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Abstract

Using a dynamic efficiency wage model, where a Phillips curve appears because worker morale depends on the unemployment rate and a change in nominal wages, we analyze the effects of fiscal and monetary expansions and of an employment subsidy on unemployment in two steady states. In one steady state, only structural unemployment occurs. In the other, not only structural unemployment but also Keynesian unemployment arises. We find that the effects obtained in the former steady state contrast strongly with those obtained in the latter.

Keywords: Efficiency wage, Phillips curve, Unemployment

JEL Classification Codes: E12, E24

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1 Introduction

Solow (1979), an early and typical contribution to the area of efficiency wage theory, simply considers that worker morale (i.e., labor productivity) is an increasing function of wages. Since the study, however, many researchers have dealt with various factors that influence worker morale, in other words, they have assumed various types of effort functions. For example, Agell and Lundborg (1992) adopt economy-wide unemployment as one of arguments of an effort function and assume that an increase in unemployment causes workers to provide greater effort. Furthermore, several researchers regard past wages as a factor that influences worker morale. Collard and de la Croix (2000) and Danthine and Kurmann (2004) construct dynamic general equilibrium models where worker morale depends on current real wages, past real wages, and the employment level.[1] Campbell (2008) proposes a model where a rise in the unemployment rate and a rise in the ratio of current wages to a reference level, including previous wages, stimulate workers to make greater efforts.[2] Shafir et al. (1997) consider that because of money illusion worker morale is affected not only by current real wages but also by the ratio of current nominal wages to previous nominal wages.

The present paper develops a money-in-the-utility-function model where worker morale (i.e., a worker's effort) is given by an increasing function of the unemployment rate and of the ratio of current nominal wages to previous nominal wages. The idea that an increase in the unemployment rate

[1] See Danthine and Donaldson (1990) and de la Croix et al. (2009) for similar dynamic general equilibrium models.

[2] See also Campbell (2010).

boosts worker morale is adopted by many studies, including the above-cited papers, and there are empirical studies that support this idea (e.g., Blinder and Choi (1990) and Agell and Lundborg (1995, 2003)). At the same time, following Shafir et al. (1997), we assume that because of money illusion an increase in current nominal wages against previous nominal wages induces workers to provide more effort. In this setting, previous nominal wages are used as a reference when workers judge whether they are treated fairly by their employers. This setting is also supported by several empirical studies. Kahneman et al. (1986) and Blinder and Choi (1990) find that money illusion affects people's judgments of fairness, and Shafir et al. (1997) report survey results that it affects people's perception of fairness and, consequently, worker morale. Moreover, Bewley (1999) and Kawaguchi and Ohtake (2007) find that a cut in nominal compensation hurts worker morale.

A firm's profit maximization subject to this effort function gives rise to a Phillips curve, as discussed in Akerlof (1982) and Campbell (2008, 2010). The present paper analyzes unemployment under the sluggish adjustment of nominal wages represented by this Phillips curve and examines the effects of fiscal and monetary expansions in two steady states. In one steady state, the imperfect adjustment of nominal wages is the only cause of unemployment. In this steady state, an increase in the money growth rate raises the rate of change in prices and hence the rate of change in nominal wages, which boosts worker morale and hence labor productivity, which expands employment and production. Meanwhile, an increase in government purchases has no effect on unemployment and crowds out private consumption. This implies that there is no Keynesian unemployment despite the presence of rigidity of nominal wages. Thus, unemployment in this steady state is considered to be structural unemployment.

In the other steady state, not only structural unemployment but also Keynesian unemployment occurs. In this case, following Ono (1994, 2001), we assume that the household's appetite for money is insatiable, in other words, the marginal utility of money has a positive lower bound. Because of this insatiable liquidity preference, households want to save money even by reducing their consumption. This reduction in consumption creates a deficiency of aggregate demand, which leads to unemployment.[3] In contrast to the first steady state, in the second steady state, an increase in government purchases crowds in private consumption and reduces unemployment, whereas an increase in the money growth rate has no effect on the real side of the economy such as unemployment and consumption despite the sluggish adjustment of nominal wages. These effects of fiscal and monetary expansions are consistent with those obtained in the Keynesian liquidity trap (i.e., the case where the LM curve is horizontal in the IS–LM analysis). These effects are also found by Ono and Ishida (2013) in a model where a Phillips curve is endogenously derived, but they do not adopt an efficiency wage model. In their model, therefore, there is no structural unemployment and their mechanism that produces the effects that are consistent with the Keynesian liquidity trap differs somewhat from that of the present paper.

[3]Many recent papers have adopted the idea of Ono (1994, 2001) and have analyzed the deficiency of aggregate demand and unemployment. See for example, Matsuzaki (2003), Hashimoto (2004, 2011), Johdo (2006, 2008a, 2008b), Ono (2006, 2013), Johdo and Hashimoto (2009), Murota and Ono (2010), and Hashimoto and Ono (2011). However, in contrast to the present paper, they assume an exogenous adjustment process of nominal wages, such as a Phillips curve without a microeconomic foundation.

Further, we examine the effects of an employment subsidy on unemployment in the two steady states. The subsidy affects unemployment through shifting the Phillips curve. This is a striking difference from the fiscal and monetary expansions and is the reason why we examine this policy. We also obtain the contrasting effects of an increase in the subsidy on unemployment. It improves unemployment in the steady state with only structural unemployment, whereas its effect is ambiguous in the steady state where structural unemployment and Keynesian unemployment occur. In the latter case, it worsens unemployment if it shifts the Phillips curve substantially.

The remainder of the present paper is organized as follows. Section 2 shows the structure of the model. In section 3, the Phillips curve is derived. Section 4 analyzes the steady state where only structural unemployment arises, and section 5 analyzes the steady state where structural unemployment and Keynesian unemployment arise. Section 6 investigates the effect of the employment subsidy on unemployment in the two steady states. Section 7 concludes the paper.

2 The Model

Following Collard and de la Croix (2000), Danthine and Kurmann (2004), and de la Croix et al. (2009), we construct a dynamic general equilibrium model. In particular, as in de la Croix et al., we introduce the idea of fair wage into a money-in-the-utility-function model. However, there is a key difference between the present model and their models.[4] In the present model, worker

[4]They focus on the business cycle implications of fair wage, whereas the present paper focuses on the effects of fiscal and monetary expansions and of an employment subsidy on structural unemployment and Keynesian unemployment.

morale hinges not upon real wages but upon nominal wages.[5]

2.1 The Household Sector

There is a continuum of identical households, the size of which is normalized to unity. Each household consists of a continuum of identical workers, the size of which is also assumed to be unity. Therefore, the aggregate population size equals unity.

The lifetime utility of a typical household, U, is given by

$$U = \sum_{t=0}^{\infty} \frac{u(c_t) + v(m_t) + n_t \chi(e_t)}{(1+\rho)^t},$$

where ρ (> 0) is the subjective discount rate, $u(c_t)$ is the utility of consumption c_t, $v(m_t)$ is the utility of real money holdings m_t, n_t is the number of employed workers or the fraction of them, and $\chi(e_t)$ is the disutility of the effort e_t provided by an employed worker.[6] As usual, we assume that

$$\begin{aligned} &u'(c_t) > 0, \quad u''(c_t) < 0, \quad u'(0) = \infty, \quad u'(\infty) = 0; \\ &v'(m_t) > 0, \quad v''(m_t) < 0, \quad v'(0) = \infty, \quad v'(\infty) = 0. \end{aligned} \tag{1}$$

[5] Collard and de la Croix (2000) suggest an extension that changes in nominal wages affect worker morale.

[6] To be precise, the period utility of the household is

$$\int_0^1 [u(c_t(j)) + v(m_t(j))]dj + \int_0^{n_t} \chi(e_t(j))dj,$$

where j denotes an index of workers belonging to the household. As in Danthine and Kurmann (2004), we assume that the household chooses aggregate consumption c_t and aggregate money holdings m_t and distributes them equally among the workers. Taking into account the fact that the number of workers is unity, we have $c_t(j) = c_t$ and $m_t(j) = m_t$ for $\forall j$. Moreover, the workers provide the same effort: $e_t(j) = e_t$, because the firms are identical and pay them the same wage: $W_t(j) = W_t$. Therefore, we obtain the following expression:

$$\int_0^1 [u(c_t(j)) + v(m_t(j))]dj + \int_0^{n_t} \chi(e_t(j))dj = u(c_t) + v(m_t) + n_t\chi(e_t).$$

Following Akerlof (1982), Collard and de la Croix (2000), Danthine and Kurmann (2004), Campbell (2006), and de la Croix et al. (2009), we simply assume that the disutility of the effort is given by a quadratic function:

$$\chi(e_t) = -(e_t - \tilde{e}_t)^2, \tag{2}$$

where $\tilde{e}_t$ is the norm of effort. However, the norm $\tilde{e}_t$ depends not on real wages but on nominal wages, and it is given by

$$\tilde{e}_t = e\left(W_t/W_{t-1}^s, 1 - n_t^a\right),$$

where W_t is the nominal wage received by a worker in period t, W_{t-1}^s is the social average of nominal wages in period $t-1$, and n_t^a is the aggregate amount of employment, all of which are taken as given. It satisfies

$$\frac{\partial \tilde{e}_t}{\partial (W_t/W_{t-1}^s)} > 0, \quad \frac{\partial^2 \tilde{e}_t}{\partial (W_t/W_{t-1}^s)^2} < 0; \quad \frac{\partial \tilde{e}_t}{\partial (1-n_t^a)} > 0, \quad \frac{\partial^2 \tilde{e}_t}{\partial (1-n_t^a)^2} < 0. \tag{3}$$

Note that $1 - n_t^a$ is the economy-wide unemployment rate because the aggregate population is unity.

The household faces the following budget constraint:

$$\frac{M_{t+1} - M_t}{P_t} = w_t n_t - c_t - \tau_t, \tag{4}$$

where M_t is nominal money holdings, P_t is a commodity price, w_t ($\equiv W_t/P_t$) is a real wage, and τ_t is a lump-sum tax. Although each worker inelastically supplies his/her one-unit labor endowment, unemployment can arise. Therefore, the number of employed workers is n_t (≤ 1) and the labor income of the household equals $w_t n_t$.

The household maximizes U subject to (4). Taking (2) and $m_t = M_t/P_t$ into account, we obtain the first-order conditions with respect to c_t, m_{t+1}, and e_t:

$$u'(c_t) = \lambda_t,$$
$$\frac{v'(m_{t+1}) + \lambda_{t+1}}{1+\rho} = \lambda_t(1+\pi_{t+1}), \tag{5}$$
$$e_t = \tilde{e}_t = e\left(W_t/W^s_{t-1}, 1-n^a_t\right),$$

where λ_t is the Lagrange multiplier associated with (4) and π_{t+1} ($\equiv (P_{t+1} - P_t)/P_t$) is the rate of change in the price. In addition, the transversality condition is

$$\lim_{t\to\infty} \frac{\lambda_t(1+\pi_{t+1})m_{t+1}}{(1+\rho)^t} = 0. \tag{6}$$

From the first and second equations of (5), we derive

$$(1+\rho)(1+\pi_{t+1})\frac{u'(c_t)}{u'(c_{t+1})} - 1 = \frac{v'(m_{t+1})}{u'(c_{t+1})}, \tag{7}$$

where the left-hand side (LHS) denotes the marginal benefit of spending money to consume a commodity and the right-hand side (RHS) denotes the marginal benefit of saving money. This equation implies that an increase in the rate of change in the price encourages consumption and discourages saving because it lowers the future purchasing power of money or equivalently it increases the cost of holding money.

From (3) and the third equation of (5), we have

$$\begin{aligned} &\frac{\partial e_t}{\partial(W_t/W^s_{t-1})} \equiv e_1 > 0, \quad \frac{\partial^2 e_t}{\partial(W_t/W^s_{t-1})^2} \equiv e_{11} < 0; \\ &\frac{\partial e_t}{\partial(1-n^a_t)} \equiv e_2 > 0, \quad \frac{\partial^2 e_t}{\partial(1-n^a_t)^2} \equiv e_{22} < 0. \end{aligned} \tag{8}$$

Following the partial gift exchange model of Akerlof (1982) and the fair wage-effort hypothesis of Akerlof and Yellen (1990), we discuss the implication of (8). The gift from a firm to a worker is the nominal wage, whereas the gift from the worker to the firm is the effort. Therefore, the more that the firm raises the current nominal wage compared with the previous nominal wage, the more effort the worker provides. Note that the previous nominal wage is used as a reference when the worker judges whether he/she is treated fairly by the firm. Moreover, the worse the employment situation becomes (i.e., the higher the unemployment rate $1-n_t^a$ is), the more the worker appreciates being hired by the firm and paid the wage. That is, the gift from the firm to the worker becomes more valuable. Thus, an increase in unemployment leads to an increase in the effort.

2.2 The Firm Sector

The firm sector is composed of a continuum of identical firms, the size of which is normalized to unity. Each firm produces a homogeneous commodity according to the following linear technology:

$$y_t = e_t n_t^d, \tag{9}$$

where y_t denotes production of the commodity, the effort e_t, given by the third equation of (5), denotes labor productivity, and n_t^d denotes labor input. The firm sets n_t^d and W_t to maximize profits:

$$P_t e\left(W_t/W_{t-1}^s, 1-n_t^a\right) n_t^d - W_t n_t^d,$$

where P_t, W_{t-1}^s, and n_t^a are taken as given. This profit maximization yields

$$e\left(W_t/W_{t-1}^s, 1-n_t^a\right) = \frac{W_t}{P_t}, \quad \frac{P_t e_1\left(W_t/W_{t-1}^s, 1-n_t^a\right)}{W_{t-1}^s} = 1. \tag{10}$$

By eliminating P_t from (10), we obtain the modified Solow (1979) condition:

$$\frac{\left(W_t/W_{t-1}^s\right) e_1\left(W_t/W_{t-1}^s, 1-n_t^a\right)}{e\left(W_t/W_{t-1}^s, 1-n_t^a\right)} = 1. \tag{11}$$

2.3 The Government

The budget equation of the government is

$$\frac{M_{t+1} - M_t}{P_t} + \tau_t = g, \tag{12}$$

where g is government purchases. The nominal money supply is adjusted at a constant rate μ ($> -\rho/(1+\rho)$):

$$\frac{M_{t+1} - M_t}{M_t} = \mu,$$

which implies that real money balances evolve according to

$$\frac{m_{t+1}}{m_t} = \frac{1+\mu}{1+\pi_{t+1}}. \tag{13}$$

3 The Dynamics

Since the households and the firms are identical and the sizes of both equal unity, we obtain

$$W^s_{t-1} = W_{t-1}, \quad n^d_t = n^a_t = n_t \text{ for any } t. \tag{14}$$

From (11) and (14), we find

$$\frac{(W_t/W_{t-1})\, e_1\left(W_t/W_{t-1}, 1-n_t\right)}{e\left(W_t/W_{t-1}, 1-n_t\right)} = 1, \tag{15}$$

which yields W_t/W_{t-1} as a function of $1-n_t$:

$$\frac{W_t}{W_{t-1}} = \psi(1-n_t). \tag{16}$$

Hence, the rate of change in the nominal wage is given by

$$\frac{W_t - W_{t-1}}{W_{t-1}} = \psi(1-n_t) - 1. \tag{17}$$

Following Campbell (2008), we assume that[7]

$$\frac{\partial^2 e_t}{\partial(W_t/W_{t-1})\partial(1-n_t)} \equiv e_{12} < 0.$$

Then, from (8), (15), and (17), we find a Phillips curve, namely, a negative relationship between the rate of change in the nominal wage $(W_t - W_{t-1})/W_{t-1}$ and the unemployment rate $1 - n_t$:

$$\frac{d((W_t - W_{t-1})/W_{t-1})}{d(1-n_t)} = \psi'(1-n_t) = \frac{e_2 - (W_t/W_{t-1})e_{12}}{(W_t/W_{t-1})e_{11}} < 0. \tag{18}$$

This Phillips curve is drawn in Figure 1, which illustrates the case where $\psi(0) - 1$ is positive and $\psi(1) - 1$ is negative. Note that both of them can be positive or negative, depending on the form of $\psi(\cdot)$, i.e., the effort function. This Phillips curve implies the following effect of unemployment on firm behavior. An increase in unemployment extracts greater effort from the workers, so that the firms have less incentive to raise the nominal wage.

From (4), (9), the first equation of (10), (12), (14), and (16), the commodity market equilibrium is

$$c_t + g = y_t = e\left(\psi(1-n_t), 1-n_t\right) n_t, \tag{19}$$

where it is naturally assumed that an increase in employment n_t leads to an increase in production y_t:

$$\frac{dy_t}{dn_t} = e - e_1\psi' n_t - e_2 n_t > 0. \tag{20}$$

From the first equation of (10), (14), and (16), the rate of change in the price π_{t+1} is given by a function of the unemployment rates $1 - n_t$ and $1 - n_{t+1}$:

$$\pi_{t+1} = \frac{\psi(1-n_{t+1})e\left(\psi(1-n_t), 1-n_t\right)}{e\left(\psi(1-n_{t+1}), 1-n_{t+1}\right)} - 1. \tag{21}$$

[7]See Campbell (2008) for the validity of the assumption.

4 Structural Unemployment

In this section, we analyze a steady state where structural unemployment occurs because of the sluggish adjustment of the nominal wage represented by the Phillips curve. From (7), (13), (19), and (21), we obtain

$$\begin{aligned} (1+\rho)(1+\pi^*)-1 &= \frac{v'(m^*)}{u'(c^*)}, \\ \pi^* &= \mu, \\ c^*+g = y^* &= e\left(\psi(1-n^*), 1-n^*\right) n^*, \\ \pi^* &= \psi(1-n^*)-1, \end{aligned} \tag{22}$$

where the asterisk is attached to endogenous variables in this steady state. The last equation of (22) shows that the price as well as the nominal wage obeys the Phillips curve relationship.

From the second and last equations of (22), we have

$$\mu = \psi(1-n^*)-1. \tag{23}$$

From (23), if the money growth rate μ satisfies

$$\psi(1)-1 < \mu \leq \psi(0)-1, \tag{24}$$

then n^* is determined so as to satisfy

$$0 < n^* \leq 1.$$

That is, unemployment (or the unemployment rate) in this steady state is

$$1-n^* \ (\geq 0),$$

and full employment ($n^*=1$) is reached only if $\mu=\psi(0)-1$. Once n^* is obtained, from the third equation of (22), y^* is determined and then c^* $(= y^*-g)$

is determined. Furthermore, from the first, second, and third equations of (22), m^* is determined so as to satisfy

$$(1+\rho)(1+\mu)-1=\frac{v'(m^*)}{u'(y^*-g)}. \tag{25}$$

We summarize the above results in the following proposition.

Proposition 1. *If (24) is valid, there exists the steady state represented by (22).*

Let us examine the effects of expansionary fiscal and monetary policies on unemployment and consumption in order to understand the properties of this steady state. From (18), (20), the third equation of (22), and (23), we obtain the following proposition.

Proposition 2. *In the steady state represented by (22), an increase in the money growth rate μ boosts employment n^* and private consumption c^*, whereas an increase in government purchases g has no effect on employment and completely crowds out private consumption:*

$$\frac{dn^*}{d\mu}=-\frac{1}{\psi'}>0,\quad \frac{dc^*}{d\mu}=\frac{dy^*}{dn^*}\cdot\frac{dn^*}{d\mu}>0;\quad \frac{dn^*}{dg}=0,\quad \frac{dc^*}{dg}=-1<0.$$

The effects of government purchases are the same as those obtained in many New Classical models. This implies that there is no deficiency of aggregate demand despite the presence of the nominal wage rigidity, and that unemployment is not Keynesian but is considered to be structural. Moreover, in contrast to Keynesian economics, an increase in the money growth rate affects employment and consumption not through the demand side but through the supply side as follows. An increase in the money growth rate raises the rate of change in the price π^* and hence the rate of change in the nominal wage $(W_t - W_{t-1})/W_{t-1}$, which enhances labor productivity e, which causes the firms to increase their labor demand. In consequence, employment expands and production increases, which leads to an increase in consumption. Note that if the worker's effort depends not on the nominal wage but on the real wage, then this effect of the monetary expansion disappears, that is, the superneutrality of money holds.

5 Structural Unemployment and Keynesian Unemployment

In this section, we analyze a steady state where aggregate demand determines output, as in Keynesian economics, and where not only structural unemployment but also Keynesian unemployment occurs. We show that if the liquidity preference is insatiable and strong, then aggregate demand becomes insufficient and unemployment becomes worse than $1 - n^*$. For this purpose, we abandon the assumption that $v'(\infty) = 0$ given in (1). Instead, following Ono (1994, 2001), we assume that the household's appetite for money is insatiable, namely, the marginal utility of money has a positive lower bound β as follows:[8]

$$\lim_{m \to \infty} v'(m) = \beta \ (> 0). \tag{26}$$

[8] Ono (1994, chapter 1) discusses the validity of the assumption, quoting statements by Keynes, Marx, and Simmel. Based on recent findings in neuroscience, Ono and Ishida (2013) also argue for its validity. Ono et al. (2004) empirically support the assumption. Murota and Ono (2011) show that the marginal utility of money remains positive in the presence of status preference, while Murota and Ono (2012) show that it reaches a positive lower bound under zero nominal interest rates if the liquidity of deposits is considered.

Because even introducing money into a utility function is often criticized, this assumption may also be criticized. However, this assumption has the great advantage that we are able to analyze the deficiency of aggregate demand and Keynesian unemployment even in a framework where households dynamically optimize their lifetime utility. Due to this assumption, we do not need the conventional Keynesian consumption function, which lacks microeconomic foundations.

If β given in (26) is large enough to satisfy

$$(1+\rho)(1+\mu)-1 < \frac{\beta}{u'(y^*-g)}, \tag{27}$$

then obviously there is no value of m^* that satisfies (25). Thus, the steady state represented by (22) does not exist. Instead, from (7), (13), (19), and (21), we obtain the following steady state:

$$\begin{aligned} (1+\rho)(1+\pi)-1 &= \frac{\beta}{u'(c)}, \\ \lim_{t\to\infty}\frac{m_{t+1}}{m_t} &= \frac{1+\mu}{1+\pi} > 1, \\ c+g &= e\left(\psi(1-n), 1-n\right)n, \\ \pi &= \psi(1-n)-1. \end{aligned} \tag{28}$$

The condition (27) shows that if consumption c takes y^*-g ($= c^*$), the marginal benefit of money exceeds that of consumption even when $m=\infty$. Intuitively, c^* is too much for the household, and the household wants to save more money even by decreasing consumption to less than c^*. Therefore, in this steady state, a deficiency of consumption arises ($c < c^*$), which worsens unemployment ($n < n^*$). Moreover, this deficiency of consumption depresses the rate of change in the price ($\pi < \pi^* = \mu$), which causes real money balances to persistently expand.

Let us prove the existence of this steady state. From the third and fourth equations of (28), n and π are expressed as functions of c and g:

$$n = n(c; g), \quad \pi = \pi(c; g) = \psi(1 - n(c; g)) - 1. \tag{29}$$

They satisfy

$$n(c^*; g) = n^*, \quad \pi(c^*; g) = \pi^* = \mu, \tag{30}$$

where c^*, n^*, and π^* are the values given in (22). In addition, they satisfy

$$\frac{\partial n}{\partial c} = \frac{\partial n}{\partial g} = \frac{1}{e - e_1\psi' n - e_2 n} > 0, \quad \frac{\partial \pi}{\partial c} = \frac{\partial \pi}{\partial g} = -\frac{\psi'}{e - e_1\psi' n - e_2 n} > 0, \tag{31}$$

where the inequalities are obtained from (18) and (20). Substituting the second equation of (29) into the first equation of (28) yields

$$(1 + \rho)[1 + \pi(c; g)] - 1 = \frac{\beta}{u'(c)}. \tag{32}$$

If (27) is valid, the LHS of (32) is smaller than the RHS when $c = y^* - g$ $(= c^*)$. Therefore, if the LHS is larger than the RHS when $c = 0$:

$$(1 + \rho)[1 + \pi(0; g)] - 1 > 0, \tag{33}$$

then the value of c satisfying (32) lies between 0 and c^*. Furthermore, if the slope of the LHS is smaller than that of the RHS at the value of c satisfying (32):

$$(1 + \rho)\frac{\partial \pi}{\partial c} < -\frac{\beta u''}{(u')^2}, \tag{34}$$

then the value of c is uniquely determined. We denote it by $\tilde{c}$, as illustrated by Figure 2. Then, from (29), the values of n and π are uniquely determined:

$$\tilde{n} = n(\tilde{c}; g), \quad \tilde{\pi} = \pi(\tilde{c}; g). \tag{35}$$

Since $\tilde{c} < c^*$, from the second equations of (30), (31), and (35), as mentioned above, the rate of change in the price is lower than the money growth rate:

$$\tilde{\pi} < \pi^* = \mu,$$

and real money balances continue to expand. Therefore, from the first equation of (5), (6), and (13), in order for the transversality condition to be satisfied:

$$\lim_{t\to\infty} \frac{\lambda_t(1+\pi_{t+1})m_{t+1}}{(1+\rho)^t} = u'(\tilde{c})(1+\mu) \lim_{t\to\infty} \frac{m_t}{(1+\rho)^t} = 0,$$

real money balances must expand at a rate less than ρ, namely, the money growth rate μ must be low enough to satisfy

$$\frac{1+\mu}{1+\tilde{\pi}} < 1+\rho. \tag{36}$$

We summarize the above results in the following proposition.

Proposition 3. *If (27), (33), (34), and (36) are valid, there exists the steady state characterized by (28).*

We now describe the properties of this steady state. As mentioned above, when (27) is valid, the household wants to save money even by reducing consumption to less than c^*. In consequence, consumption is reduced to $\tilde{c}$ ($< c^*$) so that (32) holds (the marginal benefit of money equals that of consumption), and this deficiency of consumption worsens unemployment. From the first equations of (30), (31), and (35), we indeed find that $\tilde{n} < n^*$, i.e.,

$$1 - \tilde{n} > 1 - n^*.$$

Unemployment in this steady state, $1-\tilde{n}$, is the sum of structural unemployment caused by the imperfect adjustment of the nominal wage, $1-n^*$, and Keynesian unemployment caused by the deficiency of consumption, $n^* - \tilde{n}$. Note that there is unemployment in this steady state even when there is no structural unemployment ($n^* = 1$).

Moreover, as shown by the fourth equation of (28), the rate of change in the price $\tilde{\pi}$ is not affected by the money growth rate μ and it depends only on the shape of the Phillips curve. This implies that $\tilde{\pi}$ can be positive or negative independently of μ. Therefore, as recently seen in Japan, deflationary stagnation can occur even when money is expanded.[9]

To understand further the properties of this steady state, we explore the effects of fiscal and monetary expansions. From (31), (32), and (34), we find the following effects of fiscal and monetary expansions consistent with those obtained in the Keynesian liquidity trap (i.e., the case where the LM curve is horizontal in the IS–LM analysis).

Proposition 4. *In the steady state characterized by (28), an increase in government purchases g increases consumption $\tilde{c}$ and employment $\tilde{n}$, whereas an increase in the money growth rate μ has no effect on them:*

$$\frac{d\tilde{c}}{dg} = \frac{(1+\rho)\partial\tilde{\pi}/\partial g}{-[\beta u''/(u')^2] - (1+\rho)\partial\tilde{\pi}/\partial\tilde{c}} > 0, \quad \frac{d\tilde{n}}{dg} = \frac{\partial\tilde{n}}{\partial\tilde{c}} \cdot \frac{d\tilde{c}}{dg} + \frac{\partial\tilde{n}}{\partial g} > 0;$$

$$\frac{d\tilde{c}}{d\mu} = 0, \quad \frac{d\tilde{n}}{d\mu} = 0.$$

An increase in g directly creates employment, which raises the rate of change in the nominal wage and hence the rate of change in the price along

[9] Japan has experienced a long-lasting stagnation, called the Lost Decade or now the Lost Two Decades, since the 1990s. During this period, deflation continued even though the monetary base was increased. See Murota and Ono (2012) for the deflation and the monetary expansion in the Japanese stagnation.

the Phillips curve. This increases the cost of holding money, and therefore consumption is stimulated and further employment is created.[10] Note that if g is large enough to violate (27), Keynesian unemployment is eliminated and the economy can reach the steady state of the preceding section. Meanwhile, although the adjustment of the nominal wage is sluggish, an increase in μ has no effect on the real side of the economy such as unemployment and consumption. This is because the real balance effect does not work. Even if real money holdings increase, the appetite for money does not diminish (the marginal utility of money remains at β) and thus the appetite for consumption is not stimulated. However, if μ is increased so much that (27) and (36) are violated, then the economy can move from the steady state of section 5 to that of section 4 and unemployment can decrease from $1 - \tilde{n}$ to $1 - n^*$.

6 An Employment Subsidy

This section analyzes the effects of an employment subsidy on unemployment in the two steady states. In contrast to the fiscal and monetary expansions, it affects unemployment through shifting the Phillips curve. When it is considered, the profit that a typical firm seeks to maximize is

$$P_t e(W_t/W_{t-1}^s, 1 - n_t^a)n_t^d - W_t n_t^d + P_t z n_t^d,$$

[10]Since an increase in g crowds in private consumption, a multiplier-like effect arises:

$$\frac{d\tilde{y}}{dg} = \frac{d\tilde{c}}{dg} + 1 > 1,$$

where $\tilde{y}$ is output in this steady state. This multiplier mechanism works not through an increase in disposable income but through an increase in the rate of change in the price. See Murota and Ono (2010) for the detail of this mechanism.

where z denotes the subsidy in real terms.[11] The profit maximization yields

$$e\left(W_t/W^s_{t-1}, 1-n^a_t\right) + z = \frac{W_t}{P_t}, \quad \frac{P_t e_1\left(W_t/W^s_{t-1}, 1-n^a_t\right)}{W^s_{t-1}} = 1. \tag{37}$$

The first equation of (37) shows that an increase in z is taken as an increase in marginal productivity of labor. Alternatively, it can be viewed as a decrease in the marginal cost of labor if the equation is arranged as follows:

$$e\left(W_t/W^s_{t-1}, 1-n^a_t\right) = \frac{W_t}{P_t} - z.$$

From (37), the modified Solow condition is rewritten as

$$\frac{(W_t/W^s_{t-1})e_1\left(W_t/W^s_{t-1}, 1-n^a_t\right)}{e\left(W_t/W^s_{t-1}, 1-n^a_t\right) + z} = 1. \tag{38}$$

From (14) and (38), W_t/W_{t-1} is given by a function of $1-n_t$ and z:

$$\frac{W_t}{W_{t-1}} = \phi(1-n_t; z).$$

It satisfies

$$\begin{aligned} \frac{\partial(W_t/W_{t-1})}{\partial(1-n_t)} &\equiv \phi_1 = \frac{e_2 - (W_t/W_{t-1})e_{12}}{(W_t/W_{t-1})e_{11}} < 0, \\ \frac{\partial(W_t/W_{t-1})}{\partial z} &\equiv \phi_2 = \frac{1}{(W_t/W_{t-1})e_{11}} < 0, \end{aligned} \tag{39}$$

where the inequalities are obtained from (8) and (18). Thus, a Phillips curve is also obtained but its shape depends on the subsidy z. As shown by the second property of (39), an increase in z shifts the Phillips curve downward (see Figure 3). This shift implies the following influence of the subsidy on

[11] In this case, the budget equation of the government is

$$\frac{M_{t+1} - M_t}{P_t} + \tau_t = g + zn^d_t.$$

firm behavior. An increase in z works like an increase in labor productivity. Therefore, it is less important for the firms to induce worker effort, which means that the firms are reluctant to raise the nominal wage. Note that in the steady states, where the price changes in synchronization with the nominal wage, a price Phillips curve also holds:

$$\pi = \phi(1 - n; z) - 1. \tag{40}$$

We first examine the effect of the employment subsidy in the steady state with only structural unemployment. In this steady state, from the second equation of (22) and (40), the unemployment rate $1 - n$ is determined by

$$\mu = \phi(1 - n; z) - 1.$$

By differentiating this equation and taking (39) into account, we obtain the following proposition.

Proposition 5. *In the steady state with only structural unemployment, an increase in an employment subsidy improves unemployment:*

$$\frac{dn}{dz} = \frac{\phi_2}{\phi_1} > 0.$$

This result simply arises as follows. An increase in the subsidy works like a reduction in the marginal cost of labor, so that the firm's demand for labor increases and unemployment decreases.

We next examine the effect in the steady state with both structural unemployment and Keynesian unemployment. To begin with, we show that n is expressed as a function of z in this steady state. When the subsidy is considered, instead of (19), the following equation holds:

$$c + g = y = e(\phi(1 - n; z), 1 - n)n, \tag{41}$$

where it is also assumed that $dy/dn > 0$:

$$e - e_1\phi_1 n - e_2 n > 0. \tag{42}$$

From (41), n is expressed as a function of c and z:

$$n = n(c; z). \tag{43}$$

From (40) and (43), π is also expressed as a function of c and z:

$$\pi = \pi(c; z) = \phi(1 - n(c; z); z) - 1. \tag{44}$$

Therefore, (32) is rewritten as follows:

$$(1 + \rho)[1 + \pi(c; z)] - 1 = \frac{\beta}{u'(c)}, \tag{45}$$

which implies that c is given by a function of z: $c = c(z)$. Substituting it into (43) yields n as a function of z:

$$n = n(c(z); z).$$

Differentiating this equation with respect to z, we find

$$\frac{dn}{dz} = \frac{\partial n}{\partial c} \cdot \frac{dc}{dz} + \frac{\partial n}{\partial z}. \tag{46}$$

Let us explore the sign of dn/dz. Using (41) and taking (8), (39), and (42) into account, we derive

$$\frac{\partial n}{\partial c} = \frac{1}{e - e_1\phi_1 n - e_2 n} > 0, \quad \frac{\partial n}{\partial z} = -\frac{e_1\phi_2 n}{e - e_1\phi_1 n - e_2 n} > 0,$$

where the first equation implies that an increase in consumption creates employment, and the second one implies that since an increase in the subsidy lowers labor productivity through negatively affecting the rate of change in the nominal wage, more labor is needed for producing a given amount of the commodity. By totally differentiating (45), we obtain

$$\frac{dc}{dz} = \frac{(1+\rho)\partial\pi/\partial z}{-[\beta u''/(u')^2] - (1+\rho)\partial\pi/\partial c},$$

where from (34) the denominator on the RHS is positive and from (44) $\partial\pi/\partial z$ is

$$\frac{\partial\pi}{\partial z} = -\phi_1\frac{\partial n}{\partial z} + \phi_2.$$

The first term $-\phi_1(\partial n/\partial z)$ shows that the increase in n caused by an increase in z positively affects π along the Phillips curve, whereas the second term ϕ_2 shows that an increase in z negatively affects π through shifting the Phillips curve downward. Thus, the sign of $\partial\pi/\partial z$ is ambiguous. However, if the negative effect dominates the positive effect, the total effect on π is negative:

$$\frac{\partial\pi}{\partial z} < 0.$$

This reduction in π urges the household to save money, which leads to a decrease in consumption:

$$\frac{dc}{dz} < 0. \tag{47}$$

From (46), if this decrease in consumption is sufficiently large, the total effect of an increase in z on employment is negative. We summarize these results in the following proposition.

Proposition 6. *In the steady state with structural unemployment and Keynesian unemployment, the effect of an increase in an employment subsidy z on unemployment is ambiguous. However, if the negative effect given in (47)* *is sufficiently large, then it worsens unemployment:*

$$\frac{dn}{dz} = \frac{\partial n}{\partial c} \cdot \frac{dc}{dz} + \frac{\partial n}{\partial z} < 0.$$

7 Concluding Remarks

We develop a money-in-the-utility-function model where a worker's effort depends positively on the unemployment rate and on a change in nominal wages. We show that the firm's profit maximization subject to this effort function gives rise to a Phillips curve, and we analyze the effects of fiscal and monetary expansions under the sluggish adjustment of nominal wages represented by this Phillips curve in two steady states.

In the steady state where only structural unemployment occurs, an increase in the money growth rate reduces unemployment, whereas an increase in government purchases has no effect on unemployment and crowds out private consumption. In contrast, in the steady state where not only structural unemployment but also Keynesian unemployment arises, an increase in government purchases reduces unemployment and crowds in private consumption, whereas an increase in the money growth rate has no effect on unemployment and consumption.

Furthermore, we obtain the contrasting effects of an increase in an employment subsidy on unemployment. It improves unemployment in the steady state with only structural unemployment. However, in the steady state with both structural unemployment and Keynesian unemployment, it may produce an unintended consequence. If it shifts the Phillips curve substantially downward, it depresses consumption and aggravates unemployment. Thus, we conclude that when Keynesian unemployment occurs, "creating" employment by government purchases is more effective and helpful for reducing unemployment than "promoting" employment by an employment subsidy.

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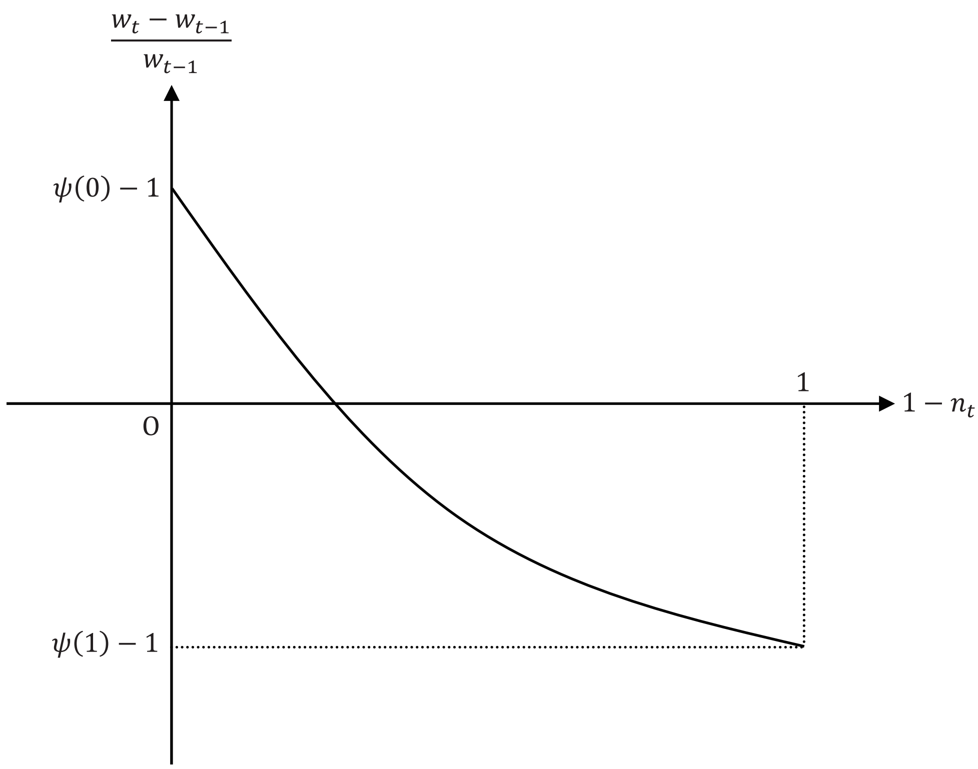

Figure 1: A Phillips curve

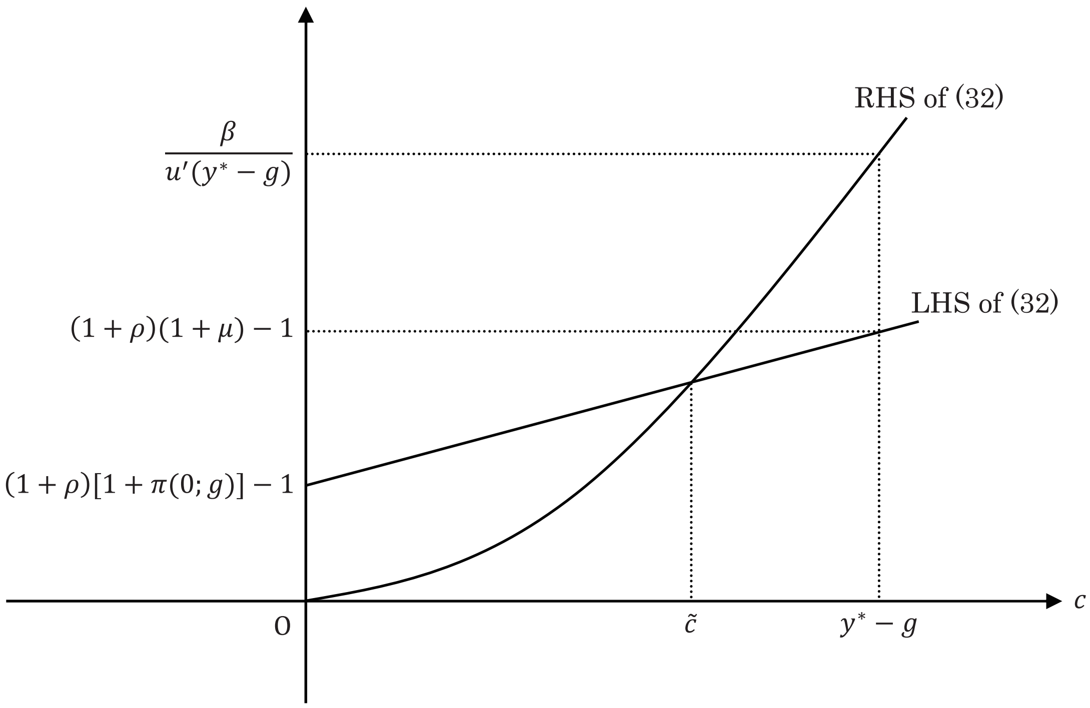

Figure 2: The existence of a unique value of c that satisfies (32)

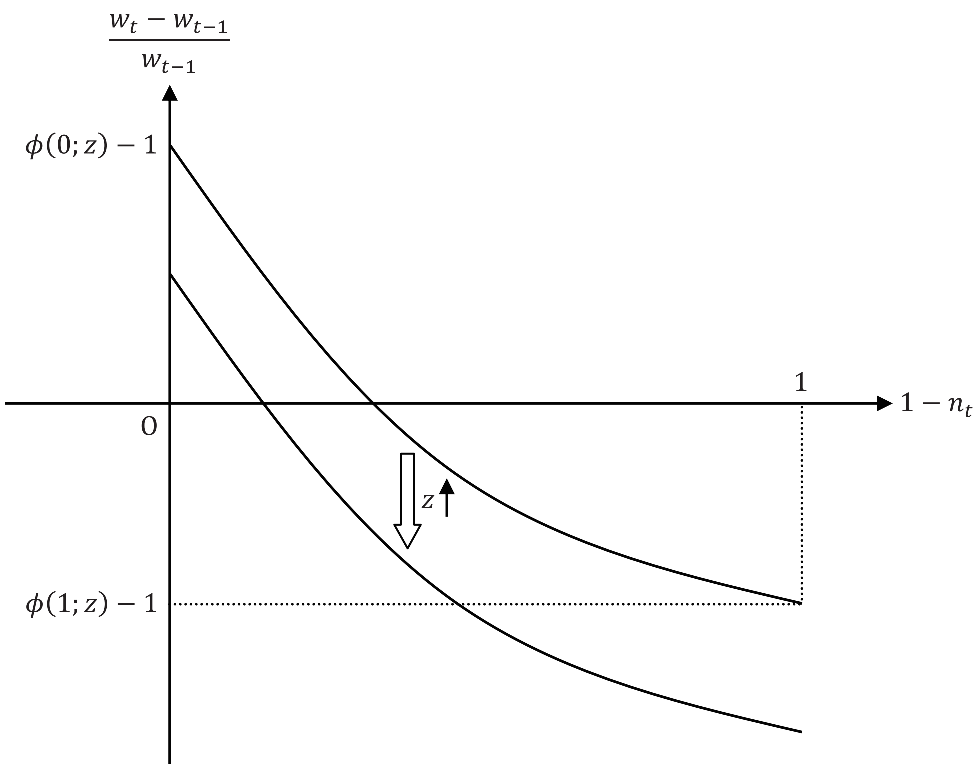

Figure 3: The effect of an increase in z on a Phillips curve