0$, $1-\gamma_2 = p_2/q_2 > 0$, $\Psi > 0$, and $\Lambda > 0$, (20) proves (iii). ∎

Corollary 1(c)-(i) is the Corlett–Hague rule applying to the total tax rate on consumption, comprising the commodity tax and tariff. Taxation on consumption creates an incentive for the overconsumption of the untaxed good (good 0). The higher tax rate on the consumption of the good that is less elastic for the untaxed good, accompanied by the lower tax rate on the other nonnumeraire good, amounts to the imposition of the consumption tax on the numeraire good and partially represses the incentive for the overconsumption of the untaxed good. Corollary 1(c)-(ii) is the Corlett–Hague rule for the optimal tariff rates, and its intuition is analogous to that for Corollary 1(c)-(i). Corollary 1(c)-(iii) is the generalized Corlett–Hague rule for the optimal commodity tax rates, which depend on the cross-price elasticities of both compensated demand and supply.

Even when $\eta_{10} = \eta_{20}$, the optimal commodity tax rates are definitely nonuniform if $\sigma_{10} \neq \sigma_{20}$. This clearly shows that the optimal commodity tax structure depends on the elasticities of supply, along with those of compensated demand. If $\eta_{10} = \eta_{20}$ and $\sigma_{10} = \sigma_{20}$, then the optimal commodity taxes and tariffs are uniform, i.e., $\phi_1 = \phi_2$ and $\gamma_1 = \gamma_2$. This result holds in the model with more than three goods.

4 A Large Country

This section analyzes optimal taxation in a large country that manipulates its terms of trade under the destination principle for commodity taxation. Consider an economy where there are two countries (a home country and a foreign country) and there are no international transfers between the two countries. The home country follows the setting described in Section 2. We assume that the foreign country imposes no commodity tax and no tariff. The expenditure and revenue functions of the foreign country are given by $e^*(\mathbf{w}, u^*)$ and $r^*(\mathbf{w})$, where u^* is the utility level of a representative consumer in the foreign country. These functions have properties analogous to those of the home country's expenditure and revenue functions. Let $\mathbf{e}^*_{\mathbf{N}} \equiv \partial e^*/\partial \mathbf{w}_{\mathbf{N}}$, $\mathbf{e}^*_{\mathbf{NN}} \equiv \partial \mathbf{e}^*_{\mathbf{N}}/\partial \mathbf{w}'_{\mathbf{N}}$, $e^*_{u^*} \equiv \partial e^*/\partial u^*$, $\mathbf{e}^*_{\mathbf{N}u^*} \equiv \partial \mathbf{e}^*_{\mathbf{N}}/\partial u^*$, $\mathbf{r}^*_{\mathbf{N}} \equiv \partial \mathbf{r}^*/\partial \mathbf{w}_{\mathbf{N}}$, and $\mathbf{r}^*_{\mathbf{NN}} \equiv \partial \mathbf{r}^*_{\mathbf{N}}/\partial \mathbf{w}'_{\mathbf{N}}$.

The budget constraint of the private sector in the foreign country is

$$e^*(\mathbf{w}, u^*) = r^*(\mathbf{w}). \tag{21}$$

The world market-clearing condition is

$$\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \mathbf{e}^*_{\mathbf{N}}(\mathbf{w}, u^*) - \mathbf{r}^*_{\mathbf{N}}(\mathbf{w}) = \mathbf{0}_N. \tag{22}$$

That of good 0 is obtained by Walras' law. Equations (1), (2), (3), (21), and (22) describe the economy.

The following proposition provides the optimal commodity taxes and tariffs in a large country.

Proposition 2. *The optimal commodity taxes and tariffs in a large country are given by*

$$\mathbf{t}'_{\mathbf{N}} = -\alpha(\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}} - \mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}}), \tag{23}$$

$$\boldsymbol{\tau}'_{\mathbf{N}} = -\alpha\mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}} + \boldsymbol{\theta}'_{\mathbf{N}}, \tag{24}$$

where

$$\boldsymbol{\theta}'_{\mathbf{N}} \equiv \beta(\mathbf{e}^*_{\mathbf{N}} - \mathbf{r}^*_{\mathbf{N}})'(\mathbf{e}^*_{\mathbf{NN}} - \mathbf{r}^*_{\mathbf{NN}})^{-1}, \tag{25}$$

and β is a scalar.

Proof. The welfare maximization problem for the home country is

$$\max_{\mathbf{t}_{\mathbf{N}},\, \boldsymbol{\tau}_{\mathbf{N}},\, \mathbf{w}_{\mathbf{N}},\, u,\, u^*} \quad u$$

$$\begin{aligned} \text{s.t.} \quad & e(\mathbf{q}, u) - r(\mathbf{p}) = 0, \\ & e^*(\mathbf{w}, u^*) - r^*(\mathbf{w}) = 0, \\ & \mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] = g, \\ & \mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \mathbf{e}^*_{\mathbf{N}}(\mathbf{w}, u^*) - \mathbf{r}^*_{\mathbf{N}}(\mathbf{w}) = \mathbf{0}_N. \end{aligned}$$

The Lagrangian is

$$\begin{aligned} \Im^{ld} = u &- \mu[e(\mathbf{q}, u) - r(\mathbf{p})] - \pi[e^*(\mathbf{w}, u^*) - r^*(\mathbf{w})] \\ &- \lambda\{\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] - g\} \\ &- \boldsymbol{\psi}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \mathbf{e}^*_{\mathbf{N}}(\mathbf{w}, u^*) - \mathbf{r}^*_{\mathbf{N}}(\mathbf{w})], \end{aligned} \tag{26}$$

where π and $\boldsymbol{\psi}'_{\mathbf{N}} \equiv (\psi_1, \ldots, \psi_n)$ are Lagrangian multipliers. The FOCs with respect to $\mathbf{t}_{\mathbf{N}}$, $\boldsymbol{\tau}_{\mathbf{N}}$, $\mathbf{w}_{\mathbf{N}}$, u, and u^* are, respectively,

$$-\mu\mathbf{e}'_{\mathbf{N}} - \lambda(\mathbf{e}'_{\mathbf{N}} + \mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} + \boldsymbol{\tau}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}}) - \boldsymbol{\psi}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} = \mathbf{0}'_N, \tag{27}$$

$$-\mu(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})' - \lambda[\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} + \mathbf{e}'_{\mathbf{N}} - \mathbf{r}'_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}})] - \boldsymbol{\psi}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}}) = \mathbf{0}'_N, \tag{28}$$

$$\begin{aligned} -\mu(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})' - \pi(\mathbf{e}^*_{\mathbf{N}} - \mathbf{r}^*_{\mathbf{N}})' - \lambda[\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}})] \\ - \boldsymbol{\psi}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}} + \mathbf{e}^*_{\mathbf{NN}} - \mathbf{r}^*_{\mathbf{NN}}) = \mathbf{0}'_N, \end{aligned} \tag{29}$$

$$1 - \mu e_u - \lambda(\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}u} + \boldsymbol{\tau}'_N\mathbf{e}_{\mathbf{N}u}) - \boldsymbol{\psi}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}u} = 0, \tag{30}$$

$$-\pi e^*_{u^*} - \boldsymbol{\psi}'_{\mathbf{N}} \mathbf{e}^*_{\mathbf{N}u^*} = 0. \tag{31}$$

Subtracting (28) from (29), utilizing $\mathbf{e_N} - \mathbf{r_N} = -(\mathbf{e}^*_{\mathbf{N}} - \mathbf{r}^*_{\mathbf{N}})$, and defining $\beta \equiv (\pi + \lambda)/\lambda$, we obtain

$$\frac{\boldsymbol{\psi}'_{\mathbf{N}}}{\lambda} = -\beta(\mathbf{e}^*_{\mathbf{N}} - \mathbf{r}^*_{\mathbf{N}})'(\mathbf{e}^*_{\mathbf{NN}} - \mathbf{r}^*_{\mathbf{NN}})^{-1}. \tag{32}$$

From (25), (27), (32), and $\alpha \equiv (\mu + \lambda)/\lambda$, we have

$$\alpha \mathbf{e}'_{\mathbf{N}} = -(\mathbf{t_N} + \boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}})'\mathbf{e_{NN}}. \tag{33}$$

Subtracting (27) from (28) and utilizing (25) and (32) yields

$$\alpha \mathbf{r}'_{\mathbf{N}} = -(\boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}})'\mathbf{r_{NN}}, \tag{34}$$

which immediately leads to (24). From (33) and (34), we obtain (23).[29] ■

An important and interesting property is that the expression for the optimal commodity tax vector in the large country is identical to that in the small country (see (4) and (10)). This leads to there being no difference in the optimal commodity tax rules between small and large countries, as shown below.

The optimal tariff vector in a large country consists of $-\alpha \mathbf{r}'_{\mathbf{N}} \mathbf{r}^{-1}_{\mathbf{NN}}$ and $\boldsymbol{\theta}'_{\mathbf{N}}$. The former reflects the price-distortion effects in production generated by a revenue constraint, and it is the form identical to the expression for the optimal tariff vector in a small country. The latter represents the optimal tariffs that maximize welfare through improvement in the terms of trade (Bond 1990 and Ogawa 2007b). This can be verified by considering a home country that uses a lump-sum tax L, because this optimal tariff problem arises even without any revenue constraint. In this case, minor changes are made to (2) and (3):

$$e(\mathbf{q}, u) = r(\mathbf{p}) - L,$$

$$\mathbf{t}'_{\mathbf{N}} \mathbf{e_N}(\mathbf{q}, u) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e_N}(\mathbf{q}, u) - \mathbf{r_N}(\mathbf{p})] + L = g.$$

[29] From (30), we have

$$\mu = \frac{1}{e_u} - \lambda \left[\frac{(\mathbf{t_N} + \boldsymbol{\tau}_{\mathbf{N}})'\mathbf{e}_{\mathbf{N}u}}{e_u}\right] - \frac{\boldsymbol{\psi}'_{\mathbf{N}} \mathbf{e}_{\mathbf{N}u}}{e_u},$$

which shows that μ is the social marginal utility of income in a country with monopoly power in trade. The last term on the RHS is the welfare impact through the terms-of-trade effects.

From (31), we have $\pi/\lambda = -\boldsymbol{\psi}'_{\mathbf{N}} \mathbf{e}^*_{\mathbf{N}u^*}/\lambda e^*_{u^*}$. By using $\boldsymbol{\theta}'_{\mathbf{N}} = -\boldsymbol{\psi}'_{\mathbf{N}}/\lambda$, this can be rewritten as

$$\beta \left(\equiv \frac{\pi + \lambda}{\lambda}\right) = \left(\frac{1}{e^*_{u^*}}\right) \left[\mathbf{e}^*_{0u^*} + (\mathbf{w_N} + \boldsymbol{\theta}_{\mathbf{N}})' \, \mathbf{e}^*_{\mathbf{N}u^*}\right],$$

which shows that β is the weighted sum of the foreign country's income effects, where the weight is (1, $\mathbf{w}'_{\mathbf{N}} + \boldsymbol{\theta}'_{\mathbf{N}}$). This corresponds to Equation (8) in Ogawa (2012).

Solving the welfare maximization problem subject to these constraints, (21), and (22), we obtain[30]

$$\boldsymbol{\tau}'_{\mathbf{N}} = \beta(\mathbf{e}^*_{\mathbf{N}} - \mathbf{r}^*_{\mathbf{N}})'(\mathbf{e}^*_{\mathbf{NN}} - \mathbf{r}^*_{\mathbf{NN}})^{-1}. \tag{35}$$

It is readily seen from (25) and (35) that $\boldsymbol{\theta}_{\mathbf{N}}$ represents the optimal tariffs that maximize welfare through the terms-of-trade effects.

Before proceeding further, let us examine the sign of α. There is no guarantee that α is positive in the large-country case, in contrast to the small-country case. It follows from (33) and (34) that

$$\begin{aligned} &\alpha[\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}}) - \boldsymbol{\theta}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})] \\ &\quad = -(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}})'\mathbf{e}_{\mathbf{NN}}(\mathbf{t}_{\mathbf{N}} + \boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}}) + (\boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}})'\mathbf{r}_{\mathbf{NN}}(\boldsymbol{\tau}_{\mathbf{N}} - \boldsymbol{\theta}_{\mathbf{N}}) \\ &\quad > 0, \end{aligned} \tag{36}$$

where the inequality follows from property (iii) of the expenditure and revenue functions. The first two terms in the square brackets, $\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})$, represents the required revenue, which is therefore positive; the third term, $\boldsymbol{\theta}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})$, represents the revenue from the optimal tariffs that maximize welfare through the terms-of-trade effects. The sign of the expression in the square brackets in (36) is ambiguous because of the existence of $\boldsymbol{\theta}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})$, and hence the sign of α is also ambiguous. To avoid this problem, we make the following plausible assumption.

Assumption. $\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}}) > \boldsymbol{\theta}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})$.

This means that the required tax revenue must exceed the revenue from the optimal tariffs that maximize welfare through the terms-of-trade effects. Under this assumption, $\alpha > 0$ from (36).

We can readily expect Corollary 1(a)-(iii), (b)-(iii), and (c)-(iii) to hold even in the large country, because the optimal commodity tax vector in the large country takes a form identical to that in the small country. The following corollary concerns the optimal commodity taxes in the large country.

Corollary 2. *Suppose that the assumption is satisfied. Then the following optimal commodity tax rules hold in a large country: (i)* $\mathbf{t}_{\mathbf{N}} > \mathbf{0}_N$ *if all goods are substitutes for each other in both consumption and production; (ii) when* $e_{ij} = r_{ij} = 0$ *for* $i, j = 1, \ldots, n$ *and* $i \neq j$, $\gamma_i < \gamma_j$ *if* $-\eta_{ii} \geq -\eta_{jj}$ *and* $\sigma_{ii} \geq \sigma_{jj}$ *with at least one strict inequality; and (iii) in the three-good case, when taxed goods are substitutes for each other in production,* $\gamma_i > \gamma_j$ *if* $\eta_{j0} \geq \eta_{i0}$ *and* $-\sigma_{j0} \geq -\sigma_{i0}$ *with at least one strict inequality.*

[30] Replacing the first and third constraints in (26) with the above two conditions, we obtain (27), (28), (29), and $-\mu - \lambda = 0$ as the FOCs with respect to $\mathbf{t}_{\mathbf{N}}$, $\boldsymbol{\tau}_{\mathbf{N}}$, $\mathbf{w}_{\mathbf{N}}$, and L, respectively. These, after some manipulations, lead to (35) and $\mathbf{t}_{\mathbf{N}} = \mathbf{0}_N$.

Proof. Applying the proofs of Corollaries 1(a)-(iii) and (b)-(iii), we can immediately prove (i) and (ii) in this corollary, respectively.

Next, we prove (iii). From (33), we obtain $\kappa_i = (-\eta_{jj} + \eta_{ij})\Psi$ for $i, j = 1, 2$ and $i \neq j$, where $\kappa_i \equiv (t_i + \tau_i - \theta_i)/q_i$. Using this and $\eta_{i0} + \eta_{ii} + \eta_{ij} = 0$, we obtain

$$\kappa_1 - \kappa_2 = (\eta_{20} - \eta_{10})\Psi. \tag{37}$$

From (34), we obtain

$$\nu_i = (-\sigma_{jj} + \sigma_{ij})\Psi, \qquad i, j = 1, 2 \text{ and } i \neq j, \tag{38}$$

where $\nu_i \equiv (\tau_i - \theta_i)/p_i$. This and $\sigma_{i0} + \sigma_{ii} + \sigma_{ij} = 0$ lead to

$$(-\nu_1) - (-\nu_2) = [(-\sigma_{20}) - (-\sigma_{10})]\Lambda. \tag{39}$$

From the definitions of κ_i, γ_i, and ν_i, we obtain $\kappa_i = \gamma_i + (1 - \gamma_i)\nu_i$. This, (37), and (39) yield

$$(\gamma_1 - \gamma_2)(1 - \nu_2) = (\eta_{20} - \eta_{10})\Psi + [(-\sigma_{20}) - (-\sigma_{10})](1 - \gamma_1)\Lambda. \tag{40}$$

From the definition of γ_i, we have $1 - \gamma_i = p_i/q_i > 0$. As $\alpha > 0$ from the assumption, $\Psi > 0$ and $\Lambda > 0$ hold.[31] It follows from (38) that $\nu_i < 0$ as $\sigma_{ij} < 0$ from the substitution condition between taxed goods. Thus, $1 - \nu_i > 0$.[32] As $1 - \gamma_i > 0$, $1 - \nu_i > 0$, $\Psi > 0$, and $\Lambda > 0$, (40) proves (iii). ■

An intuition for Corollary 2 is analogous to that for Corollaries 1(a)-(iii), (b)-(iii), and (c)-(iii). An interesting feature is that the optimal commodity tax rules are independent of the foreign country's elasticities of compensated excess demand, which relates to the terms-of-trade effects. This is because the optimal tariffs are set so that the terms-of-trade effects do not affect the optimal commodity tax structure. The optimal tariff structure consequently depends on the terms-of-trade effects, as discussed below.

Proposition 3 shows that tariffs are required at the optimum, because it is not generally the case that $\alpha \mathbf{r}'_{\mathbf{N}} \mathbf{r}^{-1}_{\mathbf{NN}} = \boldsymbol{\theta}'_{\mathbf{N}}$. As the producer prices differ between the two countries, global production efficiency does not hold in this economy. The sign of the optimal tariff on good i is, however, ambiguous, It is positive when the impact of the tariff on the improvement in the terms of trade is greater than the cost of the price distortions generated by a revenue constraint; i.e., $\alpha \mathbf{r}'_{\mathbf{N}} \mathbf{r}^{-1}_{\mathbf{NN}} \boldsymbol{\varsigma} < \boldsymbol{\theta}'_{\mathbf{N}} \boldsymbol{\varsigma}$, where $\boldsymbol{\varsigma}$ is the n-dimensional vector whose i-th element is 1 and whose other elements are zero.

The relative optimal tariff rates are characterized by the foreign country's elasticities of compensated excess demand, along with the domestic elasticities of supply. We define

[31] See the proof of Corollary 1(c) for the expressions of Ψ and Λ.

[32] This substitution condition in production, which is not required to prove Corollary 1(c)-(iii), is used to satisfy $1 - \nu_i > 0$.

the foreign country's elasticity of compensated excess demand as $\epsilon_{ij} \equiv (e^*_{ij} - r^*_{ij})w_j/(e^*_i - r^*_i)$. To simplify the analysis, we consider a three-good case in which goods 1 and 2 are exported goods for the foreign country and suppose that the goods 1 and 2 are substitutes for each other in both consumption and production in the foreign country and that they have no income effects (i.e., $\mathbf{e}^*_{\mathbf{N}u^*} = \mathbf{0}_N$).[33] When $\mathbf{e}^*_{\mathbf{N}u^*} = \mathbf{0}_N$, (31) yields $\pi = 0$ and hence $\beta = 1$. From homogeneity of the expenditure and revenue functions of the foreign country, we have $\epsilon_{i0} + \epsilon_{ii} + \epsilon_{ij} = 0$ for $i, j = 1, 2$ and $i \neq j$. Using this and $\beta = 1$, (25) leads to

$$\vartheta_1 = -(\epsilon_{20} + \epsilon_{12} + \epsilon_{21})\Xi, \quad \vartheta_2 = -(\epsilon_{10} + \epsilon_{12} + \epsilon_{21})\Xi, \tag{41}$$

where $\vartheta_i \equiv \theta_i/w_i$ and $\Xi \equiv 1/(\epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{12})$. Since $\epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{12} = [(e^*_{11} - r^*_{11})(e^*_{22} - r^*_{22}) - (e^*_{12} - r^*_{12})^2][w_1 w_2/(e^*_1 - r^*_1)(e^*_2 - r^*_2)]$, we have $\Xi > 0$ from $(e^*_1 - r^*_1)(e^*_2 - r^*_2) > 0$ and property (iii) of the expenditure and revenue functions. As goods 1 and 2 are substitutes in both consumption and production, $\epsilon_{ij} < 0$ for $i, j = 1, 2$ and $i \neq j$. From (41), we have $\vartheta_i = (\epsilon_{jj} - \epsilon_{ij})\Xi$ for $i, j = 1, 2$ and $i \neq j$. From this, $\Xi > 0$, and $\epsilon_{ij} < 0$, we have $\vartheta_i > 0$. Thus, $1 + \vartheta_i > 0$ holds under the substitution condition. Note that $\nu_i (\equiv (\tau_i - \theta_i)/p_i) = \phi_i - (1 - \phi_i)\vartheta_i$ as $1 - \phi_i = w_i/p_i$. This yields

$$[(-\phi_1) - (-\phi_2)](1 + \vartheta_1) = [(-\nu_1) - (-\nu_2)] + [(-\vartheta_1) - (-\vartheta_2)](1 - \phi_2).$$

Using (39) and (41), this can be rewritten as

$$[(-\phi_1) - (-\phi_2)](1 + \vartheta_1) = [(-\sigma_{20}) - (-\sigma_{10})]\Lambda + [(-\epsilon_{10}) - (-\epsilon_{20})](1 - \phi_2)\Xi. \tag{42}$$

Note that $1 - \phi_2 = w_2/p_2 > 0$. As $1 + \vartheta_1 > 0$, $\Lambda > 0$ and $\Xi > 0$, (42) shows that $-\phi_i > -\phi_j$ if $-\sigma_{j0} \geq -\sigma_{i0}$ and $-\epsilon_{i0} \geq -\epsilon_{j0}$ with at least one strict inequality. The second condition corresponds to the condition provided in Ogawa (2007b) and (2012), which determines the relative level of the optimal tariff rates that maximize welfare through the terms-of-trade effects.

5 Origin Principle

This section considers origin-based commodity taxes, under which the country that produces the goods imposes commodity taxes, and revenues accrue to that country. With the exception of the origin principle, the models are the same as those described in Sections 2 and 3. Under the origin principle, the difference between the consumer price vector in the home country and that in the foreign country is a tariff vector.[34] Then, (1) is replaced with

$$\mathbf{q} = \boldsymbol{\tau} + \mathbf{w}, \ \ \mathbf{p} = \mathbf{q} - \mathbf{t}, \tag{43}$$

[33] When $\mathbf{e}^*_{\mathbf{N}u^*} = \mathbf{0}_N$, $e^*_{u^*} = e^*_{0u^*}$.

[34] If the home country imposes no tariffs, there is no difference between the consumer's price vector in the home country and that in the foreign country (Lopez-Garcia, 1996).

which shows that a commodity tax under the origin principle is equivalent to a production tax. By replacing $\mathbf{t}'_{\mathbf{N}}\mathbf{e}_{\mathbf{N}}$ on the LHS in (3) with $\mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}}$, the government's budget constraint under the origin principle is given by

$$\mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] \geq g. \tag{44}$$

We can therefore regard the analysis in this section as the analysis of optimal production taxes and tariffs.[35]

It should be noted that the combination of tariffs and commodity taxes under the origin principle leads to the same allocation of resources as under the destination principle, because both the consumer and producer prices are adjustable under the destination and origin principles. Setting $\mathbf{q}$, $\mathbf{p}$, and $\mathbf{w}$ in (43) to be the consumer and producer prices under the destination principle and utilizing (1), we obtain

$$\mathbf{t}^{o}_{\mathbf{N}} = \mathbf{t}^{d}_{\mathbf{N}}, \quad \boldsymbol{\tau}^{o}_{\mathbf{N}} = \mathbf{t}^{d}_{\mathbf{N}} + \boldsymbol{\tau}^{d}_{\mathbf{N}} \ , \tag{45}$$

where $\mathbf{t}^{o}_{\mathbf{N}}$ and $\boldsymbol{\tau}^{o}_{\mathbf{N}}$ ($\mathbf{t}^{d}_{\mathbf{N}}$ and $\boldsymbol{\tau}^{d}_{\mathbf{N}}$) denote the commodity tax and tariff vectors under the origin (destination) principle.[36] Using (45) and Propositions 1 and 2, we immediately obtain the optimal commodity taxes and tariffs under the origin principle.

Proposition 3. *The optimal commodity taxes and tariffs in a small country adopting origin-based commodity taxes are given by*

$$\mathbf{t}'_{\mathbf{N}} = -\alpha(\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}} - \mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}}), \tag{46}$$

$$\boldsymbol{\tau}'_{\mathbf{N}} = -\alpha\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}}, \tag{47}$$

where $\alpha > 0$.

The optimal commodity taxes and tariffs in a large country adopting origin-based commodity taxes are given by

$$\mathbf{t}'_{\mathbf{N}} = -\alpha(\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}} - \mathbf{r}'_{\mathbf{N}}\mathbf{r}^{-1}_{\mathbf{NN}}), \tag{48}$$

$$\boldsymbol{\tau}'_{\mathbf{N}} = -\alpha\mathbf{e}'_{\mathbf{N}}\mathbf{e}^{-1}_{\mathbf{NN}} + \boldsymbol{\theta}'_{\mathbf{N}}, \tag{49}$$

where $\boldsymbol{\theta}'_{\mathbf{N}} \equiv \beta(\mathbf{e}^{*}_{\mathbf{N}} - \mathbf{r}^{*}_{\mathbf{N}})'(\mathbf{e}^{*}_{\mathbf{NN}} - \mathbf{r}^{*}_{\mathbf{NN}})^{-1}$, *and* α *and* β *are scalars.*

[35] Dasgupta and Stiglitz (1974) analyze the revenue-constrained optimal production taxes and tariffs in a small open economy. They do not, however, provide the expression for the optimal production tax vector that corresponds to (46) in the current paper and do not derive the optimal tariff and production tax rules. In addition, they do not treat the large country case.

[36] The first equation in (1) and the second equation in (43) yield $\mathbf{t}^{o}_{\mathbf{N}} = \mathbf{t}^{d}_{\mathbf{N}}$. Given that the consumer and producer price vectors are the same under the destination and origin principles, and hence the amounts of demand and supply are also the same, the world price vector becomes identical for the two principles. Then, from (1) and the first equation in (43), we obtain $\boldsymbol{\tau}^{o}_{\mathbf{N}} = \mathbf{t}^{d}_{\mathbf{N}} + \boldsymbol{\tau}^{d}_{\mathbf{N}}$.

See the Appendix for the proof of this proposition. The expression for the optimal commodity tax vector is identical across all four cases (see (4), (23), (46), and (48)). The optimal commodity tax vector under the origin principle consists of $-\alpha \mathbf{e}'_{\mathbf{N}} \mathbf{e}_{\mathbf{NN}}^{-1}$, in place of the optimal tariffs, and $-\alpha \mathbf{r}'_{\mathbf{N}} \mathbf{r}_{\mathbf{NN}}^{-1}$, which relates to its own price-distortion impact on production. The optimal commodity tax vector in a small country consequently takes an identical expression under the destination and origin principles. In the large-country case, the optimal tariffs are set so that the terms-of-trade effects do not affect the optimal commodity tax structure. Thus, identical forms express the optimal commodity taxes across all four cases.

By contrast, the optimal tariff vector takes different forms depending on the type of country and the nature of the commodity tax system (see (5), (24), (47), and (49)). As the optimal tariff vector is expressed in terms of the substitution terms on either the compensated demand or supply side (wherever the commodity taxes do not yield the price distortions), the optimal tariff vector takes a form that differs between the destination and origin principles. In a large country, the term $\boldsymbol{\theta}_{\mathbf{N}}$, which relates to the terms-of-trade effects, is added to the optimal tariff vector obtained in a small country. Thus, different forms express the optimal tariffs across the four cases.

The identical expression for the optimal commodity tax vector provided in (4), (23), (46), and (48) immediately shows that Corollaries 1(a)-(iii), (b)-(iii), (c)-(iii), and 2 hold even under the origin principle.

Corollary 3(a). *In a small country adopting the origin principle, the following optimal commodity tax rules hold: (i)* $\mathbf{t}_{\mathbf{N}} > \mathbf{0}_N$ *if all goods are substitutes for each other in both consumption and production; (ii) when* $e_{ij} = r_{ij} = 0$ *for* $i, j = 1, \ldots, n$ *and* $i \neq j$, $\gamma_i < \gamma_j$ *if* $-\eta_{ii} \geq -\eta_{jj}$ *and* $\sigma_{ii} \geq \sigma_{jj}$ *with at least one strict inequality; and (iii) in a three-good case,* $\gamma_i < \gamma_j$ *if* $\eta_{i0} \geq \eta_{j0}$ *and* $-\sigma_{i0} \geq -\sigma_{j0}$ *with at least one strict inequality. Under the immediately preceding Assumption, the optimal commodity tax rules (i) and (ii) hold, and (iii) holds when taxed goods are substitutes for each other in production, even in a large country.*

Proof. Applying the proofs of Corollaries 1(a)-(iii), (b)-(iii), (c)-(iii), and 2 to (46) and (48), this proposition is immediately proved. ■

We should note that the optimal production taxes have the same rules as Corollary 3(a) because the origin-based commodity tax is identical to the production tax.

Unlike the optimal tariff rules under destination-based commodity taxes in a small country, the optimal tariff rules under origin-based commodity taxes are evaluated by the elasticities of compensated demand. Applying the proofs of Corollaries 1(a)-(ii), (b)-(ii), and (c)-(ii) to (47), the following corollary immediately results.

Corollary 3(b). *In a small country adopting origin-based commodity taxes, the following optimal tariff rules hold: (i) $\boldsymbol{\tau}_{\mathbf{N}} > \mathbf{0}_N$ if all goods are substitutes for each other in consumption; (ii) when $e_{ij} = 0$ for $i, j = 1, \ldots, n$ and $i \neq j$, $\phi_i \lesseqgtr \phi_j$ if $-\eta_{ii} \gtreqless -\eta_{jj}$; and (iii) in a three-good case, $\phi_1 \gtreqless \phi_2$ if $\eta_{20} \gtreqless \eta_{10}$.*

Unlike Corollary 1(a)-(ii), the optimal tariffs are positive under the substitution condition. It follows from (46) and (47) that $\mathbf{t}'_{\mathbf{N}} - \boldsymbol{\tau}'_{\mathbf{N}} = \alpha \mathbf{r}'_{\mathbf{N}} \mathbf{r}^{-1}_{\mathbf{NN}} > 0$ under the substitution condition, which means that a tax is imposed on production. As a positive tariff is equivalent to a consumption tax in the consumption side, Corollary 3(b) shows that taxes are imposed on consumption and production at the optimum.

6 Conclusion

This paper analyzes the optimal tax mix of commodity taxes and tariffs under a revenue constraint in an open economy. The revenue constraint hedge creates an interaction among the optimal commodity taxes and tariffs, through the price-distortion effects on the common tax base (on either the consumption or production side) and through the terms-of-trade effects. This leads us to expect the optimal tax structure to be very complex and does not lead to useful suggestions for actual policy. This paper, however, provides very simple and intuitive expressions for the optimal commodity tax and tariff vectors, as offered by Propositions 1, 2, and 3. In addition, these expressions yield various useful optimal commodity tax and tariff rules, which are provided in Corollaries 1, 2, and 3.

The results obtained in this paper suggest some useful policy implications. First, this paper expands the possibility of predicting some parts of the optimal commodity tax structure for a country by reference to the elasticities of other countries, because the same optimal commodity tax rules hold irrespective of differences in the country's type (i.e., whether it is a small or large country) and in the commodity taxation system (i.e., whether it follows the destination or origin principle). This is useful for countries that find it difficult to evaluate their own elasticities because of insufficient information, such as developing countries.

Second, our results regarding the optimal tariffs in a small country adopting destination-based commodity taxes suggest a strong recommendation for tariff reductions. International institutions such as the World Bank and the WTO recommend tariff reductions for many countries because they believe that free trade is beneficial for them. However, our finding that optimal tariffs under a revenue constraint are negative implies that a positive tariff creates a larger welfare loss rather than when a zero tariff is optimal. That is, we find stronger support for reducing tariffs in a country facing a revenue constraint, which should result in a more rapid transition to a free trade regime.

Finally, our optimal tax solution provides useful information for piecemeal policy reforms aimed at international coordination. For instance, it is difficult to change the present tax levels, at a stroke, to those levels achieving a Pareto-efficient allocation among countries because of pressure from political organizations and vested-interest groups. Thus, piecemeal policy reforms are a useful prescription. The conditions for welfare-improving piecemeal reforms require information about the initial equilibrium (see, for example, Turunen-Red and Woodland 1990). As the optimal tax solution in this paper may reflect the actual tax structure in the real world rather than merely Pareto-efficient international taxation, it provides useful information in determining the direction for welfare-improving tax and tariff reforms.

Appendix: The Proof of Proposition 3

The case of a small country: by using the constraints (2) and (44), the Lagrangian for welfare maximization is

$$\Im^{so} \equiv u - \mu[e(\mathbf{q}, u) - r(\mathbf{p})] - \lambda\{\mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] - g\}.$$

By noting (43), the FOCs with respect to $\mathbf{t}_{\mathbf{N}}$ and $\boldsymbol{\tau}_{\mathbf{N}}$ are[37]

$$-\mu\mathbf{r}'_{\mathbf{N}} - \lambda(\mathbf{r}'_{\mathbf{N}} - \mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}} + \boldsymbol{\tau}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}}) = \mathbf{0}'_{N},$$

$$-\mu(\mathbf{e}_{\mathbf{N}} - \mathbf{r}_{\mathbf{N}})' - \lambda[\mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}} + \mathbf{e}'_{\mathbf{N}} - \mathbf{r}'_{\mathbf{N}} + \boldsymbol{\tau}'_{\mathbf{N}}(\mathbf{e}_{\mathbf{NN}} - \mathbf{r}_{\mathbf{NN}})] = \mathbf{0}'_{N}.$$

From these, we obtain

$$\alpha\mathbf{r}'_{\mathbf{N}} = (\mathbf{t}_{\mathbf{N}} - \boldsymbol{\tau}_{\mathbf{N}})'\mathbf{r}_{\mathbf{NN}}, \tag{A-1}$$

$$\alpha\mathbf{e}'_{\mathbf{N}} = -\boldsymbol{\tau}'_{\mathbf{N}}\mathbf{e}_{\mathbf{NN}} \ . \tag{A-2}$$

From (A-1) and (A-2), we have (46). Equation (A-2) easily leads to (47). Using the proof of Proposition 1, it is proved that $\alpha > 0$.

The case of a large country: by using the constraints (2), (20), (21), and (44), the Lagrangian for welfare maximization is

$$\begin{aligned}\Im^{lo} \;\equiv\; & u - \mu[e(\mathbf{q}, u) - r(\mathbf{p})] - \pi[e^*(\mathbf{w}, u^*) - r^*(\mathbf{w})] \\ & - \lambda\{\mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \boldsymbol{\tau}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p})] - g\} \\ & - \boldsymbol{\psi}'_{\mathbf{N}}[\mathbf{e}_{\mathbf{N}}(\mathbf{q}, u) - \mathbf{r}_{\mathbf{N}}(\mathbf{p}) + \mathbf{e}^*_{\mathbf{N}}(\mathbf{w}, u^*) - \mathbf{r}^*_{\mathbf{N}}(\mathbf{w})] \ .\end{aligned}$$

Noting (43), the FOCs with respect to $\mathbf{t}_{\mathbf{N}}$, $\boldsymbol{\tau}_{\mathbf{N}}$, and $\mathbf{w}_{\mathbf{N}}$ are[38]

$$-\mu\mathbf{r}'_{\mathbf{N}} - \lambda(\mathbf{r}'_{\mathbf{N}} - \mathbf{t}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}} + \boldsymbol{\tau}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}}) - \boldsymbol{\psi}'_{\mathbf{N}}\mathbf{r}_{\mathbf{NN}} = \mathbf{0}'_{N}, \tag{A-3}$$

[37] The FOC with respect to u does not affect Proposition 3.

[38] The FOCs with respect to u and u^* do not affect Proposition 3.

$$-\mu(\mathbf{e_N}-\mathbf{r_N})'-\lambda[\mathbf{t}'_\mathbf{N}\mathbf{r_{NN}}+\mathbf{e}'_\mathbf{N}-\mathbf{r}'_\mathbf{N}+\boldsymbol{\tau}'_\mathbf{N}(\mathbf{e_{NN}}-\mathbf{r_{NN}})]-\boldsymbol{\psi}'_\mathbf{N}(\mathbf{e_{NN}}-\mathbf{r_{NN}})=\mathbf{0}'_N, \quad \text{(A-4)}$$

$$-\mu(\mathbf{e_N}-\mathbf{r_N})'-\pi(\mathbf{e}^*_\mathbf{N}-\mathbf{r}^*_\mathbf{N})'-\lambda[\mathbf{t}'_\mathbf{N}\mathbf{r_{NN}}+\boldsymbol{\tau}'_\mathbf{N}(\mathbf{e_{NN}}-\mathbf{r_{NN}})] \quad \text{(A-5)}$$
$$-\boldsymbol{\psi}'_\mathbf{N}(\mathbf{e_{NN}}-\mathbf{r_{NN}}+\mathbf{e}^*_\mathbf{NN}-\mathbf{r}^*_\mathbf{NN})=\mathbf{0}'_N.$$

From (22), (A-4) and (A-5), we obtain (32). From (25), (32), and (A-3), we have

$$\alpha\mathbf{r}'_\mathbf{N}=(\mathbf{t_N}-\boldsymbol{\tau}_\mathbf{N}+\boldsymbol{\theta}_\mathbf{N})'\,\mathbf{r_{NN}}. \quad \text{(A-6)}$$

From (A-3) and (A-4), we have

$$\alpha\mathbf{e}'_\mathbf{N}=-\left(\boldsymbol{\tau}_\mathbf{N}-\boldsymbol{\theta}_\mathbf{N}\right)'\mathbf{e_{NN}}, \quad \text{(A-7)}$$

which yields (49). From (A-6) and (A-7), we obtain (48).

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