Working Paper Series　No.E-23

Financial Contagion through the Repo

Mami Kobayashi

February, 2012

Faculty of Economics, Kinki University
3-4-1 Kowakae, Higashi-Osaka, Osaka 577-8502, Japan.

近畿大学経済学部　〒577-8502　東大阪市小若江 3-4-1

Financial Contagion through the Repo*

Mami Kobayashi †
Associate Professor, Faculty of Economics, Kinki University

February, 2012

*This research was supported by a Grant-in-Aid for Scientific Research (C) No. 21530179.

†Correspondence: Mami Kobayashi, Faculty of Economics, Kinki University, 3-4-1, Kowakae, Higashi-osaka city, Osaka, 577-8502, Japan. Tel: +81-6-6721-2332. E-mail address: mm0kbys@pb3.so-net.ne.jp. Back-up e-mail address: mmkbys@kindai.ac.jp.

Financial Contagion through the Repo

Abstract

One of the important roles of the banking system is to undertake borrowing similar to demand deposits. Modern banks undertake such debts by pledging their holding assets in a secured short-term borrowing known as repurchase agreements (repos). We show that repos expose a bank to a risk that does not directly affect its fundamental value. We also show that a future liquidity crisis instigated by the bankruptcy of banks induces a bank to sell its holding asset at a fire sale price, even though that crisis does not necessarily affect the fundamental value of the asset. Our results provide theoretical explanations for, and shed new light on, the observations and findings of the 2007–2009 financial crisis. We also analyze the policies employed during the crisis.

Keywords Financial contagion; Repo; Fire sale; Credit ratings; Debt rollover.
JEL Codes G01, G21, G24, G32, G38.

1 INTRODUCTION

One of the important roles of the banking system is risk transformation. Banks finance risky investing in assets and projects with *information-insensitive* debt (referred by Holmstrom (2008), Dang et al. (2009), and Gorton (2010)) which makes creditors immune to the creditworthiness of the banks. Traditional demand deposits are the leading example of such a debt, because deposit contracts under deposit insurance protect depositors from the bankruptcy of banks, so that the value of deposits is then insensitive to information on the creditworthiness of the banks. Throughout this paper, we define information-insensitive debt as bank debt which pays creditors the predetermined returns regardless of a bank's creditworthiness. [1] [2] Growing numbers of institutional depositors, including banks, hedge funds, and insurance companies, are purchasing information-insensitive debt through money markets, especially in a *repo* (sale and repurchase agreement) market. In this paper, we show that trading in repos may work as a vehicle for spreading market shocks to a bank that was not itself directly affected by the shock.

A repo is a secured short-term loan that is defined as the sale of an asset under agreement on repurchase at a predetermined interest rate (i.e., repo rate) on the specified and usually fixed maturity date.[3] A repo is regarded as information insensitive, because the creditors receive the predetermined returns regardless of the creditworthiness of the bank. A massive increase in securitization by the pooling and tranching of loans and mortgages since the late 1990s enabled banks to pledge these securities as collateral assets in repo transactions.[4] [5]

However, to the extent that repo trading requires marketable assets as collateral, the information-insensitive feature of the repo trading may transfer the crisis in the markets to a bank through increasing its borrowing costs. If the realized crisis is more severe than

[1]We owe this definition of information-insensitive debt to Gorton (2010).

[2]Traditional theory implies that issuing of debt similar to demand deposits stems from the banks ' role in protecting their creditors from speculation associated with systemic shock to the financial system. See Gorton and Pennacchi (1990) and Gorton (2010). For a rationalization of banks ' acceptance of demand deposits, see Jacklin (1987), and for the role of the banking system in general, see Gorton and Winton (2003).

[3]See Choudhry (2006) and Krishnamurthy (2010) for more detail on the repo.

[4]For details of the securitization, see Brunnermeier (2009), Jaffee et al. (2009).

[5]The modern banking system with securitization is denoted in various ways. Gorton and Metrick (2011) denote a banking system with securitization and repo trading backed by the securitized assets as securitized banking. The securitized risky assets are often segregated from the original balance sheets of the financial institutions. Coval et al. (2009), Shin (2009), Pozsar et al. (2010), and Gennaioli et al. (2011) focus more on such a feature and denote it as shadow banking. Shleifer and Vishny (2010a) argue that securitization makes a bank fragile to investor sentiment and denote that as unstable banking.

expected, the credit risk of the bank increases because the increase in the cost of debt finance after the crisis makes repayment of the existing debt more difficult. Moreover, if the crisis is expected to be amplified in future, the bank could do better by delevering itself by liquidating the asset even for a price that is below its fundamental value, rather than rolling over the existing debt. We argue that a modern banking system characterized by repo trading and securitization may spread financial shocks even to a bank that was not directly connected to the shock. The purpose of this paper is to analyze the mechanism of the *financial contagion*, by which we mean propagation of the financial shock, by formally building a repo trading model.

To this purpose, we develop a three-date (Date 0, 1, 2) model of a financial institution (henceforth referred as a "bank"). The bank invests the security asset whose value is uncertain until Date 2, but its fundamental value is constant throughout the course of the project. We focus on the role of the bank in taking on information-insensitive debt, which in principle provides predetermined returns to lenders regardless of the solvency of the bank. At date 1, a crisis hits the economy, and the interbank rate increases. If the bank decides to let the creditors withdraw, the bank delevers by liquidating the asset for a price in the asset market. Alternatively, if the bank decides to roll over the existing debt, it keeps providing the information-insensitive debt through the repo by pledging the asset as collateral. The bank chooses either asset liquidation or debt rollover, whichever is cheaper at Date 1. Anticipating the choice at Date 1, the bank decides its leverage ratio at Date 0.

Beside its information-insensitive debt, the bank can also raise its own less rigid capital. Such capital is a substantial long-term claim. Once investors provide capital to a bank, that capital is held at the bank's discretion and the investors can neither renew the contract nor withdraw it during the project's term. In exchange for transferring the control right of the funds to the bank, investors can seek and receive a premium. Because a premium makes the capital costly, the bank chooses its investments by facing the trade-off between rigid but less costly debt and less rigid but more costly capital.[6] Throughout our analysis, we assume that all investors enter the market for providing funds to the bank competitively. The competitive entrance of the investors allows the bank to decide its leverage ratio for maximizing its own surplus.

Under the setting described above, we show that the bank decides its leverage ratio as

[6]As to the trade-off between a demand deposit like debt and capital, we follow the setting in Diamond and Rajan (2000).

a decreasing function of the expected interbank rate. Because the information-insensitive creditors stay in the bank to the extent that they gain more than the interbank lending, the bank incurs the higher cost of the debt finance the higher the interbank rate is. Thus the prospect of increases in the future interbank rate induces the bank to decrease its leverage ratio. Moreover, because the repo makes the returns of the creditors indifferent regardless of the solvency of the bank, the increase of the interbank rate induces the creditors to require more collateral. However, banks are more fragile the more assets they pledge as repo collateral. Therefore, credit risk is derived endogenously as an increasing function of the interbank rate.

In the benchmark model (Section 4.1), the interbank rate realized at Date 1 is known at Date 0. Because the expected interbank rate is equal to the Date-1 interbank rate, the debt contract is equivalent to the long-term contract under which the bank commits to roll over the debt under the Date-1 interbank rate. The bank decides the leverage ratio as decreasing in the Date-1 interbank rate. Because the realized interbank rate is known at Date 0, the ex ante credit risk is the same as the ex post one and is increasing in the Date-1 interbank rate.

In contrast, if the Date-1 interbank rate is uncertain and only its distribution is known at Date 0 (Section 4.2), we show that the bank decides its leverage ratio as the decreasing function of the expected value of the Date-1 interbank rate. Because of the uncertainty in the future interbank rate, we show that the ex post credit risk of the bank is higher than the ex ante one, if the interbank rate increases more than expected.

So far, we have assumed that the collateral asset is traded for its fundamental value at Date 2, regardless of the solvency of the bank engaged in repo trading. However, bankruptcy of the borrowing bank may reduce the market price of the collateral asset. For example, bankruptcy of the bank may trigger the liquidity crisis, where the potential bidders in the Date-2 asset market refrain from providing liquidity. In fact, the true value of the collateral asset is hardly known by market participants, even though repo trading requires backing by assets with only a high credit rating in practice. This is because even though the market participants know that collateral assets are AAA-rated securities, they cannot distinguish whether or not these assets are "virtually" safe securities that are synthesized by tranching the pooled portfolios of loans and mortgages. If the bank becomes bankrupt, especially during a downturn of the prices of loans and mortgages, the participants of the asset market may abstain from providing liquidity for fear of further deterioration of the quality of the assets. Following the mark-to-market pricing model in Brunnermeier and Pedersen (2009) and Adrian and Shin (2010), the evaporation of market liquidity reduces the market price of the asset at Date 2. Alternatively, creditors are subject to the risk of receiving less of the collateral if the bank pledges the security asset that the other investors lend to the bank in exchange for its cash borrowing. Pledging the borrowed security as a collateral is referred to as *rehypothecation*, which is ubiquitous in repo transactions. According to Singh and Aitken (2009a), investors who lend securities to banks are subject to the risk of losing the securities if the banks become bankrupt. This is because if the banks engage in rehypothecation by using the investors' securities as collateral in repo trading, repo creditors can seize their securities in case of the bankruptcy of the banks. In order to avoid such a situation, investors may seek to retrieve their securities from the bankrupted bank before the repo creditors seize collateral. As a result, the repo creditors may receive less when the bank becomes bankrupt, as in the case of the decline in the market price of the collateral asset due to the liquidity crisis.

Analyzing the extended model (Section 5), we show that the reduction in the price of the collateral asset in the Date-2 asset market increases the cost of the debt finance to the bank, because information-insensitive creditors require more collateral to guard against bankruptcy of the bank. Pledging more collateral makes the rollover of the existing debt more costly, which induces the bank to delever by liquidating the asset in the Date-1 market. Because the market price of the asset is discounted by the Date-1 interbank rate, the bank responds by liquidating the asset for a price that is below its fundamental value, or a *fire-sale price*.

Our results provide theoretical support for, and shed new light on, the empirical findings and observations of the 2007–2009 financial crisis. First, our results provide a theoretical explanation of the path of the financial contagion observed in the 2007–2009 financial crisis. Starting from the downturn of the U.S. housing market in 2007, the deterioration in prices of the securities that are associated with the subprime loans and mortgages spread to the non-subprime-related securities that are supposed not to be directly connected to the deteriorated housing market. Gorton and Metrick (2011) found in their empirical research that the decline in the price of the non-subprime-related assets is highly correlated with the increase in the counterparty risk that is represented by the LIB-OIS spread, rather than the increase in the risk of the subprime-related securities.[7] According to their argument, the precipitation in the price of subprime-related securities in August 2007 increased the counterparty risk

[7] The LIB-OIS (or LIBOR-OIS) spread is the spread between the London Interbank Offered Rate (LIBOR) and Overnight Index Swap (OIS); it represents the counterparty risk in the interbank market.

among banks. Particularly, because the prevailing off-balancing technologies made the true amount of the damaged assets held and supported by the banks unclear, increasing doubt about the solvency of banks in the interbank market precipitated reduced "willingness to provide liquidity" to the market. The empirical findings of Afonso et al. (2011) confirm that there was a drying up of liquidity during the financial crisis that was caused mainly by increased counterparty risk. Although Gorton and Metrick (2011) suggest that the repo market works as a nexus of spreading the crisis to the non-subprime-related securities, the mechanism for the spread of the financial crisis to the non-subprime-related securities has not so far been determined. By formally modeling repo trading, our results provide the missing link to the path of the financial contagion; the information-insensitive feature of the repo trading transfers the increased counterparty risk in the interbank market to the collateral asset pledged by a bank that was not directly connected with the risk that increased the counterparty risk.

Second, our results shed new light on the sudden increase in the credit risk of financial products including the collateral assets observed in the 2007–2009 financial crisis. The sudden fall in security prices during the financial crisis generated criticism of the credit rating agencies (CRAs), because they failed to warn investors of their looming risks by timely downgrading of these securities.[8] The existing literature, such as White (2002), pointed out that the latent agency problem between CRAs and client firms may inflate the credit ratings of the firms.[9] In contrast to the agency-problem view of the credit rating system as above, our results suggest that as long as the information-insensitive feature of the repo trading transfers the counterparty risk to the bank, the credit risk of the bank increases ex post if the counterparty risk increases more than expected. In particular, if the credit rating provided by the CRAs is less frequent than the change in the counterparty risk in the market, the credit rating is likely to fail in incorporating the rise in the counterparty risk to the credit risk of the bank. Our results suggest that the ex ante distribution of the future counterparty risk affects the occurrence of the sudden increase in the credit risk. If the probability of the increase in the counterparty risk is estimated to be lower under the ex ante distribution of the counterparty risk than under the true one, credit risk is more likely to increase sharply ex post. This argument is in a similar vein to the idea in Gennaioli et al. (2011) that if certain unlikely

[8]Benmelech and Dlugosz (2010) found in their empirical research that the financial institutions that held the AAA-rated securities made significant losses during the financial crisis.

[9]The latent problem of the credit rating system is elaborated in Section 4.2.2.

risk is neglected, financial fragility increases.

Lastly, our results suggest that although the interbank borrowing condition is the key to the path of the financial contagion, relaxing tension in the interbank market by providing liquidity does not always work to stop the contagion. In particular, if bankruptcy of a bank is anticipated to trigger a liquidity crisis in the market for the collateral asset, the liquidity provision cannot stop asset fire sales. Liquidity provision can mitigate the deterioration in the asset price in the fire sale, but it cannot stop the fire sale itself. These results support the argument of Shleifer and Vishny (2011), where the financial crisis after the bankruptcy of the Lehman Brothers investment bank was much worse than it had been before that bankruptcy. They argue that the liquidity provision that worked well for mitigating the crisis before the Lehman bankruptcy did not work well enough, because the Lehman bankruptcy triggered the liquidity crisis by inducing potential participants to step back from bidding in the asset market. Our results provide theoretical support to their argument by explicitly modeling the liquidity crisis in an asset market that is triggered by the bankruptcy of the borrowing bank. Furthermore, our results suggest that asset purchase works to prevent fire sales, to the extent that authorities correctly select the securities they purchase in the asset market. Otherwise, the fire sale can be aggravated, and the asset purchase may only result in expanding the balance sheet of the central bank.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explicitly models the repo trading based on the model. Section 4 shows that the leverage ratio and credit risk of banks depend on the expected interbank rate. Section 5 extends the basic setting in Section 4 and investigates the case in which the bankruptcy of the bank in a repo trading is anticipated to trigger the liquidity crisis in the asset market. Section 6 suggests policy implications. Section 7 discusses the related literature. Section 8 concludes. Proofs are relegated to the Appendix.

2 THE MODEL

We model a modern financial institution that creates short-term trading or transaction of securities backed by a long-term investment.[10]

Consider a financial institution ("bank" hereafter) that seeks to invest in a financial asset backed by a long-term risky project. The project requires 1 unit of an initial investment at

[10] This definition of modern financial institutions is due to Gorton (2010).

Date 0 and matures at Date 2. The return of the project at Date 2 is given by

$$y_2 = y_1^F + a\epsilon,$$

where $a(\in (0, y_1^F))$ is a parameter and ϵ is a random variable that follows $\epsilon \sim U[-1, 1]$. The distribution of ϵ is publicly known at Date 0, and the uncertainty is resolved at Date 2.

The bank has no initial wealth and raises funds to invest the asset at Date 0. There are different types of investors in the market. One is the short-term creditors who are ready to provide funds at Date 0 and Date 1. To be precise, after they invest the funds at Date 0 and redeem them at Date 1, they can refund to the bank under the renewed contract, according to the financial condition realized at Date 1. In particular, when they renew the debt contract at Date 1, they require the same returns regardless of their information about the solvency of the bank at Date 2. Otherwise, they do not refund, and the short-term debt is withdrawn. This feature of the debt is embedded in traditional demand deposits and is referred to as *information insensitiveness* (Gorton and Pennacchi (1990), Holmstrom (2008), and Dang et al. (2009)).

The other is the investors who provide funds with a less rigid claim on the bank than the information-insensitive short-term debt. Under this claim, once the investors provide their funds under the initial financial condition at Date 0, they can neither renew the contract nor withdraw their funds in the course of the investing project of the bank. We refer to such a claim as *capital*, because the funds provided by the investors are substantially at the bank's discretion. In exchange for using discretion, the bank must incur a higher marginal cost than on short-term debt for inducing the investors to provide their funds. We refer to the additional marginal cost of the capital as the premium. In sum, although the capital is a less rigid claim, the capital is costly; thus, the bank needs to depend on the rigid claim to finance the project. Hence, when investing in the asset, the bank faces a trade-off between the softness of the claim and the financing cost, as in Diamond and Rajan (2000). Under such a trade-off, the bank decides the ratio of the information-insensitive short-term debt to the total investing cost, known as the leverage ratio, denoted by $\beta \in [0, 1)$.

Short-term creditors invest in the bank as long as they gain not less than the returns under the unsecured interbank rate. Suppose that the interbank rate between Date 0 and Date 1 is 0. The condition under which the short-term creditors provide funds at Date 0 is given by the following assumption.

Assumption 1 $1 < y_1^F$.

Assumption 1 shows that the fundamental value of A_Y at Date 1 covers the investment cost, so the short-term creditors are fully repaid regardless of the size of β at Date 1. Because y_1^F is publicly observable and verifiable, the bank cannot default strategically at Date 1. Hence, Assumption 1 ensures that the creditors finance the bank by the information-insensitive short-term debt at Date 0.

At Date 0, if the bank decides to invest in a project, both creditors and investors enter competitively. Because of the competitive entrance, the bank decides its leverage ratio so as to maximize its own surplus. Because its capital is a soft claim on banks, they pay the cost of capital to their investors in exchange for receiving the capital at the bank's discretion.[11]

At Date 1, the interbank rate between Date 1 and Date 2 is realized to be $r > 0$. Observing r and the market price per unit of A_Y, as denoted by p, the bank redeems the short-term debt that matures at Date 1. The bank repays the short-term debt by (i) delevering itself by liquidating a fraction of A_Y in the asset market, or (ii) rolling over the information-insensitive short-term debt by pledging A_Y as collateral in the repo, or both. To be precise, the bank decides to liquidates $\alpha \in [0, 1)$ of A_Y and delevers $\alpha p y_1^F$ of the short-term debt. The bank then rolls over the remaining $\beta - \alpha p y_1^F$ of the debt by pledging $(1 - \alpha)$ of A_Y as collateral by deciding the repo rate, r_R.

At Date 2, the value of A_Y is realized. The asset A_Y is liquidated in the asset market for its fundamental value. If the bank repays all of the short-term debt at Date 1, the bank gains the value of the remaining A_Y. If the short-term debt is rolled over in the repo at Date 1, the solvency of the bank depends on the realized value of ϵ. Let us denote a threshold value of ϵ below which the bank becomes bankrupt in the repo trade as a *default threshold*. Denoting the default threshold as $\epsilon_0 \in (-1, 1)$, the bank is solvent if $\epsilon \geq \epsilon_0$. The bank liquidates A_Y and makes a repayment to the short-term creditors under the repo rate that is determined at Date 1. The remaining value goes to the bank. On the other hand, if $\epsilon < \epsilon_0$, the bank becomes bankrupt, and the short-term creditors retain the collateral. The bank gains nothing. As we show in Section 4.1.2, ϵ_0 is endogenously determined.

The timing and events are summarized in Figure 1.

[Insert Figure 1 about here]

[11] Put differently, the bank "purchases" the capital from the investors at Date 0. In this way, the capital in our model is different from the standard setting of capital in theoretical models, where capital represents residual claims such as equity. However, as long as the bank learns its cost of capital through the competitive entrance of the investors at Date 0, the purchase of capital in our setting is not different from standard equity finance.

3 THE REPO TRANSACTION

We analyze the bank's investment decisions by following the backward induction. In this section, we consider the decision made by the bank at Date 1. At Date 1, the bank needs to repay β to the creditors who provide the short-term debt at Date 0. Observing the realized value of p and r, the bank reduces the amount of the short-term debt from β to $\beta - \alpha p y_1^F$ by liquidating $\alpha (\in [0, 1))$ of A_Y. If $\beta - \alpha p y_1^F > 0$, the bank rolls over the remaining amount of the debt by pledging $1 - \alpha$ of A_Y in a repo market.

3.1 Rollover stage

Suppose that $\beta - \alpha p y_1^F > 0$. The bank can roll over $\beta - \alpha p y_1^F$ of the debt in a repo market if the short-term creditors accept a repo rate r_R at Date 1. The repo rate is offered by the bank in a take-it-or-leave-it manner, as creditors in the market are small and dispersed; thus, it is practically impossible for them to negotiate with the bank.

A repo trade is such that Date-1 creditors who accept r_R provide $\beta - \alpha p y_1^F$ of the short-term debt in exchange for forcing the bank to pledge $1 - \alpha$ of A_Y as collateral. Under the repo transaction, the rights to the pledged asset depend on the solvency of the bank at Date 2. Having the default threshold ϵ_0 as given, suppose that the value of ϵ realized at Date 2 satisfies $\epsilon \geq \epsilon_0$. The bank is solvent and thus retains its rights over the pledged asset. The bank liquidates the asset and repays $(\beta - \alpha p y_1^F)(1 + r_R)$ to Date-1 creditors. In contrast, if $\epsilon < \epsilon_0$, the bank becomes bankrupt, and control over the pledged asset shifts to Date-1 creditors. Date-1 creditors seize the pledged asset and sell it in the market at Date 2. We assume that the asset is liquidated for its fundamental value in the Date-2 asset market. In Section 5, we consider the case in which the bankruptcy negatively affects the Date-2 market liquidity of the collateral asset.

The repo makes the creditors information insensitive in that they gain the same returns regardless of the solvency of the bank. For providing such returns, the bank must offer r_R, which satisfies the following equation.

$$\frac{1}{2}\int_{\epsilon_0}^{1} (\beta - \alpha p y_1^F)(1 + r_R) d\epsilon = \frac{1}{2}\int_{-1}^{\epsilon_0} (1 - \alpha)(y_1^F + a\epsilon) d\epsilon. \tag{1}$$

The left-hand side of the above equation is the expected gains if the bank is solvent in the repo trade, whereas the right-hand side is the expected value of the collateral that is seized by the creditors if the bank becomes bankrupt. Let us denote the condition given by the above equation as the *indifference constraint* of the repo trading.

Creditors invest in the bank through repos as long as they gain from their interbank trading. Hence, in order to induce the creditors to finance the bank, the following constraint must be satisfied.

$$\frac{1}{2}\Big[\int_{-1}^{\epsilon_0}(1-\alpha)(y_1^F+a\epsilon)d\epsilon+\int_{\epsilon_0}^{1}(\beta-\alpha p y_1^F)(1+r_R)d\epsilon\Big]\geq(\beta-\alpha p y_1^F)(1+r). \qquad (2)$$

The inequality (2) shows that the creditors' expected gains from the repo trading (the left-hand side) are not less than those from the investment under the interbank rate (the right-hand side). We refer to this inequality as the *repo-participation constraint.*

The bank that rolls over the existing short-term debt at Date 1 gains from the investment if and only if the bank is solvent at Date 2. Denoting π as the expected value of the asset held in the bank, it is given by

$$\begin{aligned}\pi &= \frac{1}{2}\int_{\epsilon_0}^{1}\{(1-\alpha)(y_1^F+a\epsilon)-(\beta-\alpha p y_1^F)(1+r_R)\}d\epsilon \\ &= (1-\alpha)y_1^F-\Big\{\int_{-1}^{\epsilon_0}(1-\alpha)(y_1^F+a\epsilon)d\epsilon+\int_{\epsilon_0}^{1}(\beta-\alpha p y_1^F)(1+r_R)d\epsilon\Big\}. \qquad (3)\end{aligned}$$

Notice that the second term of the second line of (3), which is in curly brackets, is the expected gains of creditors from a repo as given in the left-hand side of the repo-participation constraint. Because π is decreasing in the creditors' expected gains from the repo, the bank decides r_R to bind the repo-participation constraint given by (2) for maximizing π. We obtain the following lemma.

Lemma 1 *Suppose that the bank rolls over the short-term debt in a repo market at Date 1. For a given ϵ_0, the bank decides the repo rate as*

$$r_R=\frac{r+\epsilon_0}{1-\epsilon_0}.$$

Lemma 1 shows that other things being equal, r_R is increasing in ϵ_0. According to the definition of the default threshold, the higher ϵ_0 indicates that the bank is more likely to bankrupt in the repo trade. Because the repo rate is decided so as to meet the expected returns of creditors who accept the debt rollover indifferently regardless of the solvency of the bank, it is natural that the repo rate increases as the bank is more likely to be bankrupted. As we show in the next section, the repo rate is endogenously determined, because we derive ϵ_0 endogenously.

3.2 Liquidation stage

At the beginning of Date 1, the bank decides to liquidate $\alpha \in [0,1)$ of A_Y for redeeming the short-term debt that is matured at Date 1. Substituting the binding repo-participation constraint into π, we derive

$$\pi = (1-\alpha)y_1^F - (\beta - \alpha p y_1^F)(1+r). \tag{4}$$

Deriving the first-order condition of π with respect to α, we have

$$\frac{\partial \pi}{\partial \alpha} \begin{cases} \leq 0 & \text{if } p \leq \frac{1}{1+r} \\ > 0 & \text{if } p > \frac{1}{1+r}. \end{cases} \tag{5}$$

The partial derivative of π on α, which is given by (5), indicates that the bank has an incentive to decide $\alpha = 0$ and rolls over all the existing debt if p is not higher than $1/(1+r)$. Otherwise, the bank has an incentive to liquidate A_Y for redeeming the short-term debt at Date 1.

We now suppose that investors enter the asset market competitively at Date 1. The competitive entrance sets the price per unit of A_Y as

$$p = \frac{1}{1+r}.$$

Therefore, the bank decides $\alpha = 0$ according to (5). Substituting $\alpha = 0$ into (4), the expected value of A_Y retained by the bank is

$$\pi = y_1^F - \beta(1+r). \tag{6}$$

For a given β, the condition under which the bank can roll over all of the existing debt is such that π in (6) is strictly positive. Because $\beta \in [0,1)$, the condition under which the bank does not fail at Date 1 is given by

$$0 \leq \beta < \min\Big(\frac{y_1^F}{1+r}, 1\Big). \tag{7}$$

Before proceeding with our analysis, notice that the second term of the right-hand side of (6) indicates that the repo trading makes the marginal cost of the information-insensitive short-term debt equal to the interbank rate. Because the creditors accept the debt rollover to the extent that they receive the same returns from investing in interbank trading, (6) indicates that the marginal cost of the repo is $(1+r)$, which is equal to the cost of rolling over the unsecured short-term debt under the interbank rate. To the extent that both the bank and the creditors can liquidate the asset for its fundamental value at Date 2, the marginal cost of the repo depends only on the interbank rate. In Section 5, we will investigate the case in which the repo costs the bank more than its interbank trading.

4 LEVERAGE RATIO AND THE CREDIT RISK OF THE BANK

We now investigate the decisions made by the bank at Date 0. At Date 0, investors competitively enter the market and seek to provide capital to the bank. The bank needs to pay the investors for the capital, in exchange for taking discretion over the funds they provide. Let us define the cost of that capital as follows.

$$\text{(The cost of the capital)} = \frac{(1-\beta)^2}{2k},$$

where $(1-\beta)$ is the capital ratio and $1/k$ is the marginal cost for the bank to acquire capital. Because of the bank taking discretion over its capital, investors require a higher marginal cost than on information-insensitive short-term debt. To be precise, to the extent that the investors gain the premium, $\nu(>0)$, in addition to the marginal cost of the information-insensitive short term debt that is derived as $(1+r)$, they provide funds to the bank. Because we assume that the investors (capital providers) and the creditors (purchasers of the information-insensitive debt) are two different types of market investors, we assume that the value of ν is independent of r and is given exogenously.[12] The competitive entrance of the investors results in deciding the marginal cost of the capital as

$$\frac{1}{k} = 1 + r + \nu.$$

Denoting Π as the expected gain of the bank from the investment, it is then given by

$$\Pi = E[\pi] - \frac{(1-\beta)^2}{2}(1 + r + \nu), \tag{8}$$

where the first term of the right-hand side of (8) is the expected value of π at Date 0, whereas the second term is the cost of the capital. The bank decides β in order to maximize Π at Date 0.

[12]We argue the case in which ν is increasing in the expected value of the Date-1 interbank rate in Section 4.1.1. However, as long as the elasticity of ν on r is sufficiently high, our results derived under the fixed ν are robust.

In the following analysis, we first consider the benchmark case, where the value of r realized at Date 1 is certain and publicly known at Date 0. We next investigate the uncertain case where only the ex ante distribution of r is known at Date 0.

4.1 The benchmark case

4.1.1 Decision of the leverage ratio

Let us consider the benchmark case where r is certain and publicly known at Date 0. Denoting Π_B as Π in the benchmark case, it is given by

$$\Pi_B = y_1^F - \beta(1+r) - \frac{(1-\beta)^2}{2}(1+r+\nu),$$

because $E[\pi]$ in (8) is equal to π in (6) in the benchmark case.

The maximization problem of the bank at Date 0 with respect to β (denoted as Problem $[\mathcal{B}]$) is then given by

$$\max_{\beta} \Pi_B,$$

subject to

$$\Pi_B > 0, \tag{BC0}$$

$$\min\Big(\frac{y_1^F}{1+r}, 1\Big) > \beta \geq 0. \tag{BC1}$$

The constraint (BC0) is the participation constraint of the bank. The constraint (BC1) is given by (7), which ensures that the bank does not fail in rolling over its existing short-term debt in the repo at Date 1.

The solution to Problem $[\mathcal{B}]$ is summarized in the following proposition.

Proposition 1 *Suppose that the value of r is certain and publicly known at Date 0. Denoting Ω_B as*

$$\Omega_B \equiv \frac{(1+r)(\nu + \frac{1+r}{2})}{1+r+\nu},$$

the decision of the bank at Date 0 is given as follows.

(i) If $y_1^F \leq \Omega_B$, the bank does not invest the asset.

(ii) If $\Omega_B < y_1^F$, the bank invests the asset, and the leverage ratio is

$$\beta^* = \frac{\nu}{1+r+\nu} \in (0,1).$$

Proof: See the Appendix.

Proposition 1 shows that the bank decides its leverage ratio as if it offered long-term debt with the rollover cost $1 + r$, because r realized at Date 1 is publicly known at Date 0. Otherwise, creditors withdraw their funds from the bank at Date 1.

Proposition 1 derives the threshold on the fundamental value of A_Y as Ω_B, above which the bank decides to invest in the project at Date 0. As derived in the Appendix, because (BC1) is satisfied as long as (BC0) holds, Ω_B is the lowest required value of y_1^F that satisfies (BC0). This implies that the bank does not fail in rolling over the existing debt at Date 1 to the extent that the bank gains from investing A_Y. The threshold value of Ω_B is increasing in r and ν. Because both r and ν are the marginal costs of the external finance, these results imply that costly external finance requires the bank to invest A_Y with the higher fundamental value, y_1^F.

The leverage ratio given by β^* in Proposition 1(ii) is decreasing in r while increasing in ν. This indicates that the trade-off between the short-term debt and the capital stems from the relative marginal costs of these financing measures. The bank depends more on the short-term debt the lower is r. In contrast, the higher is ν, the bank increases its leverage ratio because the capital costs the bank more.

Before proceeding with the analysis, let us consider the case in which ν is an increasing function of the expected value of the Date-1 interbank rate at Date 0. Because the value of r is certain, the ν is given by $\nu(r)$, $\nu'(r) > 0$. In this case, the partial derivative of β^* on r is derived as

$$\frac{\partial \beta^*}{\partial r} = \frac{1}{(1 + r + \nu(r))^2}\Big[\nu'(r)(1 + r) - \nu(r)\Big].$$

As long as the following assumption holds, β^* is strictly increasing in r.

Assumption 2:

$$\frac{\nu'(r)/r}{1/(1 + r)} < 1.$$

Assumption 2 indicates that the elasticity of the premium on the rollover cost of the information-insensitive short-term debt is less than 1. To be precise, a 1-percent increase in the marginal cost of the short-term debt that is rolled over through the repo increases the premium on the capital by less than 1 percent. As long as this assumption holds, our results in Proposition 1 hold.

4.1.2 Repo rate and the credit risk of the bank

We continue our analysis by focusing on the case in which the bank invests in the project at Date 0. We first investigate the credit risk of the bank in a repo market. The bank becomes bankrupt in a repo trade if the value of A_Y realized at Date 2, which is given by y_2, satisfies

$$y_2 = y_1^F + a\epsilon < y_1^F + a\epsilon_0,$$

where ϵ_0 is the default threshold.

Having β as given, the default threshold, ϵ_0, is derived by substituting $\alpha = 0$ and r_R in Lemma 1 into the indifference constraint of the repo trading, which is given by (1), as follows.

$$\epsilon_0 = \frac{-y_1^F + \sqrt{(y_1^F - a)^2 + 2a(1+r)\beta}}{a}. \tag{9}$$

Denoting the default threshold in the benchmark case as ϵ_0^*, it is derived by substituting β^* in Proposition 1(ii) into (9) as follows.

$$\epsilon_0^* = \frac{-y_1^F + \sqrt{(y_1^F - a)^2 + 2a(1+r)\frac{\nu}{1+r+\nu}}}{a}.$$

The partial derivatives of ϵ_0^* on r and ν are respectively given as [13]

$$\frac{\partial \epsilon_0^*}{\partial r} > 0, \quad \frac{\partial \epsilon_0^*}{\partial \nu} > 0.$$

We immediately derive the following lemma.

Lemma 2 *Suppose that the value of r is certain at Date 0. The default threshold given by ϵ_0^* is increasing in both r and ν.*

The increases of r and ν increase the value of ϵ_0^* through those different paths. One we denote as the *interbank-rate effect*, where the increase in r at Date 1 increases the rollover

[13]In particular, the result of ϵ_0^* as increasing in r holds even if ν is given by an increasing function of r because of

$$\begin{aligned}\frac{\partial \epsilon_0^*}{\partial r} &= \frac{1}{\sqrt{(y_1^F - a)^2 + 2a(1+r)\frac{\nu}{1+r+\nu}}} \frac{\partial}{\partial r}\left[\frac{\nu(r)(1+r)}{1+r+\nu(r)}\right] \\ &= \frac{1}{\sqrt{(y_1^F - a)^2 + 2a(1+r)\frac{\nu}{1+r+\nu}}}[(1+r)^2\nu'(r) + r^2] > 0.\end{aligned}$$

cost of the existing debt held by the bank, because the repo trading adjusts the rollover cost equal to r. The other we denote as the *leverage effect*, where the higher ν induces the bank to be more leveraged, because the relative cost of its capital increases for a given r. Other things being equal, the bank with the higher leverage ratio must pledge more as repo collateral. This increases the default threshold.

Because $\epsilon \sim U[-1, 1]$, the default probability of the bank in a repo at Date 2 is given by

$$d = \frac{1 + \epsilon_0}{2}. \tag{10}$$

Let us denote d as the *credit risk* of the bank. Denoting the credit risk of the bank in the benchmark case as d^*, it is derived by substituting ϵ_0^* into (10) by

$$d^* = \frac{1 + \epsilon_0^*}{2}.$$

Because ϵ_0^* is increasing in both r and ν, the credit risk of the bank is also given by the increasing function of both r and ν.

Denoting the repo rate in the benchmark case as r_R^*, it is derived by substituting ϵ_0^* into r_R in Lemma 1. Because ϵ_0^* is increasing in r and ν, we derive the result that the repo rate is also increasing in r and ν as follows.

$$\frac{\partial r_R^*}{\partial r} = \frac{r + \epsilon_0^*}{1 - \epsilon_0^*} = \frac{1}{(1 - \epsilon_0^*)^2}\left[1 - \epsilon_0^* + \frac{\partial \epsilon_0^*}{\partial r}(1 + r)\right] > 0,$$

$$\frac{\partial r_R^*}{\partial \nu} = \frac{1 + r}{(1 - \epsilon_0^*)^2}\frac{\partial \epsilon_0^*}{\partial \nu} > 0.$$

We summarize the above results in the following proposition.

Proposition 2 *In the benchmark case, the credit risk of the bank and the repo rate are increasing in both r and ν.*

Although the credit risk and the repo rate are decided according to the realized value of r at Date 1, these are derived correctly at Date 0 because the value of r is certain and publicly known at Date 0. In the following section, we investigate the case in which the value of r is uncertain at Date 0 and see whether such uncertainty changes the results given by Proposition 2.

4.2 Uncertain case

We next consider the case in which the value of r realized at Date 1 is uncertain at Date 0. Suppose that $r \sim f(r)$ over $[r_L, r_H]$, where $0 \leq r_L < r_H \leq 1$ at Date 0.

4.2.1 Investment decision under uncertainty

The analysis follows the backward induction as in the benchmark case. The expected value of A_Y retained in the bank is given by π in (3). The bank's decision whether to liquidate A_Y or to roll over the existing short-term debt depends on the realized p in Date 1, as given by (4). Because p is given by $1/(1+r)$ for any realized value of $r(\in [r_L, r_H])$ at Date 1, the bank decides to roll over the existing debt at Date 1. Therefore, the value of π at Date 1 is given by (6) as in the benchmark case.

Because $r \in [r_L, r_H]$, the highest possible rollover cost is $1 + r_H$. Hence, the leverage ratio must satisfy the following constraint so as to allow the bank to choose the debt rollover without failing at Date 1.

$$\min\left(\frac{y_1^F}{1+r_H}, 1\right) > \beta \geq 0. \tag{11}$$

At Date 0, the expected gain of the bank from the investment is given by (8). Because the value of r is uncertain at Date 0, $E[\pi]$ is given by

$$\begin{aligned} E[\pi] &= \int_{r_L}^{r_H} (y_1^F - \beta(1+r))dr \\ &= y_1^F - \beta(1+r_e), \end{aligned}$$

where r_e is the expected value of r at Date 0. Denoting the expected gain of the bank from its investment in the uncertain case as Π_U, it is given by substituting $E[\pi]$ above into (8) as

$$\Pi_U = y_1^F - \beta(1+r_e) - \frac{(1-\beta)^2}{2}(1+r_e+\nu).$$

Note that because the investors who provide the capital to the bank require the premium at Date 0, the value of ν is supposed to be the premium on r_e. Following the discussion in the previous section, our results are not affected by the assumption of which value of ν is independent of r_e.

The maximization problem of the bank with respect to β (denoted as Problem $[\mathcal{U}]$) is

$$\max_{\beta} \Pi_U,$$

subject to

$$\Pi_U > 0, \tag{UC0}$$

$$\min\Big(\frac{y_1^F}{1+r_H}, 1\Big) > \beta \geq 0. \tag{UC1}$$

As in Problem [$\mathcal{B}$], (UC0) is the participation constraint of the bank. The constraint (UC1) is equal to (11), which ensures that the bank does not fail in rolling over the existing debt at Date 1.

The solution to Problem [$\mathcal{U}$] is summarized in the following proposition.

Proposition 3 *Suppose that the value of r is uncertain at Date 0.*

(i) If $y_1^F \leq \Omega_i$, the bank does not invest the asset.

(ii) If $\Omega_i < y_1^F$, the bank invests the asset and decides its leverage ratio as

$$\beta_U^* = \frac{\nu}{1+r_e+\nu} \in (0,1).$$

The value of Ω_i is given differently as follows.

$$\Omega_i = \begin{cases} \Omega_0 = \frac{1+r_e}{1+r_e+\nu}\left(\nu + \frac{1+r_e}{2}\right) & \text{if } r_H - \nu \leq \frac{1}{2}, \text{ or } r_H - \nu > \frac{1}{2} \text{ and } r_{e+} < r_e, \\ \Omega_1 = \frac{\nu(1+r_H)}{1+r_e+\nu} & \text{if } r_H - \nu > \frac{1}{2} \text{ and } r_e \in (0, r_{e+}], \end{cases}$$

where

$$r_{e+} = \sqrt{\nu^2 + 2(1+r_H)} - (1+\nu).$$

Proof: See the Appendix.

As in the benchmark case, the higher y_1^F makes the bank more likely to invest A_Y at Date 0. The threshold value of y_1^F above which the bank invests the asset is now given by Ω_i, which is the minimum fundamental value of A_Y required for satisfying the constraint of (UCi) ($i = 0, 1$) in the maximization problem of [$\mathcal{U}$]. Hence, the threshold of y_1^F is given by Ω_i if the bank is constrained by (UCi) rather than (UCj), where $j = 0, 1$ and $j \neq i$. In particular, differently from the results in the benchmark case given by Proposition 1, Proposition 3 derives the conditions under which the bank is more constrained by (UC1), which ensures that the bank does not fail in rolling over the debt, rather than the participation constraint given by (UC0).

Proposition 3 shows that other things being equal, the investment decision of the bank depends more on the value of Ω_1 the higher is r_H, while the lower is r_e, because the threshold value of r_e, which is given by r_{e+} in Proposition 3, is increasing in r_H. What this result shows is that the investment decision of the bank is more constrained by (UC1) the higher is r_H relative to r_e. Such conditions imply that r realized at Date 1 is more likely to be higher than r_e, because r_H is the upper limit of the ex ante distribution of r. Because β_U^* is decreasing in r_e, the lower r_e and the higher r make it more difficult for the bank to roll over debt, because the bank with a higher leverage ratio needs to make the debt rollover at a higher cost. Therefore, the bank is more constrained by (UC1).

In contrast, under the condition in which either r_H is low or both r_H and r_e are high, the threshold of y_1^F above which the bank invests in the project is given by Ω_0, above which the participation constraint of the bank is satisfied. This is because the bank under these conditions can roll over the existing debt more easily, and the participation constraint given by (UC0) is more critical for the bank when making the investment decision.

Notice that although the ex ante distribution of r is given by the range of $[r_L, r_H]$, the expected value of r_e is given differently according to the distribution function of r. The *skewness* of the distribution of r, or the distribution's third central moment, denoted by μ_3, is defined as follows.

$$\mu_3 = \int_{r_L}^{r_H} f(r)(r - r_e)^3 dr,$$

where $f(r)$ is the ex ante density function of r at Date 0.[14] The larger positive value of μ_3 indicates that the distribution of r is more skewed toward r_L, and the expected value of r given by r_e is lower. Hence, although the range of the distribution of r is fixed to be given by $[r_L, r_H]$, if the skewness of $f(r)$ is large and positive, r_e is lower; thus, the bank decides its leverage ratio of β_U^* in Proposition 3 at a higher level at Date 0.

Corollary 1 *In the uncertain case, the large and positive skewness of $f(r)$ induces the bank to decide the higher β_U^*.*

The interpretation of r and the procyclical leverage ratio One of the distinctive observations during the 2007–2009 financial crisis is the extreme increase of the interbank lending rates. As many studies on the financial crisis including Acharya and Skeie (2011) and Gorton and Metrick (2011) point out, the counterparty risk, which is represented by the

[14]See for the definition of the skewness in Levy (1998).

LIBOR-OIS spread (the spread between the London Interbank Offered Rate (LIBOR) and Overnight Index Swap (OIS) rate), widened considerably during the crisis. Interpreting r in our setting as the counterparty risk given by the LIBOR-OIS spread, the level of r_e indicates the prospect of the future counterparty risk measured at Date 0. Because Ω_i $(i = 0, 1)$ and β_U^* increase as r_e decreases, the results in Proposition 3 imply that the bank is more likely to invest A_Y with the lower fundamental value and depends more on the debt finance the lower the expected counterparty risk is.

Our results show that the bank decides the higher leverage ratio for the lower r_e, which represents that the future borrowing conditions are expected to be easier. Because booms ease borrowing conditions while busts tighten those conditions in general, our results also imply that the leverage ratio of banks is procyclical: banks set their leverage ratio higher when future borrowing conditions are expected to be easier, and vice versa. This is in similar vein to the discussion of the procyclical tendency of the leverage ratio in the banking sector, as pointed by Adrian and Shin (2010). They find that the leverage ratio of financial institutions increases during booms, where the financing condition is expected to be lax, while decreasing during busts. Because the expected fundamental value of the investing assets increases during booms, financial institutions can expand their leverage ratio by pledging assets as security on their borrowings. In contrast, the pledged value of the assets is likely to decrease during busts; thus, the institutions must delever themselves. Acharya and Vishwanathan (2011) analyze theoretically the leverage ratio of financial institutions and derive the procyclical leverage ratio. The investing asset in their model varies in quality depending on the moral hazard intensity of financial institutions. Although there is a moral hazard problem, because the quality of assets usually does not deteriorate during booms, financial institutions can increase their leverage ratio the higher the expected future value of those assets is. In Adrian and Shin (2010) as well as Acharya and Vishwanathan (2011), the ease of borrowing comes from the *funding liquidity*, which is first presented by Brunnermeier and Pedersen (2009), under which the higher future value of the asset makes the borrower expand its leverage ratio.

In contrast to the funding liquidity model, we show that the bank decides its leverage ratio according as the future ease of borrowing in the interbank market. This is because repo trading adjusts the cost of the debt finance to the future interbank borrowing conditions, which are in turn affected by counterparty risk.

4.2.2 Jump in the credit risk and the repo rate

We focus on the case in which the bank decides to invest in the project at Date 0 in the uncertain case. We first derive the default threshold by following the analysis in Section 4.1. Denoting the default threshold in the uncertain case as ϵ_U^*, it is derived by substituting β_U^* in Proposition 3 into ϵ_0 given by (9) as

$$\begin{aligned}\epsilon_U^* &= \frac{-y_1^F + \sqrt{(y_1^F - a)^2 + 2a(1+r)\beta_U^*}}{a} \\ &= \frac{-y_1^F + \sqrt{(y_1^F - a)^2 + 2a(1+r)\frac{\nu}{1+r_e+\nu}}}{a}.\end{aligned}$$

The difference between the default threshold in the uncertain case and that in the benchmark case is such that the leverage effect, which is the effect of the change in the leverage ratio on the default threshold, is segregated from the interbank-rate effect, which is the change in r on the default threshold. This is because r is realized at Date 1, and only its expected value given by r_e is publicly known at Date 0, and the bank decides its leverage ratio according to r_e and ν. Hence, the partial derivatives of ϵ_U^* on respective r_e and ν are given by

$$\frac{\partial \epsilon_U^*}{\partial r_e} < 0, \quad \frac{\partial \epsilon_U^*}{\partial \nu} > 0,$$

because β_U^* given in Proposition 3 is also decreasing in r_e while increasing in ν.

Having r_e and ν as given, ϵ_U^* is given by an increasing function of r as

$$\frac{\partial \epsilon_U^*}{\partial r} > 0.$$

However, because the value of r is uncertain until Date 1, we can describe $\epsilon_U^*(r)$ as $\epsilon_U^*(\tilde{r})$, where $\tilde{r} \sim f(r)$ over $[r_L, r_H]$. Denoting the credit risk of the bank measured at Date t as d_{Ut}^* $(t = 0, 1)$, it is derived by substituting $\epsilon_U^*(\tilde{r})$ into ϵ_0 in d given by (10) as

$$d_{Ut}^*(\tilde{r}) = \frac{1 + \epsilon_U^*(\tilde{r})}{2},$$

where

$$\tilde{r} = \begin{cases} r_e & \text{if } t = 0 \\ r & \text{if } t = 1. \end{cases} \tag{12}$$

Because d_{Ut}^* is increasing in $\epsilon_U^*(\tilde{r})$ and ϵ_U^* is increasing in r, we immediately derive

$$d_{U0}^* < d_{U1}^*,$$

if the value of r is realized to satisfy $r_e < r$ at Date 1.

The repo rate is also derived as an increasing function of $\epsilon_U^*(\tilde{r})$, because it is derived by substituting $\epsilon_U^*(\tilde{r})$ into ϵ_0 in r_R in Lemma 1, and r_R in Lemma 1 is increasing in ϵ_0. Because $\tilde{r}$ is r_e at Date 0 and r at Date 1, the repo rate measured at Date t ($t = 0, 1$) is

$$r_{Rt}^* = \frac{\tilde{r} + \epsilon_U^*(\tilde{r})}{1 - \epsilon_U^*(\tilde{r})},$$

where $\tilde{r}$ is given differently according as t, as given by (12). Because ϵ_U^* is increasing in r, $r_{R0}^* < r_{R1}^*$ holds if r is realized to satisfy $r_e < r$ at Date 1.

We summarize the discussion above in the following proposition.

Proposition 4 *In the uncertain case, if $r_e < r$, both the credit risk of the bank and the repo rate increase from Date 0 to Date 1.*

By interpreting r as the counterparty risk in the interbank market, Proposition 4 shows that if the counterparty risk is realized to be higher than its expected value, both the credit risk of the bank and the repo rate are realized to be higher than expected at Date 0.

The probability of $r > r_e$ increases if the ex ante density function of r is updated to $g(r)$, which has more weight in the upper tail of $f(r)$. The shift of the probability from the center of $f(r)$ to its tail(s) without affecting the mean is called the mean preserving spread (MPS), or "fat tails" criterion (Levy (1998), pp. 246–250).[15] If the updated density function, $g(r)$, preserves the mean as r_e while the center of probability is shifted from that of $f(r)$ toward r_H, the probability of r being higher than r_e is higher than that expected under the ex ante density function given by $f(r)$. The results of Proposition 4 suggest that if the ex ante density function of $f(r)$ is updated to $g(r)$ by reflecting changes of economic environment, the realized value of r is more likely to be higher than r_e, and consequently, both the insolvency risk and the repo rate at Date 1 are more likely to be higher than those measured at Date 0.

We summarize the above discussion in Corollary 2.

Corollary 2 *Suppose that the true distribution of r is realized to be $r \sim g(r)$ over $[r_L, r_H]$ after the bank makes the investment at Date 0. If $g(r)$ reserves the expected value of r as r_e but has more weight in the upper tail than the ex ante density function given by $f(r)$, both the credit risk of the bank and the repo rate are more likely to rise sharply at Date 1.*

[15]Rothschild and Stiglitz (1970) provide the five definitions of risk and introduce fat tail risk criterion or MPS as one of them.

We can interpret the update of the density function from $f(r)$ to $g(r)$ after Date 0 occurs because of the limited ability of agents to foresee economic conditions associated with the change in the distribution function of r. The increment in the risk not foreseen under limited ability is also interpreted as the *neglected risk* in Gennaioli et al. (2011). Although they analyze cash flows from two assets that are not pledged as collateral, they also stress that the neglected risk, which is not taken into consideration under limited ability of agents, tends to make the financial system more fragile.

Financial contagion and the sudden rise in credit risk Our results provide theoretical explanations of the financial contagion during the 2007–2009 financial crisis, which is pointed out by Gorton and Metrick (2011). By their definition, the contagion is the spread of the financial crisis from subprime-housing-related securities to non-subprime securities that have no direct connection to the deteriorated housing market. Tracing the path of the crisis, they found that there is no direct connection between the subprime and non-subprime-related securities. Moreover, they found that the LIB-OIS (or LIBOR-OIS) spread was strongly correlated with changes in credit spreads and repo rates of the non-subprime-related securities. Deterioration in the quality of subprime-related securities may increase the cost of finance in the interbank market by increasing the counterparty risk. Afonso et al. (2011) found in their empirical study that the counterparty risk dramatically increased after the bankruptcy of Lehman Brothers, which was one of the largest investment banks in the U.S. and had been vigorously engaged in investing in securitized assets including the deteriorating subprime assets. They interpreted the bankruptcy of Lehman Brothers as betraying the market beliefs of the "too big to fail" rule and resulting in increasing the counterparty risk.

These empirical findings lend support to the interpretation of our theoretical findings such that the bankruptcy of (any other) banks that hold large amounts of deteriorated subprime-related assets updates the market belief of the future counterparty risk by updating the ex ante density function of r to the one with more weight on the upper tail. This updating increases the risk of A_Y held by the bank, even though the risk of A_Y is not associated with a particular market or event.

A sudden precipitation of AAA-rated securities from the summer of 2007 also induced critiques of credit rating agencies (CRAs), because they failed to exhibit signs of distress of asset-backed securities and banks that had massively invested in such securities. [16] CRAs are

[16] Problems associated with the credit rating agencies are summarized by White (2010).

private firms and institutions that make credit ratings on securities issued by their own client firms. Because the gains of CRAs mainly consist of fees from their client firms, as White (2002) pointed out, CRAs have an incentive to make favorable ratings to attract more client firms. All the same, a high credit rating is desirable, and firms are likely to choose the CRA that provides it with the most favorable rating.[17] This problem, known as rating shopping, possibly inflates the credit ratings of the firms, as analyzed by Sketa and Veldkamp (2009). Using these aspects as building blocks, the recent study of Bolton, Freixas, and Shapiro (2011) analyzes credit ratings in equilibrium and shows that CRAs in a competitive rating market tend to relax the rating standard for attracting issuers.

Sudden increases in the credit risks of AAA-rated securities increased criticisms that CRAs' conflicts of interests may have made credit ratings sluggish. However, our results suggest that the credit risk of securities can increase in the short period, between Date 0 and Date 1 in our model, even without the agency problems of CRAs. Specifically, we show that if the ex ante expected value of r (denoted by r_e) is lower than the realized value of r, the credit risk of the bank measured at Date 0 is updated to increase at Date 1. If the credit rating is not frequent enough to be made only at Date 0, the credit risk of the bank looks like exhibiting a sudden increase at Date 1. Our results suggest that frequent rerating is necessary to update the credit risk of banks, especially if r is uncertain ex ante and the center of the ex ante density function can be shifted to its upper tail.

5 LIQUIDITY CRISIS AND ASSET FIRE SALE

So far we have assumed that A_Y is traded for a price that is equivalent to its fundamental value in the Date-2 asset market, regardless of the solvency of the bank at this date. In this section, we extend the basic model by considering the situation under which the bankruptcy of the bank engaged in repo trading reduces the market liquidity of A_Y at Date 2. We show, using the mark-to-market pricing model proposed by Brunnermeier and Pedersen (2009) and Adrian and Shin (2010), that the decrease in market liquidity results in the market price of A_Y falling below its fundamental value.

The insolvency of financial institutions is likely to reduce the market liquidity of assets that are pledged as collaterals, depending on growing doubts about the true quality of the

[17] According to the empirical findings of Tang (2009), borrowing costs of uprated firms decrease ex post compared with those of downrated ones.

pledged assets among market participants. In fact, even though credit ratings are applied to most financial assets, the true qualities of these assets are still not adequately estimated by market participants. For example, financial assets, in particular synthetic securities that may contaminate securities backed by low-quality loans and mortgages, are exotic enough for market participants to know the true quality.[18] Following the idea of limited arbitrage in Shleifer and Vishny (1997), if financial institutions fail in repo trading and are bankrupted when bad news on the collateral securities prevails, market participants may perceive the bankruptcy as being a result of the deterioration of the value in the collaterals. Consequently, a bankruptcy can mark the onset of a liquidity crisis, because potential participants in the asset market refrain from bidding for the asset held by the bankrupted bank.

In the following section, we model the situation under which a liquidity crisis is triggered by bankruptcy of the bank at Date 2. We show that anticipation of the liquidity crisis induces the bank to sell the collateral asset at a fire-sale price.

5.1 Decrease in the market liquidity

We analyze the situation under which the bankruptcy of the bank in the repo trade triggers the liquidity crisis in the asset market at Date 2. Suppose that the Date-2 market liquidity of A_Y falls to $\phi \in (0,1)$ if the bank is insolvent at Date 2. As in Section 3, we solve the problem by backward induction and begin with the decision of the bank on the repo rate.

The indifference constraint, which is given by (1) in the benchmark case, is now modified as follows.

$$\frac{1}{2}\int_{\epsilon_0}^{1}(\beta - \alpha p y_1^F)(1+r_R)d\epsilon = \frac{1}{2}\int_{-1}^{\epsilon_0}(1-\alpha)\phi(y_1^F + a\epsilon)d\epsilon. \tag{1'}$$

The only difference between (1') and (1) is the right-hand side, which shows the expected return of the creditors if the bank becomes bankrupt and is insolvent in a repo trade at Date 2. The right-hand side of (1') discounts that of (1) by ϕ, because the reduction of the liquidity of the asset market, which is triggered by the bankruptcy of the bank, decreases the market price of A_Y per unit from 1 to ϕ. Consequently, the repo-participation constraint of the creditors for rolling over the short-term debt in a repo market, which is given by (2) in

[18] Pagano and Volpin (2009) point out in their theoretical study that the quality of asset-backed securities is opaque when these securities are sold.

the benchmark case, is also modified as

$$\frac{1}{2}\Big[\int_{-1}^{\epsilon_0}(1-\alpha)\phi(y_1^F+a\epsilon)d\epsilon+\int_{\epsilon_0}^{1}(\beta-\alpha p y_1^F)(1+r_R)d\epsilon\Big]\geq(\beta-\alpha p y_1^F)(1+r). \tag{2'}$$

This inequality shows that the expected returns of the creditors from the repo trading, which are given in the left-hand side, are not less than those from the interbank trading given in the right-hand side. The first term of the left-hand side of (2') is discounted by ϕ, because this term indicates the expected return if the bank becomes bankrupt in the repo trade.

Even though the bankruptcy of the bank triggers the liquidity crisis in the asset market at Date 2, it does not affect the expected value of A_Y held by the bank. This is because if the bank becomes bankrupt, the bank trades A_Y to the creditors. Hence, the expected value of A_Y retained by the bank at Date 1 is the same as π given by (3). The bank maximizes π by deciding r_R to bind (2'), so the expected net gain of the bank from A_Y is derived as

$$\pi=(1-\alpha)y_1^F-\chi(\beta-\alpha p y_1^F),$$

where we define

$$\chi\equiv\frac{(1+r)(1+\phi)}{2\phi}. \tag{13}$$

Because $(\beta-\alpha p y_1^F)$ in the second term of the right-hand side of π given above is the amount of the rolled-over debt, χ given by (13) corresponds to the rollover cost of the debt.

In contrast, suppose that the bank decides to redeem all the existing debt by liquidating A_Y instead of rolling it over at Date 1. A liquidity crisis does not occur because the bank did not engage in repo trading. The cost of the asset liquidation is given by its market price as $p=1/(1+r)$. Comparing the rollover cost given by χ and the liquidation cost, we derive

$$1+r<\frac{(1+r)(1+\phi)}{2\phi}(\equiv\chi), \tag{14}$$

because of $\phi\in(0,1)$. This indicates that the redemption of the existing debt by liquidating A_Y is cheaper for the debt rollover in repo trading.[19]

We derive the following proposition.

[19] Equivalently, deriving the first-order condition of π with respect to α, we have

$$\frac{\partial\pi}{\partial\alpha}=(\chi p-1)y_1^F.$$

This indicates that the bank chooses to liquidate the asset because of $\frac{\partial\pi}{\partial\alpha}>0$ for $p=1/(1+r)$.

Proposition 5 *If the bankruptcy of the bank engaged in repo trading triggers a liquidity crisis in the asset market, the bank sells the asset that it pledged as collateral at a fire-sale price. The fire sale is aggravated by the increment of the counterparty risk.*

Proof: Because the selling price of A_Y per unit is given by $p = 1/(1+r)$, the increase in r reduces p. ■

Remembering that the rollover cost in the benchmark case where there is no liquidity crisis is given by $1+r$, our results show that the liquidity crisis triggered by the bankruptcy of the bank increases the rollover cost. Put differently, a future liquidity crisis reduces funding liquidity of A_Y by decreasing its value at Date 1. Therefore, the results in Proposition 4 imply that the *future* liquidity crisis in response to the bankruptcy of the bank engaged in repo trading reduces the *current* funding liquidity of the collateral asset. The liquidity crisis makes the bank do better by selling A_Y at Date 1, even though the market price of A_Y is discounted by the counterparty risk.

The deleverage of the bank induced by the future liquidity crisis results in substantially shortening the tenor of the debt in equilibrium. This result is similar to the one derived by Acharya, Gale, and Yorulmazer (2011). They argue that the bank needs to decide whether to roll over the debt before information about the quality of the investing assets materializes. Building on this idea, they find that future information about those assets can reduce debt rollover. Their results explain theoretically the market freeze observed during the 2007–2009 financial crisis. In contrast to the uncertainty of the information assumed by Acharya, Gale, and Yorulmazer (2011), information about the liquidity crisis in our model is certain and symmetric. The bank in our model stops rolling over debt, because it learns that the liquidity crisis triggered by the bankruptcy of a bank increases rollover cost.

5.2 Alternative interpretation: rehypothecation

The bankruptcy of the bank may reduce the return of the secured creditors if the bank is engaged in *rehypothecation*. Rehypothecation is the prevailing trading procedure of broker–dealers including investment banks and hedge funds. Considering the bank as a broker–dealer, the bank engages in rehypothecation by borrowing the asset from other financial firms. The bank then pledges the borrowed asset as collateral in its own secured trading.[20] However, as

[20] For detail on rehypothecation, see Aragon and Strahan (2009) and Singh and Aitken (2009a).

Duffie (2010) argues, the lending firms can retrieve their asset from the bank, in particular if they obtain bad information about the borrowing bank before the repo creditors.[21] The retrieval of the asset can reduce the return of the secured creditors of the bank. This is because the retrieval of the asset reduces the amount of the collateral received by the secured creditors of the bank if the bank becomes bankrupt.

To see this, suppose that the bank invests $1-\phi$ of A_Y by borrowing from other financial firms. If the financial firms correctly learn that the bank is going to become bankrupt before Date 2, they retrieve their lending asset from the bank. After the financial firms retrieve $1-\phi$ of A_Y, the bank goes into bankruptcy, and the repo creditors can receive only ϕ of the asset as collateral. If the repo creditors anticipate the retrieval of the rehypothecated asset, they require the higher repo rate for making their expected returns from the repo trading to be not less than that from the interbank lending. Consequently, the rollover cost of the bank increases from $1+r$ to χ, as we saw in the previous section.

In practice, according to Singh and Aitken (2009a), regulation of rehypothecation in the U.S. such as Rule 15c3-3 requires financial institutions such as hedge funds and broker–dealer banks to segregate the account for the rehypothecation from that of their customers (or the repo creditors in our setting). However, Singh and Aitken found that after the bankruptcy of Lehman Brothers, rehypothecation declined dramatically. They conclude that the collateral lenders sideline rehypothecation because they fear losing the collateral assets if the collateral borrowers become bankrupt. Their findings provide the support to our alternative interpretation of ϕ in this section such that the withdrawal of the collateral is prior to the bankruptcy of the borrowing bank, which reduces the amount of the collateral received by the repo creditors of the bank.

6 POLICY IMPLICATIONS

We now consider the financial policy that affects the interbank borrowing conditions of the bank. We first consider the impact of conventional monetary policy on the level of the interbank rate. We then consider the asset purchase implemented during the quantitative easing shortly after the 2007–2009 financial crisis.

[21] Duffie (2010) refers to the withdrawal of the asset from the prime-broker bank as the flight of prime-broker clients.

6.1 Liquidity provision

We first consider monetary policy whereby the central bank sets its target level for the short-term interest rate, r, by adjusting reserve requirements.[22] Liquidity injection to the banking system works to reduce r, because the increased solvency of the banking sector reduces counterparty risk. Based on the discussion in Section 4, in the absence of the liquidity crisis in the Date-2 asset market, liquidity provision by the central bank reduces r, which improves the borrowing conditions of the bank by increasing the funding liquidity of A_Y. In particular, if the future counterparty risk is uncertain as in Section 4.2, the announcement of liquidity provision can mitigate the future counterparty risk through updating the density function of r by shifting its center downward. According to Proposition 4, this makes the realized r more likely to be less than r_e. Having r lower than r_e mitigates the financial crisis, by making both the credit risk of the bank and the repo rate lower than expected.

In contrast, if the bankruptcy of the bank triggers a liquidity crisis in the Date-2 asset market as in Section 5, the effect of monetary policy is limited. This is because according to Proposition 5, although the reduction of r mitigates the extent of the deterioration in the selling price of A_Y by increasing p at Date 1, it does not stop the fire sale itself. This is because as long as $\phi < 1$ holds, equation (14) shows that the cost of the asset liquidation, which is given by $(1 + r)$, is strictly less than the debt-rollover cost through the repo, which is given by χ. This implies that although the pure monetary policy can increase the fire-sale price, it cannot stop the asset fire sale unless it resolves the liquidity crisis in the Date-2 asset market.

6.2 Asset purchase

We next consider the asset purchase, by which the central banks intervene in markets and purchase the financial assets including risky collateral. To investigate the impact of the asset purchase on ϕ in the case of the liquidity crisis, suppose that A_Y consists of two kinds of assets. One is a certain asset, whose value is known to be equal to its fundamental value. Hence, the market price of the certain asset is equal to the fundamental value, $y_1^F + a\epsilon$. The other is an uncertain asset. The uncertain asset has the same fundamental value as the

[22]We progress our argument by assuming that the traditional monetary policy works by adjusting the interest rate to the setting target. Recent work by Friedman and Kuttner (2011) argues that the changes in the interest rate are not associated with discernible changes in the reserves.

certain asset, but because its value is opaque to outsiders, the bidders in the market value this asset relative to the solvency of the bank at Date 2. To be precise, if the bank is solvent at Date 2, the uncertain asset is traded for a price that is equal to its fundamental price. In contrast, if the bank fails at Date 2, the market price of the asset falls to zero, because the bankruptcy of the bank casts doubt on the quality of the asset, and nobody bids for such an asset in the market.

Suppose that A_Y consists of ϕ of the certain asset and $1-\phi$ of the uncertain asset. Although the central bank decides to purchase the asset if the bank becomes bankrupt at Date 2, it does not know the quality of the asset, as the central bank cannot distinguish a certain asset from an uncertain asset. Suppose that the central bank purchases $s \in [0,1]$ of the uncertain asset and $1-s$ of the certain asset. After the asset purchase, the remaining fraction of the uncertain asset is $1-s$, while that of the certain asset is s. Hence, the total asset remaining in the market is $(1-\phi)(1-s)+\phi s$. If the central bank does not intervene, only the certain asset is traded in the market at Date 2. Denoting ϕ_G as the liquidity of the asset market at Date 2 if the central bank intervenes after the bankruptcy of the bank, it is given by

$$\phi_G = \frac{\phi s}{(1-\phi)(1-s)+\phi s}. \tag{15}$$

Subtracting ϕ from ϕ_G, we have

$$\phi_G - \phi = \frac{\phi(1-\phi)(2s-1)}{(1-\phi)(1-s)+\phi s}.$$

Because $\phi \in (0,1)$ and $s \in [0,1]$, $\phi_G > \phi$ iff $s > 1/2$. Otherwise, ϕ_G decreases below ϕ. Moreover, having ϕ as given to be fixed, $\phi_G = 1$ holds if and only if $s = 1$.

Summarizing the above discussion, we derive the following proposition.

Proposition 6 *The asset purchase prevents the liquidity crisis if and only if the central bank correctly selects the uncertain assets for purchase.*

The results of Proposition 6 highlight the importance of the selection of securities when implementing asset purchases. An asset purchase can end the liquidity crisis if and only if it reduces the rollover cost of the bank from χ to $1+r$ by increasing ϕ_G to 1. Because $\phi_G = 1$ holds under $s = 1$, the results of Proposition 6 suggest that although the central bank implements asset purchases with the intention of ending the liquidity crisis, it cannot stop the asset fire sale if asset selection is mistaken. This result holds even if the asset purchase can reduce the counterparty risk jointly with the liquidity risk. Suppose that the intervention reduces the counterparty risk from r to r_G and increases ϕ to ϕ_G. Denoting the rollover cost of the bank after the intervention as χ_G, it is given by

$$\chi_G = \frac{(1 + r_G)(1 + \phi_G)}{2\phi_G}.$$

However, as long as $\phi_G < 1$, the asset liquidation is still cheaper for the bank than the debt rollover because of $1 + r_G < \chi_G$. Therefore, as long as the central bank fails to select the purchasing asset, the intervention cannot stop the bank from selling the asset, even though the selling price is higher than the one without the intervention.

Implications for the financial crisis after the Lehman bankruptcy Shleifer and Vishny (2011) point out that the financial crisis after the bankruptcy of the Lehman Brothers investment bank in September 2008 was much worse than the one before that bankruptcy. They argue that the aggravation of the crisis stemmed from the liquidity crisis, whereby a large population of investors abstained from bidding in the security market. In line with their argument, our discussion in 6.1 based on the results of Proposition 5 implies that the liquidity provision does not end asset fire sales if the liquidity crisis occurs in the market for collateral assets. As we show in Proposition 6, appropriately selected asset purchases can end an asset fire sale by terminating the liquidity crisis in the markets for repo collateral. Shleifer and Vishny (2010b) also support asset purchases as part of monetary policy and emphasize the importance of asset purchase selection. They show theoretically that purchasing relatively safe securities mitigates financial difficulty. Note that the uncertain asset in our model has the same fundamental value as the certain one. In this way, the uncertain asset is safe, as it is not directly correlated with the risk in the market. The purchasing of an uncertain asset does not necessarily produce deterioration of the balance sheet of the central bank, but if $s < 1$, the central bank needs to keep purchasing enormous amounts of assets in order to eliminate uncertain assets from the banking sector. However, the outcome of this is that the central bank massively increases its balance sheet, and the effect of that greatly expanded balance sheet of the central bank on the financial system remains for our further research.

7 RELATED LITERATURE

The existing theoretical literature argues that the source of the vulnerability of the banking system is attributable to the simultaneous withdrawal of depositors, or a bank run, in line with the original insight of Diamond and Divbig (1983). In Allen and Gale (1994, 1998), the shock on the preference of depositors induces an aggregate shortage in liquidity through the maturity mismatch in the banking system. Diamond and Rajan (2000, 2001) argue that although a demand deposit is useful as a commitment device for a bank to use the collection skill from the borrowers for the depositors, it also makes the bank fragile if the simultaneous withdrawal of the deposit occurs. Shleifer and Vishny (2010a) argue in their unstable banking model that the banking system is fragile to the sentiment of investors, which is irrelevant to the fundamentals of the bank. Although the depositors withdraw for different reasons and causes, the occasion for withdrawals is only on an ad hoc basis in the existing literature. In contrast, we explicitly model repo trading and show that to the extent repo trading makes returns of creditors indifferent regardless of whether or not the bank becomes bankrupt, the rollover cost of the bank increases if the bankruptcy of the bank will trigger a liquidity crisis in the asset market. The increasing rollover cost induces the bank to delever itself in the midterm of the investing project (Date 1 in our model), even by liquidating the asset at a fire sale price. Hence, differently from the existing literature, the deleverage occurs not by the withdrawal of creditors, but by the early redemption with asset fire sales decided by the bank which is triggered by the crisis in the asset market.

Our model captures the main features of the modern banking system, where the bank invests its securitized asset by issuing an information-insensitive debt that is secured by the investing asset.[23] The closest model to ours is the one introduced by Acharya and Vishwanathan (2011). They also build their banking model around securitization and short-term borrowing backed by the securitized asset. In their model, there is a continuum of identical banks, and each of them invests in identical assets with the same long-term maturity. The banks decide either to sell or to buy their holding assets by pledging their assets in the midterm of the investing project. The decision of each bank depends on its funding liquidity,

[23]Although we do not formally model the process of securitization, our model implies that securitization makes information about the investing asset symmetric, as derived by De Marzo and Duffie (1999) and De Marzo (2005). Gorton and Souleles (2006) summarize the role and the mechanism of securitization. Acharya and Schnabl (2009) argue that banks increased leverage by using off-balance-sheet asset-backed commercial paper (ABCP) conduit and structured investment vehicles (SIV) before the subprime crisis in 2007.

which is the future value of the holding asset. The bank is determined to be either a buyer or a seller of the asset, because of the potential risk-shifting problem, where if the bank manager can replace the asset for the riskier one, that erodes the funding liquidity. Assuming that the asset trading is limited among the banks, the market price of the asset follows the cash-in-the-market pricing model as in Allen and Gale (1994, 1998), Acharya and Yorulmazer (2008), Acharya, Shin, and Yorulmazer (2011), and Bolton, Santos, and Scheinkman (2011), where the price of the asset is decided by the balance of the demand and the supply in the asset market. The lower is the funding liquidity, the more likely is the asset to be liquidated for a price below its fundamental value, because more banks are sellers than buyers in the asset market. This mechanism is similar to the ones introduced by Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). Interpreting the probability of success as a proxy of the business cycle, they show that the leverage ratio is procyclical, as it is endogenously determined by the business cycle.

The mechanism of the asset fire sale in Acharya and Vishwanathan (2011) follows the scheme of Shleifer and Vishny (1992). The lower funding liquidity financially constrains the potential buyers of the asset, which results in an asset fire sale. Although we also derive asset fire sales, the mechanism is quite different. First, the shortage of the funding liquidity at the stage of the debt rollover occurs in our model not because of the agency problem but because of the liquidity crisis in the asset market, which is anticipated to be triggered by the bankruptcy of the bank. Secondly, investors in our setting competitively enter the asset market. Because the fundamental value of the asset in our setting does not deteriorate by either the agency problem or idiosyncratic risk, more general investors are likely to bid in the market. This assumption links the asset price to the interbank borrowing condition, which reflects the degree of the counterparty risk. Finally and most importantly, we show that the fragility of the bank derives exogenously rather than from its own balance sheet, such as any deterioration in the quality of its holding assets.

Much of the existing literature, including Brunnermeier (2009), Gorton and Metrick (2011), and Shleifer and Vishny (2011), points out that a fire sale of financial assets, especially those related to deteriorated subprime loans and mortgages, is one of the most distinctive observations during the 2007–2009 financial crisis. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) introduced the mechanism of the spiral of deterioration of asset values. They showed that deterioration in future asset value (referenced as funding liquidity) reduces the asset liquidity in the current market (referred to as the market liquidity). The reduction of market liquidity reduces the asset price, which further decreases the value of the asset. Adrian and Shin (2010) and Shin (2010) argue that the regulatory capital requirement forces banks to delever themselves by selling holding assets for fire-sale prices. Although they successfully explain fire sales of deteriorated assets like the subprime assets, the decline in the prices of the securities that are *not* related to the subprime loans and mortgages jointly with the subprime-related ones (referred as the joint selling as in Wagner (2011)) is still in controversy. We show that repo trading spreads the counterparty risk, which reflects increasing doubt about the solvency of banks with deteriorated assets, to a bank that does not hold such assets. Our results are consistent with the findings of Gorton and Metrick (2011) and also related to those of Singh and Aitken (2009b), when they stress the impact of counterparty risk on the market value of the collateral assets.

Financial contagion, which refers to the propagation of the crisis in the financial system, has been mostly analyzed under the setting where banks are connected by the holding claims each has on the others.[24] Allen and Gale (2000) model banks that are directly connected though interregional claims. They showed that imperfectly correlated liquidity shocks spread though the banking sector via the interbank market. Cifuentes et al. (2005) state that a financial crisis that depresses prices of asset spreads through the interconnected financial assets held by banks. Diamond and Rajan (2005) focus on the asset side of the banking sector and show that a failure of one bank spreads by shrinking the common pool of liquidity. We show that even though a bank holds neither claims of other banks nor the assets that are exposed to the crisis, financial contagion occurs as long as banks engage in repo trading.

Because repo trading conveys not only counterparty risk but also any future liquidity crisis that is anticipated relative to the current borrowing condition of healthy banks, the intervention of the central bank does not always work if the intervention aims solely at controlling the interbank rate. This is because any future liquidity crisis reduces the current market liquidity of the asset, or equivalently, increases banks' borrowing costs. Purchasing correctly selected assets can ease any future liquidity crisis by eliminating the uncertainty of asset values on banks' balance sheets. In contrast, suppose that a financial contagion comes from a timing maturity mismatch caused by an unexpected liquidity shock but that available liquidity supports total liquidity needs. In this case, the intervention of the central bank may succeed in preventing financial contagion. Allen et al. (2009) introduced the role of central banks in responding to financial contagion spreading among banks through their overlapping

[24] Allen and Gale (2007) survey the literature on financial contagion.

claims.

Our literature is also related to Freixas and Holthausen (2004) with respect to formally examining the role of the repo trading in the banking system. That paper analyzes the cross-national interbank market and suggests that repo markets reduce the spread of interest rates between countries. They also showed that repo markets may mitigate asymmetric information between countries. So long as the collateral is marketable in the asset market, we show that market risk may spread to banks through the pricing of their collateral assets.

8 CONCLUSIONS

In this paper, we model a modern banking system that consists of securitization and provision of information-insensitive debt through repo trading. We show that repo plays an important role by transferring counterparty risk to the credit risk of a bank that pledges its asset holdings as repo collateral. In particular, the distribution of the counterparty risk is a key to changes in credit risks. Similar to the argument of neglected risks proposed by Gennaioli et al. (2011), if the probability of an increase in the counterparty risk is negligible ex ante but not ex post, the credit risk of the bank is more likely to rise sharply ex post. Furthermore, anticipation of a liquidity crisis triggered by bankruptcy of the bank leads to asset fire sales, by making debt rollover more costly for the bank than asset liquidation. We argue that absent from a liquidity crisis, the liquidity provision works to mitigate the financial crisis by reducing the credit risk of banks through decreasing counterparty risk. However, if a liquidity crisis is anticipated, asset purchases are required for avoiding asset fire sales. Purchasing correctly selected assets helps to make banks healthier by allowing them to dispose of assets that investors refrain from bidding for because of doubts about their fundamental value. However, if the central bank cannot correctly distinguish suspicious assets from healthy assets, it enlarges its own balance sheet in exchange for ending asset fire sales.

The most interesting implication of our analysis in regard to the 2007–2009 financial crisis is that banks that do not invest in deteriorated assets such as subprime-related securities are not immune from the crisis, to the extent that those banks are engaged in repo trading. The most plausible source of the crisis begins with the spread of banks' massive investment in deteriorating subprime securities, which eventually spread doubt about the solvency of the banks most exposed to such investments and thereby increased borrowing costs in the interbank market. Repo trading transfers the increasing counterparty risk to the assets that are not directly related to the subprime ones, by forcing banks to pledge more as collateral. These explanations provide the missing link to the recent empirical findings of Afonso et al. (2011) and Gorton and Metrick (2011) and increase the importance of the role of repo trading in the banking system. As introduced by Shleifer and Vishny (2011), it is plausible that the prevailing banks' setting up of off-balance-sheet ABCP conduits and SIVs and opacity of the true quality of exotic financial products may have triggered the liquidity crisis in response to the bankruptcy of some major investment banks, like Lehman Brothers. Such an inference together with our theoretical results explain that the liquidity crisis in the asset market triggered by the bankruptcy of Lehman Brothers in September 2008 largely contributed to the aggravation of the crisis. Although our results provide support for the reserve banks' asset purchases after the Lehman bankruptcy, we also emphasize the importance of their appropriate asset selection in limiting the fallout from the collapse of Lehman.

References

Acharya, Viral V., Douglas Gale, and Tanju Yorulmazer, 2011. Rollover Risk and Market Freeze, Journal of Finance 66, 1177–1209.

Acharya, Viral V., and Philipp Schnabl, 2009. How Banks Played the Leverage Game, in V. Viral Acharya and Matthew Richardson (eds.), Restoring Financial Stability: How to Repair a Failed System, New York University and John Wiley & Sons, Hoboken, New Jersey.

Acharya, Viral V., and Skeie, David, 2011. A Model of Liquidity Hoarding and Term Premia in Inter-bank Markets, Journal of Monetary Economics 58, 436–447.

Acharya, Viral V., and Tanju Yorulmazar, 2008, Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures, Review of Financial Economics 21, 2705–2742.

Acharya, Viral V., Hyun Song Shin, and Tanju Yorulmazer, 2011. Crisis Resolution and Bank Liquidity, Review of Financial Studies 24, 2166–2205.

Acharya, Viral V., and S. Vishwanathan, 2011. Leverage, Moral Hazard, and Liquidity, Journal of Finance 66, 99–138.

Adrian, Tobias, and Hyun Song Shin, 2010. Liquidity and Leverage, Journal of Financial Intermediation 19, 418–437.

Afonso, Gara, Anna Kovner, and Antoinette Schoar, 2011. Stressed, Not Frozen: The Federal Funds Market in the Financial Crisis, Journal of Finance 66, 1109–1139.

Allen, Franklin, and Douglas Gale, 1994.Limited Market Participation and Volatility of Asset Prices, American Economic Review, 84, 933–955.

Allen, Franklin, and Douglas Gale, 1998. Optimal Financial Crisis, Journal of Finance 53, 1245–1284.

Allen, Franklin, and Douglas Gale, 2000. Financial Contagion, Journal of Political Economy 108, 1–33.

Allen, Franklin, and Douglas Gale, 2007. Understanding Financial Crisis, Oxford University Press, Oxford.

Allen, Franklin, Elena Carletti, and Douglas Gale, 2009. Interbank Market Liquidity and Central Bank Intervention, Journal of Monetary Economics 56, 639–652.

Aragon, George O., and Philip E. Strahan, 2009. Hedge Funds as Liquidity Providers: Evidence from the Lehman Bankruptcy, NBER Working Paper No. 15336.

Benmelech, Efraim, and Jennifer Dlugosz, 2010. The Credit Rating Crisis, NBER Macro Annual, 161-207, National Bureau of Economic Research, Cambridge, MA.

Bolton, Patrick, Xavier Freixas, and Joel Shapiro, 2011. The Credit Rating Game, Journal of Finance, forthcoming.

Bolton, Patrick, Tano Santos, and Jose A. Scheinkman, 2011. Outside and Inside Liquidity, Quarterly Journal of Economics 126, 259–321.

Brunnermeier, Markus K., 2009. Deciphering the Liquidity and Credit Crunch 2007–2008, Journal of Economic Perspectives 23, 77–100.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009. Market Liquidity and Funding Liquidity, Review of Financial Studies 22, 2201–2238.

Cifuentes, Rodrigo,Gianluigi Ferrucci, and Hyun Song Shin, 2005. Liquidity Risk and Contagion, Journal of the European Economic Association 3, 556–566.

Choudhry, Moorad, 2006. An Introduction to Repo Markets, Third edition, Securities and Investment Institute, John Wiley & Sons, West Sussex, England.

Coval, Joshua, Jakub Jurek, and Erik Stafford, 2009. The Economics of Structured Finance, Journal of Economic Perspectives 23, 3–25.

Dang, Tri Vi, Gary Gorton, and Bengt Holmstrom, 2009. Ignorance and the Optimality of Debt for the Provision of Liquidity, mimeo.

De Marzo, 2005. The Pooling and Tranching of Securities: A Model of Informed Intermediation, Review of Financial Studies 18, 1–35.

De Marzo, Peter, and Darrell Duffie, 1999. A Liquidity-based Model of Security Design, Econometrica 67, 65–99.

Diamond, Douglas, and Philip Divbig, 1983. Bank Runs, Deposit Insurance, and Liquidity, Journal of Political Economy 91, 401–419.

Diamond, Douglas, and Raghuram G. Rajan, 2000. A Theory of Bank Capital, Journal of Finance 55, 2431–2465.

Diamond, Douglas, and Raghuram G. Rajan, 2001. Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking, Journal of Political Economy 109, 287–327.

Diamond, Douglas, and Raghuram G. Rajan, 2005. Liquidity Shortages and Banking Crises, Journal of Finance 60, 615–647.

Duffie, Darrell, 2010. The Failure Mechanics of Dealer Banks, Journal of Economic Perspectives 24, 51–72.

Freixas, Xavier, and Cornelia Holthausen, 2004. Interbank Market Integration under Asymmetric Information, Review of Financial Economics 18, 459–490.

Friedman, Benjamin M., and Kenneth N. Kuttner, 2011. Implementation of Monetary Policy: How Do Central Banks Set Interest Rates?, in Benjamin M. Friedman and Michael Woodford (eds.), Handbook of Monetary Economics, Elsevier, 1345–1438.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2011. Neglected Risks, Financial Innovation, and Financial Fragility, Journal of Financial Economics, forthcoming.

Gorton, Gary B., and Nicholas S. Souleles, 2006. Special Purpose Vehicles and Securitization, in Mark Carey and Rene M. Stulz (eds.), The Risks of Financial Institutions, University of Chicago Press, Chicago, 549–602.

Gorton, Gary, 2010. Slapped by the Invisible Hand, Oxford University Press.

Gorton, Gary, and Andrew Metrick, 2011. Securitized Banking and the Run on Repo, Journal of Financial Economics, forthcoming.

Gorton, Gary, and George Pennacchi, 1990. Financial Intermediaries and Liquidity Creation, Journal of Finance 45, 49–72.

Gorton, Gary, and Andrew Winton, 2003. Financial Intermediation, in George C. Constantinides, Milton Harris, and Rene M. Stulz (eds.), Handbook of the Economics of Finance: Corporate Finance, Elsevier Science, 432–552.

Gromb, Denis, and Dimitri Vayanos, 2002. Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs, Journal of Financial Economics 66, 361–407.

Holmstrom, Bengt, 2008. Discussion of 'The Panic of 2007' by Gary Gorton, in Maintaining Stability in a Changing Financial System, Proceedings of the 2008 Jackson Hole Conference, Federal Reserve Bank of Kansas City, 2008.

Jacklin, Charles J., 1987. Demand Deposits, Trading Restrictions, and Risk Sharing, in E. Prescott and N. Wallace (eds.), Contractual Arrangements for Intertemporal Trade, University of Minnesota Press, 26–47.

Jaffee Dwight, Anthony W. Lynch, Matthew Richardson, and Stijn Van Nieuwerburgh, 2009. Mortgage Origination and Securitization in the Financial Crisis, in Viral V. Acharya and Matthew Richardson (eds.), Restoring Financial Stability: How to Repair a Failed System, New York University and John Wiley & Sons, Hoboken, New Jersey.

Krishnamurthy, Arvind, 2010. How Debt Markets Have Malfunctioned in the Crisis, Journal of Economic Perspectives 24, 3–28.

Levy, Haim, 1998. Stochastic Dominance: Investment Decision Making Under Uncertainty, Kluwer Academic Publishers.

Pagano, Marco, and Paolo Volpin, 2010. Securitization, Transparency, and Liquidity, London Business School working paper.

Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky, 2010. Shadow Banking, Working Paper, Federal Reserve Bank of New York.

Rothschild, Michael, and Joseph E. Stiglitz, 1970. Increasing Risk: I. A Definition, Journal of Economic Theory 3, 225–243.

Shin, Hyun Song, 2009. Reflections on Northern Rock: The Bank Run that Heralded the Global Financial Crisis, Journal of Economic Perspectives 23, 101–120.

Shin, Hyun Song, 2010. Risk and Liquidity, Oxford University Press, Oxford.

Singh, Manmohan, and James Aitken, 2009a. Deleveraging after Lehman. Evidence from Reduced Rehypothecation, IMF Working Paper 09/42.

Singh, Manmohan, and James Aitken, 2009b. Counterparty Risk, Impact on Collateral Flows, and a Role for Central Counterparties, IMF Working Paper 09/173.

Shleifer, Andrei, and Robert Vishny, 1992. Liquidation Values and Debt Capacity: A Market Equilibrium Approach, Journal of Finance 47, 1343–1366.

Shleifer, Andrei, and Robert Vishny, 1997. The Limits of Arbitrage, Journal of Finance 52, 35–55.

Shleifer, Andrei, and Robert Vishny, 2010a. Unstable Banking, Journal of Financial Economics 97, 306–318.

Shleifer, Andrei, and Robert Vishny, 2010b. Asset Fire Sales and Credit Easing, American Economic Review: Papers & Proceedings 2010, 100, 46–50.

Shleifer, Andrei, and Robert Vishny, 2011. Fire Sales in Finance and Macroeconomics, Journal of Economic Perspectives 25, 29–48.

Sketa, Vasiliki, and Laura Veldkamp, 2009. Rating Shopping and Asset Complexity: A Theory of Rating Inflation, NBER Working Paper No. 14761.

Tang, Tony T., 2009. Information Asymmetry and Firms’ Credit Market Access: Evidence from Moody’s Credit Rating Format Refinement, Journal of Financial Economics 93, 325–351.

Wagner, Wolf, 2011. Systemic Liquidation Risk and the Diversity–Diversification Trade-Off, Journal of Finance 66, 1141–1175.

White, Lawrence J., 2002. The Credit Rating Industry: An Industrial Organization Analysis, in Richard M. Levich, Giovanni Majnoni and Carmen Reinhart (eds.), Ratings, Rating Agencies and the Global Financial System, Kluwer Academic Publishers, Boston.

White, Lawrence J., 2010. Markets: The Credit Rating Agencies, Journal of Economic Perspectives 24, 211–226.

APPENDIX

Proof of Proposition 1 Let us form the Lagrangian for the maximization problem of the bank

$$\mathcal{L} = \Pi_B + \lambda\beta,$$

where λ is the multiplier of (BC1). Because Π_B is concave in β, $\mathcal{L}$ is also concave in β. Optimizing with respect to β yields

$$\frac{\partial \mathcal{L}}{\partial \beta} = -(1+r) + (1+r+\nu)(1-\beta) + \lambda = 0. \tag{A1}$$

If $\lambda > 0$, $\beta = 0$. However, substituting $\beta = 0$ into (A1), we derive $\frac{\partial \mathcal{L}}{\partial \beta} > 0$. Hence, $\lambda = 0$. The optimal β that makes (A1) equal to zero is then derived as

$$\beta = 1 - \frac{1+r}{1+r+\nu} = \frac{\nu}{1+r+\nu} \in (0,1). \tag{A2}$$

According to (BC1), the upper limit of β depends on whether $\frac{y_1^F}{1+r}$ is larger or smaller than 1. We first suppose that $1 + r \leq y_1^F$. Because $\min(\frac{y_1^F}{1+r}, 1) = 1$, (BC1) is given by $1 > \beta > 0$. Therefore, β given by (A2) always satisfies (BC1).

In contrast, suppose that $y_1^F < 1+r$. In this case, (BC1) is given by $\frac{y_1^F}{1+r} > \beta > 0$. Hence, the condition under which β given as in (A2) satisfies (BC1) is given by

$$\frac{y_1^F}{1+r} > \frac{\nu}{1+r+\nu},$$

or equivalently

$$y_1^F > \frac{\nu(1+r)}{1+r+\nu}. \tag{A3}$$

We now derive the condition under which β given by (A2) satisfies (BC0) under the constraint of (A3). Substituting (A2) into Π_B, we derive

$$\Pi_B = y_1^F - \frac{1+r}{1+r+\nu}\Big(\nu + \frac{1+r}{2}\Big). \tag{A4}$$

As long as Π_B in (A4) is strictly positive, (BC0) is satisfied and the bank invests in the project. Because $r > 0$, we have

$$\frac{\nu(1+r)}{1+r+\nu} < \frac{1+r}{1+r+\nu}\Big(\nu + \frac{1+r}{2}\Big).$$

The above inequality indicates that the bank invests in the project if and only if

$$\frac{1+r}{1+r+\nu}\Big(\nu+\frac{1+r}{2}\Big)<y_1^F$$

and decides the optimal β as given in Proposition 1(ii), which is equal to (A2). Otherwise, the bank does not invest in the asset at Date 0. ■

Proof of Proposition 3 Applying the same logic to the proof of Proposition 1, we form the Lagrangian for the Problem $[\mathcal{U}]$ as

$$\mathcal{L}_U=\Pi_U+\lambda_U\beta,$$

where λ_U is the multiplier of (UC1). Optimizing with respect to β yields

$$\frac{\partial\mathcal{L}_U}{\partial\beta}=-(1+r_e)+(1+r_e+\nu)(1-\beta)+\lambda_U=0. \tag{A5}$$

Following the same logic as in the proof of Proposition 1, we derive $\lambda_U=0$; thus, $\beta>0$. Deriving β from (A5), we have

$$\beta_U=\frac{\nu}{1+r_e+\nu}\in(0,1). \tag{A6}$$

The upper limit of β_U is differently given whether $\frac{y_1^F}{1+r_H}$ is larger or smaller than 1, because of (UC1). First, suppose that $1+r_H\le y_1^F$. The constraint (UC1) is given by $1>\beta>0$. Because $\beta_U\in(0,1)$ from (A5), (UC1) is always satisfied.

In contrast, suppose that $1<y_1^F<1+r_H$. In this case, $\min(\frac{y_1^F}{1+r_H},1)=\frac{y_1^F}{1+r_H}$. Thus, the condition under which β_U in (A5) satisfies (UC1) is $\frac{y_1^F}{1+r_H}>\beta_U$, or equivalently,

$$y_1^F>\frac{\nu(1+r_H)}{1+r_e+\nu}\equiv\Omega_1. \tag{A7}$$

The condition under which β_U in (A6) satisfies (UC0) under the constraint of (A7) is derived by substituting (A6) into Π_U as

$$\begin{aligned}\Pi_U&=y_1^F-\frac{1+r_e}{1+r_e+\nu}\Big(\nu+\frac{1+r_e}{2}\Big)\\&\equiv y_1^F-\Omega_0.\end{aligned}$$

Notice that the value of Ω_i $(i=0,1)$ is a threshold of y_1^F above which the bank decides to invest in the asset at Date 0. Hence, if $y_1^F\le\max(\Omega_0,\Omega_1)$, the bank does not invest in the asset at Date 0. In contrast, if $\max(\Omega_0, \Omega_1) < y_1^F$, the bank invests in the asset and decides its leverage ratio as (A6).

For deriving $\max(\Omega_0, \Omega_1)$, we now compare Ω_0 with Ω_1. Subtracting Ω_1 from Ω_0, we derive

$$\begin{aligned}\Omega_0 - \Omega_1 &= \frac{1}{1 + r_e + \nu}\left[(1 + r_e)\left(\nu + \frac{1 + r_e}{2}\right) - (1 + r_H)\right] \\ &\equiv \frac{1}{2(1 + r_e + \nu)} f(r_e). \end{aligned} \tag{A8}$$

Note that $f(r_e)$ in (A8) is a quadratic function of r_e as given by

$$f(r_e) = (r_e)^2 + 2(1 + \nu)r_e + 2(\nu - r_H) - 1.$$

The value of $r_e(> 0)$ that makes $\Omega_0 = \Omega_1$ is then derived as

$$r_{e+} = -(1 + \nu) + \sqrt{\nu^2 + 2(1 + r_H)}.$$

Because $f(r_e)$ is convex in r_e, we have

$$\max(\Omega_0, \Omega_1) = \begin{cases} \Omega_0 & \text{if } r_{e+} \leq r_e, \\ \Omega_1 & \text{if } r_e < r_{e+}, \end{cases}$$

if $r_{e+} > 0$. On the other hand, if $r_{e+} \leq 0$, $\Omega_0 \geq \Omega_1$ holds for any $r_e > 0$. Summarizing these conditions, we derive

$$\max(\Omega_0, \Omega_1) = \begin{cases} \Omega_0 & \text{if } r_H - \nu \leq \frac{1}{2}, \text{ or } r_H - \nu > \frac{1}{2} \text{ and } r_{e+} < r_e, \\ \Omega_1 & \text{if } r_H - \nu > \frac{1}{2} \text{ and } r_e \in (0, r_{e+}]. \end{cases}$$

The results are summarized as in Proposition 3. ■

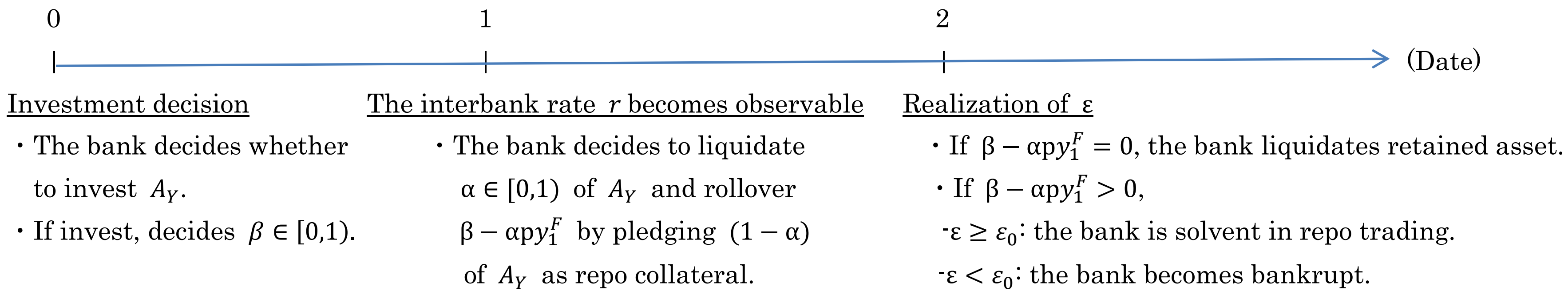

Figure 1: Timing and events