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Unemployment Equilibria and Economic Fluctuations in a Monetary Production Economy

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Abstract

This paper presents a simple model of effective demand in the monetary production economy. The important features of the paper compared to previous studies are discrimination between factor income distribution and factor price determination and application of *the principle of effective demand* to all markets. In addition, the real wage depends on the expected inflation rate. The employment function provides the aggregate supply curve, which is affected by the inflation rate. Consequently, the aggregate demand size affects the aggregate income and employment level. A constitutive shortage of aggregate demand brings about constitutive unemployment equilibria.

Keywords: Unemployment equilibria; Business cycle; OLG model
JEL classification:

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1 Introduction

The issue of economic fluctuation and employment has remained a matter of greatest public concern since industrialised societies were first established. Particularly the Great Depression affected people with misery and wretchedness of the economic crisis. Even today, recessions often occur and economic crises are not completely divorced from people's lives.

An influential idea among economic theories related to recessions is *the principle of effective demand* presented formerly by Kalecki (1933) and subsequently by Keynes (1936).[1] Especially, Keynes (1936) widely diffused the principle of effective demand, which is counter to the concept of Say's Law. He asserted that aggregate demand plays a key role in the determination of output and employment in an economy subject to the influences of demand.

The Keynesian models have built an era of mainstream theory on economic fluctuations (e.g., Keynes 1936; Hicks 1937; Modigliani 1944). At present, the new Keynesian macroeconomics provides microfounded general equilibrium models incorporating imperfect competition, sticky price and information (e.g., Calvo 1983; Akerlof and Yellen 1985; Mankiw 1985; Blanchard and Kiyotaki 1987; Blanchard and Fisher 1989; Yun 1996; McCallum and Nelson 1999; Mankiw and Reis 2002).[2]

The departure from perfect competition implies that a monopolistic firm, confronting a downward sloping demand curve, sets a price above marginal cost. Such distortion exists. Therefore, the profits of firms and the consumer's income are mutually reinforcing. Greater demand for goods gives rise to higher profits and consumers' income. Moreover, higher income feeds back demand for goods through consumption.

Similarly, new Keynesian monetary models also assume the presence of price adjustment costs or the infrequent opportunity of price revision, unlike the neoclassical monetary models. These assumptions engender nominal rigidities such as sticky prices or wage rates. Nominal rigidities bring about economy-wide market failure.

In new Keynesian models, the recessions or underemployment equilibria are temporary deviations from the natural equilibrium under efficient market functions. The economy arrives at full employment equilibrium as do new Keynesian models in the long run without market friction because all sources of nominal rigidities disappear in the long run.

Nevertheless, a prolonged recession or stagnation is not unusual in the real world. In that respect, new Keynesian models cannot sufficiently explain constitutive unemployment equilibria, but they contribute to provision of a micro-founded explanation of nominal rigidities and clarifying short-term economic fluctuations.

Reverting to assertions by Kalecki (1933) and Keynes (1936), the principle of effective demand is a powerful tool for clarifying constitutive unemployment equilibria. In other words, the source of constitutive unemployment equilibria is a shortage for aggregate demand, which quantities of demand determine quantities of supply, rather than imperfections of the market structure, stickiness of prices, or information.

[1] The model of effective demand developed by Kalecki (1933) is based on a Marxian economic view.

[2] See Woodford (2003) and Galí (2008) for a general review of this literature on new Keynesian monetary general equilibrium models.

That explanation leaves plenty of room for developing a model of effective demand that is separate from the influence of imperfect competition. Indeed, this paper presents development of the model based on the principle of effective demand using an overlapping generations model with money. Key attributes of our approach are separation of factor income distribution and factor price determination and applications of the principle of effective demand to all markets (i.e., goods, money and labour markets).

The principle of effective demand implies that the production schedule is subject to the effective demand in the goods market. Labour demand is subject to the aggregate demand for final good, i.e., firms cannot arbitrarily decide the distribution of factor income based on the marginal principle and households also cannot do it naturally. The distribution of factor income must be distinguished from the factor price determination under the principle of effective demand.

The separation between factor income distribution and factor price determination produces the dissociation of aggregate demand and aggregate supply. In other words, it rejects the possibility of a one-to-one correspondence between the variation of aggregate supply and of aggregate demand: Say's Law. This economic condition is a structural phenomenon rather than a short-run phenomenon that occurs during the adjustment process.

Consequently, this paper shows that a constitutive shortage of aggregate demand engenders constitutive unemployment equilibria. This paper demonstrates the existence of unemployment equilibria using an alternative approach of some recent studies (e.g., Ono 1994, 2001; Otaki 2007). Furthermore, this paper shows a possibility of generating multiple unemployment equilibria and an endogenous business cycle.

The remainder of this paper is organised as follows: Section 2 presents a full description of our basic model. Section 3 presents characterisation of the properties of dynamic equilibria, e.g., investigation of the existence of equilibria and transitional dynamics. Section 4 presents a description of analyses of the macroeconomic effects of fiscal policy. Section 5 provides welfare analysis and some discussions related to application and limitation of the basic model. Finally, Section 6 explains the conclusions of this paper.

2 The economy

Consumption and labour supply. Individuals are born at continuous density $[0, n] \times [0, n]$ in each period. They live for two periods (young and old) and supply one unit of labour. They do so only when they are young. In this economy, two types of individual indexed by i exist: employed persons ($i = e$) and unemployed persons ($i = u$). For analytical simplicity, the total mass of individual at generation t (for all t) is normalized to unity, as $n = 1$.

Individuals maximise their lifetime utility subject to their budget constraint. The lifetime utility is assumed to be $U_t^i = (c_t^y)^\beta (c_{t+1}^o)^{1-\beta}$, where c_t^y is the consumption when individuals are young and c_{t+1}^o is the consumption when individuals are old and $0 < \beta < 1$. The budget constraint for i's individual is $p_t c_t^y + p_{t+1} c_{t+1}^o = I_t^i$ where p_{t+1} is the price of goods at period $t+1$, p_t the price of goods at period t, and I_t^i the income of i individual.

Solving the optimisation problem, the consumption functions are given as

$$c_t^y = \frac{\beta I_t^i}{p_t} \text{ and } c_{t+1}^o = \frac{(1-\beta) I_t^i}{p_{t+1}}, \tag{1}$$

where

$$I_t^i = \begin{cases} W_t + R_t - H_t & \text{if } i = e \\ R_t - H_t & \text{if } i = u \end{cases}$$

In the expression above, W_t signifies the nominal labour income, R_t denotes the nominal profit share and H_t stands for the nominal tax.

Each individual provides one unit of labour input if the indirect utility when employed is sufficiently larger than the utility when unemployed. Therefore, the individuals calculate the nominal reservation wage according to their minimum permissible standard of living as a standard employee:

$$U_t^e - U_t^u = \left(\frac{\beta}{p_t}\right)^{\beta} \left(\frac{1-\beta}{p_{t+1}}\right)^{1-\beta} W_t \geq \mu,$$

where $\mu > 0$. If individuals supply labour, then the nominal wage rate is expected to satisfy

$$W_t \geq \phi p_t^{\beta} p_{t+1}^{1-\beta}, \tag{2}$$

where $\phi := \mu / [\beta^{\beta}(1-\beta)^{1-\beta}]$.[3] The term of the right-hand-side of (2) is the nominal reservation wage.

Determination of labour and profit share. Within the firm as good producer, distribution of gross income is determined after negotiation between insiders supply factors of production. Bargaining between the management and the labour is formulated as

$$\max_{R_t} \left(p_t y_t - R_t\right)^{\gamma} R_t^{1-\gamma}.$$

The first-order condition is

$$\frac{R_t}{p_t y_t} = 1 - \gamma \Leftrightarrow \frac{W_t l_t}{p_t y_t} = \gamma. \tag{3}$$

Production and employment function. The final good is producible by labour and fixed capital input. The production technology is formulated as $y_t = f(l_t)$, where the aggregate output is increasing monotonically in labour input, i.e. $f'(\cdot) \geq 0$. Assume that the marginal productivity of labour is non-positive when the labour input is sufficiently large: $f''(\infty) \leq 0$. However, we do not exclude the case in which the marginal productivity of labour is locally increasing in the labour input.

The firms must decide their production schedule subject to aggregate demand.[4] Then, the relation between output and employment is

$$l_t = \min[l(y_t^d), n], \tag{4}$$

[3] Otaki (2007) derives a similar equation by assuming an indivisible labour supply and disutility of labour supply.

[4] This approach follows Clower (1965).

where y_t^d denotes the aggregate demand, $l(y_t^d) := f^{-1}(y_t^d)$ and $y^* := f(n)$. For $l_t < n$,

$$\frac{dl(y_t^d)}{dy_t^d} = \frac{1}{f'(l_t)} > 0.$$

Equation (4) is interpreted as the employment function in Keynes's phrasing.

Furthermore, the respective managements of firms intend to minimise product costs subject to the labour supply condition (2) and the employment function (4). Then, the wage rate set by management is

$$W_t = \phi p_t^{\beta} p_{t+1}^{1-\beta}. \tag{5}$$

Equation (5) implies that the nominal wage rate depends not only on the current price level but also on the future price level.[5] In other words, the real wage rate is positively affected by the inflation rate.

Government and the central bank. The public sector consists of government and the central bank. At present, each is an independent agent. The government and central bank respectively determine the amount of government expenditure and that of the money stock. However, regarding financial accounts, the central bank is not independent of government because the central government is the government's banker.

Because the government imposes the tax on individuals and allocates the revenue for government spending, the central bank accounts (balance sheet in the nominal term) are represented as $H_t + \Delta M_t = G_t$ where G_t is the nominal government spending $\Delta M_t = M_t - M_{t-1}$ and M_t the nominal stock of money. Assuming that the central bank sets the nominal supply of money at time t to $M_t = (1 + \lambda_t)M_{t-1}$, then $\Delta M_t = \lambda_t M_{t-1}$. Consequently, the budget constraint of government is $H_t + \lambda_t M_{t-1} = G_t$.

3 Dynamic equilibria

This section describes an investigation of the properties of dynamic equilibria. We now consider the clearing conditions for goods and money markets. In the goods and money markets, the following inequalities are expected to hold.

$$y_t^d \leq y_t, \tag{6}$$

$$(1-\beta)y_t^d \leq \frac{M_t}{p_t} \tag{7}$$

Equations of (6) and (7) give clearing conditions for goods and money markets.

Applying the principle of effective demand to the money market, the quality of (7) implies that the aggregate demand size determines the nominal money supply. Consequently, the growth rate of the nominal money supply λ_t is determined endogenously

[5] Using (5), we can formulate the bargaining of determination of the labour and profit share as the determination of future price by the other way to describe it, as stated above. For example, we can set

$$\max_{p_{t+1}} \left[p_t y_t^d - \phi p_t^{\beta} p_{t+1}^{1-\beta} l(y_t^d)\right]^{1-\gamma} \left[\phi p_t^{\beta} p_{t+1}^{1-\beta} l(y_t^d)\right]^{\gamma}.$$

Then, the first-order condition is given as equation (3).

as

$$\lambda_t = \frac{g - \tau y_t}{[1-(1-\tau)\beta]y_t - g}.$$

We next consider aggregate demand and supply. Because the aggregate demand is defined as the total expenditure of the economy, it is $y_t^d = c_t + G_t/p_t = c_t + g_t$ where $c_t := c_t^y + c_t^o$ and $g_t := G_t/p_t$. By (1), and the aggregate demand function is

$$y_{t+1} = \frac{p_t}{p_{t+1}}(y_t - h_t) + \frac{g_{t+1} - \beta h_{t+1}}{1-\beta}.$$

Presuming that the tax revenue at period t depends on the aggregate income at period t and presuming also that public spending is maintained as constant by the government, that is $h_t = \tau y_t$ and $g_{t+1} = g_t = g$. Then, the aggregate demand function is rewritten as

$$y_{t+1} = \frac{(1-\beta)(1-\tau)y_t}{[1-(1-\tau)\beta]\pi_{t+1}} + \frac{g}{1-(1-\tau)\beta}, \tag{8}$$

where $\pi_{t+1} := p_{t+1}/p_t$. The AD curve implies that the current price level depends negatively on current aggregate income for given past economic variables.

The aggregate supply function represents the relation between the price level and aggregate output. Using (3), (5), and (6), we obtain the aggregate supply function as

$$\pi_{t+1} = \left[\frac{\min\{y_t, y^*\}}{\min\{l(y_t), n\}}\right]^{\frac{1}{1-\beta}} \rho, \tag{9}$$

where $\rho := (\gamma/\phi)^{1/(1-\beta)}$. The AS curve shape depends on the property of employment function.

By elimination of π_{t+1}, equations (8) and (9) determine the relation between current aggregate income and past aggregate income. Indeed, the dynamic equation of realised aggregate income is

$$y_{t+1} = \min[F(y_t), y^*] \tag{10}$$

where

$$F(y_t) := \left[\frac{\min\{l(y_t), n\}}{\min\{y_t, y^*\}}\right]^{\frac{1}{1-\beta}} \frac{(1-\beta)(1-\tau)y_t}{[1-(1-\tau)\beta]\rho} + \frac{g}{1-(1-\tau)\beta}.$$

$$F'(y_t) = \begin{cases} \frac{[\epsilon(y_t)-\beta](1-\tau)}{[1-(1-\tau)\beta]\rho}\left[\frac{l(y_t)}{y_t}\right]^{\frac{1}{1-\beta}} \gtreqless 0 \Leftrightarrow \epsilon(y_t) \gtreqless \beta & \text{if } y_t < y^*. \\ \frac{(1-\beta)(1-\tau)}{[1-(1-\tau)\beta]\rho}\left(\frac{n}{y^*}\right)^{\frac{1}{1-\beta}} > 0 & \text{if } y^* < y_t. \end{cases}$$

The steady-state that is a long-run equilibrium satisfies $y_{t+1} = y_t = y$ and $\pi_{t+1} = \pi_t = \pi$. Using these conditions, the steady-state AD curve is represented as

$$AD: \ \pi = \frac{(1-\beta)(1-\tau)y}{[1-(1-\tau)\beta]y - g}.$$

The gradient of AD curve is

$$\left.\frac{d\pi}{dy}\right|_{AD} = -\frac{(1-\beta)(1-\tau)g}{[\{1-(1-\tau)\beta\}y-g]^2} < 0.$$

However, the steady-state AS curve is represented as

$$AS:\ \pi = \left[\frac{\min\{y, y^*\}}{\min\{l(y), n\}}\right]^{\frac{1}{1-\beta}} \rho.$$

The gradient of AS curve is

$$\frac{d\pi}{dy} = \frac{\rho}{1-\beta}\left[\frac{y}{l(y)}\right]^{\frac{\beta}{1-\beta}} \frac{1-\epsilon(y)}{l(y)} \gtreqless 0 \Leftrightarrow \epsilon(y) \lesseqgtr 1,$$

where $\epsilon(y) := l'(y)y/l(y)$ stands for the elasticity of labour demand with respect to aggregate income.

The AS curve has an upward slope if $\epsilon(y) < 1$. Increasing returns to scale satisfies $\epsilon(y) < 1$. Because the AD curve has a downward slope, there exists a unique steady-state equilibrium (Fig. 1a). If $\epsilon(y) = 1$, that is the linear production function, then the AS curve is a horizontal line. In the same mode of $\epsilon(y) < 1$, there exists a unique steady-state equilibrium (Fig. 1b). If $\epsilon(y) > 1$, that is decreasing-returns to scale production function, then the AS curve has a downward slope. The existence of a steady-state equilibrium is ambiguous (Fig. 1c).

Regarding the existence and stability of long-run equilibria, we establish the following proposition.

Proposition 1. *(i) There exists at least one long-run unemployment equilibrium and no full-employment equilibrium if*

$$\pi^* > \frac{(1-\beta)(1-\tau)}{1-(1-\tau)\beta-\eta^*}.$$

In that equation, $\pi^* := (y^*)^{1/(1-\beta)}\rho$ *and* $\eta^* := g/y^*$*. Then, the long-run unemployment equilibrium is stable or a periodic cycle exists. (ii) There exists at least one long-run stable equilibrium including full-employment equilibrium if*

$$\pi^* < \frac{(1-\beta)(1-\tau)}{1-(1-\tau)\beta-\eta^*}.$$

Proof. See Appendix. □

Proposition 1 shows that the unemployment equilibria are indigenous to the economy according to the economic conditions. Furthermore, endogenous business cycles are indigenous to the economy in some cases. Figures 2a and 2b portray an unemployment equilibrium and endogenous business cycles.

According to the principle of effective demand, the labour demand is subject to the aggregate demand for final goods. As a matter of course, the distribution of factor income diverges from the factor price determination. The divergence between the factor income distribution and factor price determination also separates aggregate demand and aggregate supply schedule. Then, products are *not* paid for with products. Therefore, the possibility exists of a general glut that brings about persistent unemployment equilibria.

A key to generating a business cycle is an increasing-returns production function. The increasing returns provides the upward dynamic AS curve that exhibits a positive relation between the inflation rate and employment. The dynamic AD curve exhibits a negative correlation between the aggregate income at next period and inflation rate. The increasing returns brings about a strong negative feedback of aggregate income dynamics. Consequently, the aggregate income traces a cyclical path around the unemployment equilibrium.

The roles of government and the central bank are important to escape the ill-fated economic conditions. However, in this paper, monetary policy is subject to the demand for real money; the monetary policy does not affect the output and employment level. Therefore, we investigate the effectiveness of fiscal policy in the next sections.

4 Macroeconomic effects of fiscal policy

The effectiveness of fiscal policy on aggregate income has long been discussed since Keynes (1936). According to Keynesian economics, one unit of additional government spending will engender the additional increase in aggregate income more than unity if the government spending can be independent of tax revenue.[6]

New Keynesian models demonstrate that a balanced-budget fiscal multiplier is less than unity (e.g., Mankiw 1988; Startz 1989).[7] Molana and Montagna (2000) show that whether the multiplier effect is greater than unity or not depends on preferences for variety, monopoly power of firms, heterogeneity of firms, individuals' preference, etc.[8] Recently, Bénassy (2007) and Otaki (2007) described that the fiscal multiplier is larger than unity under the non-Ricardian framework.

We begin our analysis of macroeconomic effects of fiscal policy to calculate the short-run and long-run government expenditure multiplier. Partial differentiation (10) with respect to g gives

$$\frac{\partial y_{t+1}}{\partial g} = \frac{1}{1-(1-\tau)\beta} > 1. \tag{11}$$

The short-run effect of government expenditure on aggregate income is positive and one unit of increase in government expenditure increases aggregate income more than its expenditure.

Fiscal expansion such as government purchase of goods directly increases aggregate demand in the current period. An increase in aggregate demand increases the aggregate output, and therefore boosts the aggregate income. An increase in current aggregate income induces consumption by young people. Their marginal propensity

[6]It is also shown under a balanced-budget constraint of government that the Keynesian fiscal multiplier is equal to unity (e.g. Haavelmo 1945).

[7]See Matsuyama (1995) for a general review of these models with monopolistic competition.

[8]Molana and Moutos (1992) show non-positive multipliers for labor income tax rates.

to consume is $(1-\tau)\beta$. Therefore, the short-run government expenditure multiplier is greater than unity.

Presuming that a unique long-run equilibrium exists, then in the long run, the effect of a change in government expenditure represents a cumulative short-run effect of government expenditure. The effect of government expenditure on the inflation rate must be considered. Total differentiation of (10) and $d\tau = 0$ engender the long-run government expenditure multiplier, as

$$\frac{\partial y}{\partial g} = \frac{\pi}{\pi - (1-\tau)[(\pi-1)\beta + \epsilon]} > 0, \tag{12}$$

where the denominator of (13) is positive:

$$\pi > \frac{(\epsilon - \beta)(1-\tau)}{1-(1-\tau)\beta}$$

An increase in government expenditure raises the aggregate income through the short-run multiplier effect. Simultaneously, an increase in aggregate income affects the inflation rate and so affects the adjustment process of aggregate income in the subsequent next period. According to the form of the AS curve, i.e., ϵ and π^*, the long-run effect of government expenditure is varied. The AS curve with upward slope will weaken the effect of government expenditure, although the AS curve with downward slope will strengthen it.

The results above are summarised as the following proposition:

Proposition 2. *The short-run government expenditure multiplier is always larger than unity. In the long run, the government expenditure multiplier is larger than unity if and only if*

$$\pi > \frac{\beta - \epsilon}{\beta}.$$

We next consider the effect of a change in tax revenue. The increment of tax revenue follows $dh_t = \tau dy_t + y_t d\tau$. In the short run, y_t is taken as given; $dy_t = 0$ holds. Then, we obtain $dh_t = y_t d\tau$. The partial derivative (10) with respect to τ and $dh_t = y_t d\tau$ engender

$$\frac{\partial y_{t+1}}{\partial h_t} = -\left[\frac{(1-\beta)y_t + \beta g \pi_{t+1}}{y_t \pi_{t+1}}\right]\left(\frac{\partial y_{t+1}}{\partial g}\right)^2 < 0. \tag{13}$$

A tax increase decreases disposable income and aggregate consumption. It brings about the negative short-run multiplier process. Therefore, the short-run effect of tax on aggregate income is negative. Proposition 2 shows that an increase in government expenditure by deficit financing has a positive effect on aggregate income, which means the underwriting of Treasury Bills by the central bank or seigniorage financing. However, the financial resources of government expenditures depend on tax revenue in many cases.

Presuming that the government covers an additional increase in government expenditure by an additional increase in taxes, then $dh_t = dg_{t+1} = dg$ holds. Using (11) and (13), the short-run balanced-budget multiplier is

$$\begin{aligned}\frac{dy_{t+1}}{dg} &= \frac{\partial y_{t+1}}{\partial g} + \frac{\partial y_{t+1}}{\partial h_t}\frac{dh_t}{dg} \\ &= \frac{[1-(1-\tau+\eta_t)\beta]\pi_{t+1} - (1-\beta)}{[1-(1-\tau)\beta]\pi_{t+1}}\frac{\partial y_{t+1}}{\partial g} \gtreqless 0. \end{aligned} \tag{14}$$

A government expenditure financed by increased taxation has a positive short-run multiplier effect of government expenditure and therefore a positive short-run effect on aggregate income. Simultaneously, a government expenditure financed by increased taxation has a negative short-run effect of increased taxation, and therefore exerts a negative short-run effect on aggregate income.

The negative short-run effect of increased taxation is weakened.[9] The positive short-run effect of government expenditure dominates the negative short-run effect of increased taxation. In contrast, low inflation rates strengthen negative short-run effects of increased taxation. Then, the negative short-run effect of increased taxation dominates the positive short-run effect of government expenditure.

Finally, we investigate the long-run effect of government expenditure with increased taxation on aggregate income. The long-run effect of increased taxation on aggregate income is

$$\frac{\partial y}{\partial h} = \frac{\frac{\partial y}{\partial \tau}}{y + \frac{\partial y}{\partial \tau}}, \tag{15}$$

where

$$\frac{\partial y}{\partial \tau} = -\frac{(1-\beta)y + \beta\pi g}{\pi - (1-\tau)[(\pi-1)\beta+\epsilon]}\frac{\partial y_{t+1}}{\partial g} < 0.$$

Unlike the case of short-run effects of increased taxation, a change in the tax rate not only affects tax revenue directly but also indirectly through the long-run effect of a change in tax rate on aggregate income. As expected, the long-run effect of increased taxation is negative. Furthermore, a negative impact of increased taxation in the long run is greater than the short-run through a decrease in tax revenue by the effect of increased tax rate on aggregate income.

Combining (12) with (15), we arrive at the long-run effect of government expenditure with increased taxation on the aggregate income such as

$$\begin{aligned}\frac{dy}{dg} &= \frac{\partial y}{\partial g} + \frac{\partial y}{\partial h}\frac{dh}{dg} \\ &= \frac{\pi - [1-(1-\eta\pi)\beta]\frac{\partial y_{t+1}}{\partial g}}{\pi - (1-\tau)[(\pi-1)\beta+\epsilon] - [1-\beta(1-\eta\pi)]\frac{\partial y_{t+1}}{\partial g}}. \end{aligned} \tag{16}$$

The sign of (16) is generally ambiguous because a positive long-run multiplier effect of government expenditure and a negative one of increased taxation tend to cancel

[9] Let $g/y_t < [1-(1-\tau)\beta]/\beta$ if the inflation rate is sufficiently high. Then, the sign of (14) is

$$\operatorname{sgn}\frac{dy_{t+1}}{dg} = \pi_{t+1} - \frac{1-\beta}{1-(1-\tau+g/y_t)\beta}.$$

each other. However, one concern of this section is to investigate whether the balanced budget multiplier is greater than unity or not.

Let $0 \leq \tau < 1$. $dy/dg = 1$ holds if and only if $\pi = (\beta - \epsilon)/\beta$. Therefore, if the signs of the numerator and denominator of (16) are stable, then we obtain the following proposition.

Proposition 3. *Within $dy/dg > 0$, the long-run balanced budget multiplier is shown below.*

$$\frac{dy}{dg} \gtreqless 1 \Leftrightarrow \pi \gtreqless \frac{\beta - \epsilon}{\beta}$$

Proposition 3 shows that a key condition of the size of multiplier effect of government expenditure under a balanced budget is the same as the condition given in Proposition 2. A higher inflation rate engenders the stronger multiplier effect of government expenditure under a balanced budget. In contrast, a low inflation rate weakens the multiplier effect of government expenditure under a balanced budget.

The steady-state AD curve provides a similar Keynesian cross diagram for given π, which depends on y; dynamic equation (8) with $y_{t+1} = y_t = y$ and $\pi_{t+1} = \pi_t = \pi$. A high inflation rate engenders a high marginal propensity to consume, although low inflation leads to a low marginal propensity to consume. As is true also of standard Keynesian cross analysis, the high (low) marginal propensity to consume provides a large (small) balanced budget multiplier in the long run. Because the inflation rate depends on y, the balanced budget multiplier is not unity, in general.

This result contrasts against those presented in earlier reports including those describing some studies that show a positive balanced budget multiplier larger than unity (e.g., Mankiw 1988; Startz 1989; Molana and Montagna 2000; Bénassy 2007; Otaki 2007). The difference between this paper and others is in the divergence between dynamic AD and AS curves, as explained in the preceding section.

5 Welfare analysis and some discussion

In the preceding section, we showed that the active fiscal policy is effective to increase the aggregate income and to enhance employment. However, it does not mean that the active fiscal policy is a Pareto-improving policy. In new Keynesian models, the active fiscal policy is not Pareto-improving, although the fiscal policy is effective to increase the aggregate income (e.g., Reinhorn 1998; Tamai 2009).[10]

Therefore, we investigate the welfare effect of fiscal policy using the outcomes derived from the previous section. Presuming that the economy is in an unemployment equilibrium and that the economy monotonically converges to its long-run equilibrium, then using (1), (3), (4), (5), (9) and the utility function, the indirect utility function are the following.

$$V(y_t) = \begin{cases} \mu + (1 - \gamma - \tau) y_t^{\beta} l(y_t)^{1-\beta} \psi & \text{if } i = e. \\ (1 - \gamma - \tau) y_t^{\beta} l(y_t)^{1-\beta} \psi & \text{if } i = u. \end{cases} \tag{17}$$

[10]Reinhorn (1998) shows that optimal fiscal expenditure is equal to zero in the model developed by Mankiw (1988). Tamai (2009) develops a dynamic model with imperfect labour markets, and shows that the fiscal policy has a positive effect on aggregate income but it has a negative effect on welfare.

Therein, $\psi := \mu/\gamma$. Differentiating (17) with respect to y_t, we obtain

$$V'(y_t) = (1-\gamma-\tau)\psi y_t^{\beta-1} l(y_t)^{-\beta}[\beta l(y_t) + (1-\beta)y_t] > 0. \tag{18}$$

The indirect utility function is increasing in the aggregate income.

Assuming that the government increases its expenditure without increased taxation at period t, then according to Proposition 2, we have $\partial y_{t+1}/\partial g > 0$ and $\partial y/\partial g > 0$. Then, equation (18) shows $dV(y_{t+1})/dg > 0$, $dV(y_t)/dg = 0$ and $dV(y)/dg > 0$. The current generation does not benefit from the government expenditure, but future generations benefit from its policy. Therefore, full-employment policy is Pareto-improving.

We now assume that the government increases its expenditure with increased taxation at period t. Then, the current generation incurs a loss by its policy because they are only charged with tax burden and no benefit from its policy. Future generations might benefit from its policy if it increases the aggregate income. However, we conclude that the full-employment policy is not Pareto-improving because the existence of welfare cost of current generation.

The above results are summarised as follows:

Proposition 4. *Presuming that there exists a unique long-run unemployment equilibrium, then the full-employment policy without increased taxation is Pareto-improving for all generations. However, the full-employment policy with increased taxation is not Pareto-improving for all generations although some generations benefit from its policy.*

This proposition demonstrates the validity of full-employment policy from the viewpoint of welfare.[11] The important point of this result is that it includes not only welfare distribution between the employed and unemployed within same generation but also welfare distribution among different generations.

Full-employment policy benefits employed and unemployed people within the same generation. However, it induces conflict between the current generation and future generations according to the financial source of government expenditure. The policy maker must focus not only the size of unemployment but also on welfare distribution among different generations.

However, the approach presented in this paper has some limitations. First, a generalisation of individual preference is necessary to improve the precision of the analytical framework. Using other types of utility function, the form of dynamic AS curve will be changed. However, it will contradict the main message of this paper that the shortage of aggregate demand brings about a persistent unemployment equilibrium.

Second, the monetary policy does not affect the real economic activity because the money supply is subject to the money demand. However, the monetary policy in the real world influences the real economic activity in various ways because the central bank affects the market interest rate through open market operations in the financial market. Therefore, considering these issues, the complete model should treat the asset such as stocks and national bonds.

[11] Some studies also present a similar welfare result (e.g., Ono 1994, 2001; Otaki 2009). Ono (1994, 2001) provides a similar result in a continuous time version of the infinite horizon model. Otaki (2009) provides a welfare economics foundation for the full-employment policy using the model developed by Otaki (2007).

6 Conclusion

This paper developed an overlapping generations model with money, and showed by its use that the aggregate demand affects the aggregate income and employment. It is assumed for the analyses described in this paper that the distribution of factor income is determined by the negotiation between firms and labour. Then, firms are solely able to decide the nominal wage rate subject to aggregate demand and the households' minimum permissible standard of living as a standard employee.

The separation between factor income distribution and price determination under the principle of effective demand brings about the aggregate supply schedule showing that the aggregate output is affected by the inflation rate. Furthermore, the aggregate supply schedule is not one-to-one related to the aggregate demand schedule: Say's law no longer holds. Therefore, a constitutive shortage of aggregate demand brings about a constitutive unemployment equilibrium.

We also showed the possibility of an endogenous business cycle around the unemployment equilibrium if the production function has a part of an increasing returns to scale. The increasing returns engender the positive relation between inflation rate and employment. The dynamic aggregate demand is negatively affected by the inflation rate; the increasing returns situation brings about a strong negative feedback of aggregate income dynamics. Consequently, the aggregate income traces a cyclical path around the unemployment equilibrium.

The macroeconomic policy is important to escape the unemployment equilibrium and to stabilise the economy in some cases because the fiscal policy is effective to increase the aggregate income and to promote employment. The active fiscal policy is approved when a big push is needed under depression and stagnation.

Indeed, welfare analysis in this paper provides a theoretical background for active fiscal policy to attain the full-employment equilibrium. In some cases, the fiscal policy is effective to promote the expansion of aggregate income and employment. Increases in aggregate income and employment raise social welfare. This contrasts with the result derived from new Keynesian models.

Finally, we point out the direction of future research. First, it will be productive to consider the issue of national debt and deficits. The variety of financial source of government spending might be an important consideration when evaluating the effectiveness of economic policy. Second, as discussed in the previous section, the rules of monetary policy extend to various patterns with national debt and money. These issues all remain as subjects for future investigations.

Appendix: Proof of Proposition 1

The locus of $y_{t+1} = \min[F(y_t), y^*]$ is important to investigate the existence of steady-state equilibrium. Because $y_{t+1} = \min[F(y_t), y^*]$ is continuous for $0 \leq y_t \leq y^*$. For $y_t = 0$, we have $\min[F(0), y^*]$. Using F, $y_{t+1} = y^*$ or $y_{t+1} = g/[1-(1-\tau)\beta]$ holds for $y_t = 0$. In each case, $y_{t+1} > 0$ for $y_t = 0$.

However, we have $y_{t+1} = \min[F(y^*), y^*]$ for $y_t = y^*$. According to the property of F, we obtain $y_{t+1} = y^*$ or $y_{t+1} = F(y^*)$. Let $y^* > F(y^*)$. Then, $y_{t+1} = F(y^*)$ for $y_t = y^*$. By the continuity of $\min[F(y_t), y^*]$, there exists at least one steady-state equilibrium such that $y < y^*$. The curve $F(y_t)$ crosses the 45 degree line at $F'(y) < 1$. When $-1 < F'(y) < 1$, the steady-state equilibrium is stable. in the case in which $F'(y) < -1$, endogenous business cycle is raised because y_{t+1} has the upper limit y^* and lower limit $F(y^*)$. We next assume $y^* < F(y^*)$; $y_{t+1} = y^*$ holds for $y_t = y^*$. Then, one of the steady-state equilibria is the stable full-employment equilibrium.

The critical value of y^* is derived as the solution of $y^* = F(y^*)$. It is calculated as

$$y^* \gtreqless F(y^*) \Leftrightarrow \pi^* \gtreqless \frac{(1-\beta)(1-\tau)}{1-\theta-(1-\tau)\beta}.$$

Therefore, the case (i) holds if the above condition satisfies "$>$". Case (ii) holds if "$<$".

References

[1] Akerlof, G.A. and J.L. Yellen (1985), A near-rational model of the business cycle, with wage and price inertia, *Quarterly Journal of Economics*, 100 (5), 823-838.

[2] Bénassy, J.-P. (2007), IS-LM and the multiplier: a dynamic general equilibrium model, *Economics Letters*, 96 (2), 189-195.

[3] Blanchard, O.J. and S. Fisher (1989), *Lectures on Macroeconomics*, MIT Press, Cambridge, Massachusetts.

[4] Blanchard O.J. and N. Kiyotaki (1987), Monopolistic competition and the effect of aggregate demand, *American Economic Review*, 77 (4), 647-666.

[5] Calvo, (1983), Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics*, 12 (3), 383-398.

[6] Clower, R.W. (1965), The Keynesian counter-revolution: a theoretical appraisal, in F.H. Hahn and F.P.R. Brechling, ed., *The Theory of Interest Rates*, Macmillan, London.

[7] Diamond, P. (1965), National debt in a neoclassical growth model, *American Economic Review*, 55 (5), 1126-1150.

[8] Haavelmo, T. (1945), Multiplier effects of a balanced budget, *Econometrica*, 13 (4), 311-318.

[9] Hicks, J.R. (1937), Mr. Keynes and the "Classics": a suggested interpretation, *Econometrica*, 5 (2), 147-159.

[10] Keynes, J.M. (1936), *The General Theory of Employment, Interest and Money*, MacMillan Press, London.

[11] McCallum, B.T. and E. Nelson (1999), An optimizing IS–LM specification for monetary policy and business cycle analysis, *Journal of Money, Credit and Banking*, 31 (3) part 1, 296-316.

[12] Mankiw, N.G. (1985), Small menu costs and large business cycles: a macroeconomic model, *Quarterly Journal of Economics*, 100 (2), 529-538.

[13] Mankiw, N.G. (1988), Imperfect competition and the Keynesian cross, *Economics Letters*, 26 (1), 7-13.

[14] Mankiw, N.G. and R. Reis (2002), Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve, *Quarterly Journal of Economics*, 117 (4), 1295-1328.

[15] Matsuyama, K. (1995), Complementarities and cumulative processes in models of monopolistic competition, *Journal of Economic Literature*, 33 (2), 701-729.

[16] Modigliani, F. (1944), Liquidity preference and the theory of interest and money, *Econometrica* 12 (1) 45-88.

[17] Molana, H. and C. Montagna (2000), Market structure, cost asymmetries, and fiscal policy effectiveness, *Economics Letters*, 68 (1), 101E07.

[18] Molana, H. and T. Moutos (1992), A note on taxation, imperfect competition, and the balanced-budget multiplier, *Oxford Economic Papers*, 44 (1), 68-74.

[19] Ono, Y. (1994), *Money, Interest, and Stagnation: Dynamic Theory and Keynes's Economics*, Oxford University Press, New York.

[20] Ono, Y. (2001), A reinterpretation of Chapter 17 of Keynes's General Theory: effective demand shortage under dynamic optimization, *International Economic Review*, 42 (1), 207-236.

[21] Otaki, M. (2007), The dynamically extended Keynesian cross and the welfare-improving fiscal policy, *Economics Letters*, 96 (1), 23-29.

[22] Otaki, M. (2009), A welfare economics foundation for the full-employment policy, *Economics Letters*, 102 (1), 1-3.

[23] Reinhorn, L.J. (1998), Imperfect competition, the Keynesian cross, and optimal fiscal policy, *Economics Letters*, 58 (3), 331-337.

[24] Startz, R. (1989), Monopolistic competition as a foundation for Keynesian macroeconomic models, *Quarterly Journal of Economics*, 104 (4), 737-752.

[25] Tamai, T. (2009), Employment, fiscal policy, and oligopsonistic labour market, *Australian Journal of Labour Economics*, 12 (3), 321-337.

[26] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, Princeton, New Jersey.

[27] Yun, T. (1996), Nominal price rigidity, money supply endogeneity, and business cycles, *Journal of Monetary Economics*, 37 (2-3), 345-370.

Figures

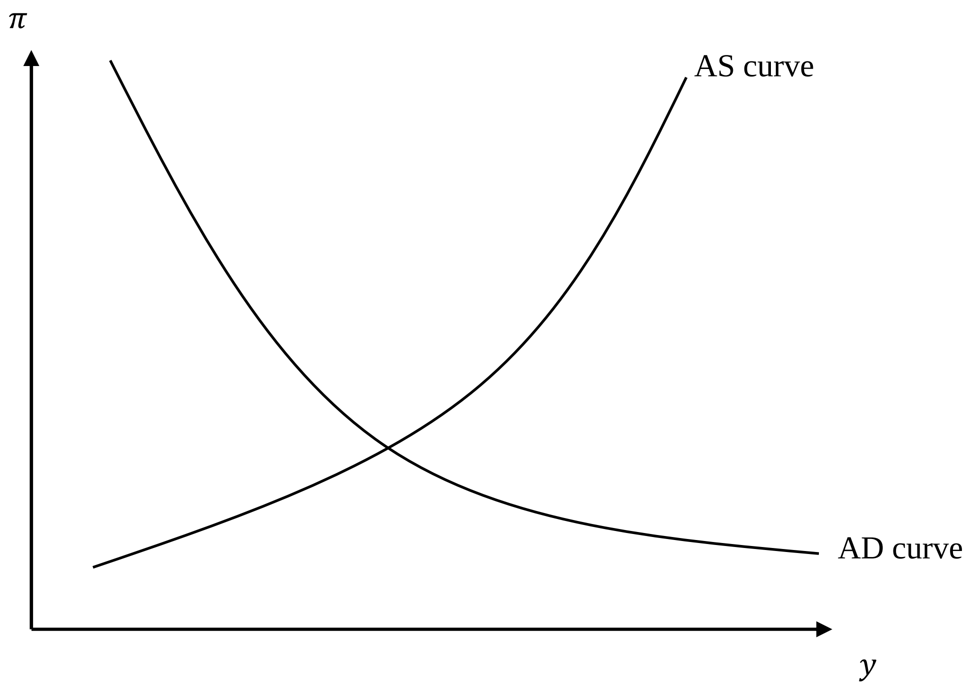

Figure 1a: The case where $\epsilon(y) < 1$

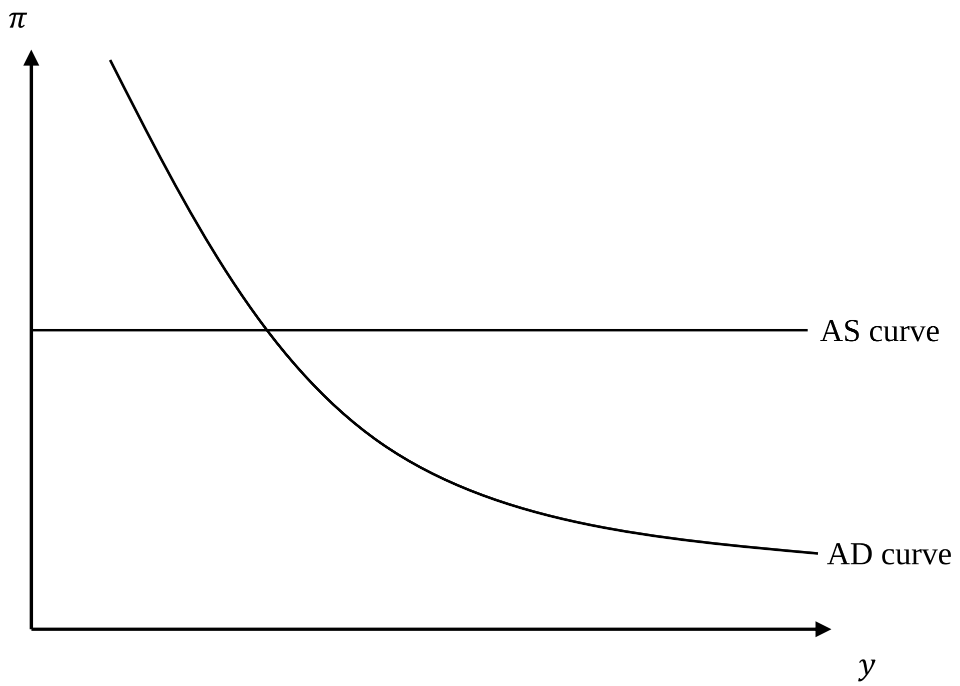

Figure 1b: The case where $\epsilon(y) = 1$

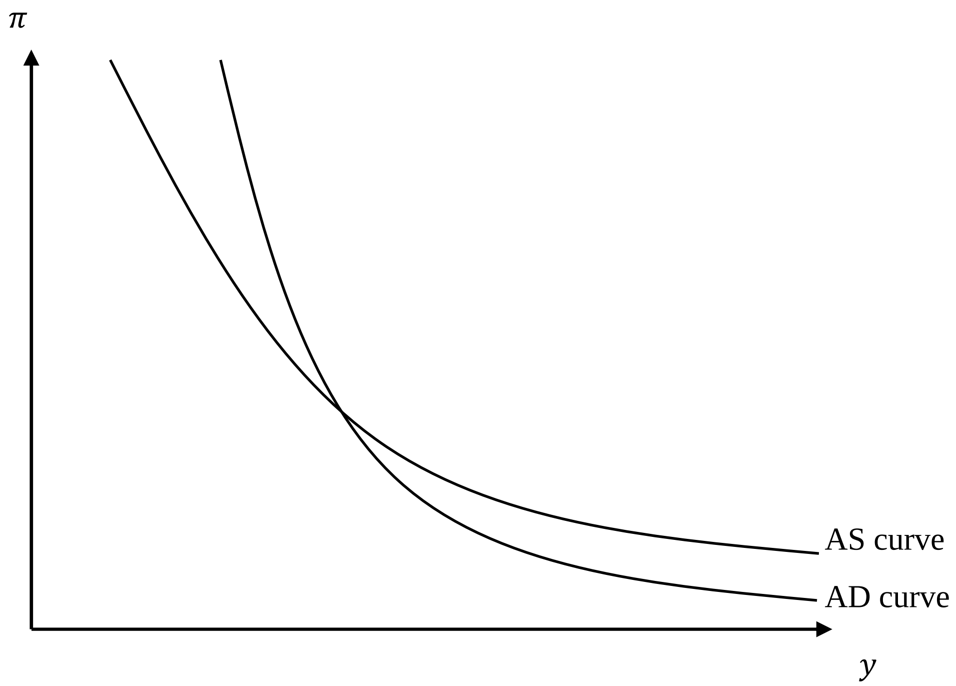

Figure 1a: The case where $\epsilon(y) > 1$

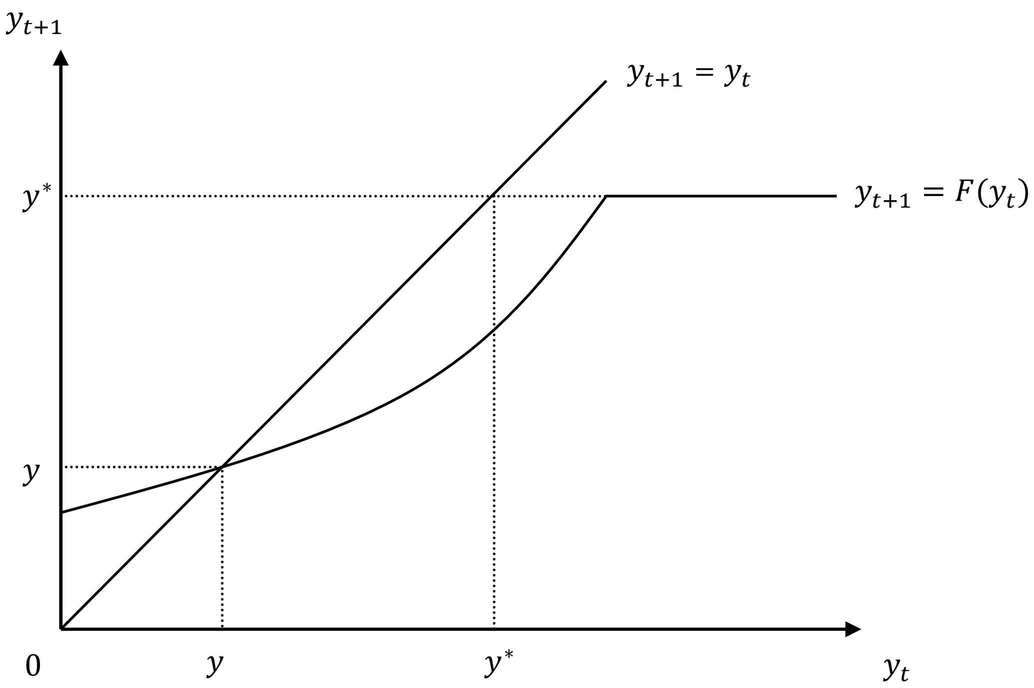

Figure 2a: The constitute unemployment equilibrium

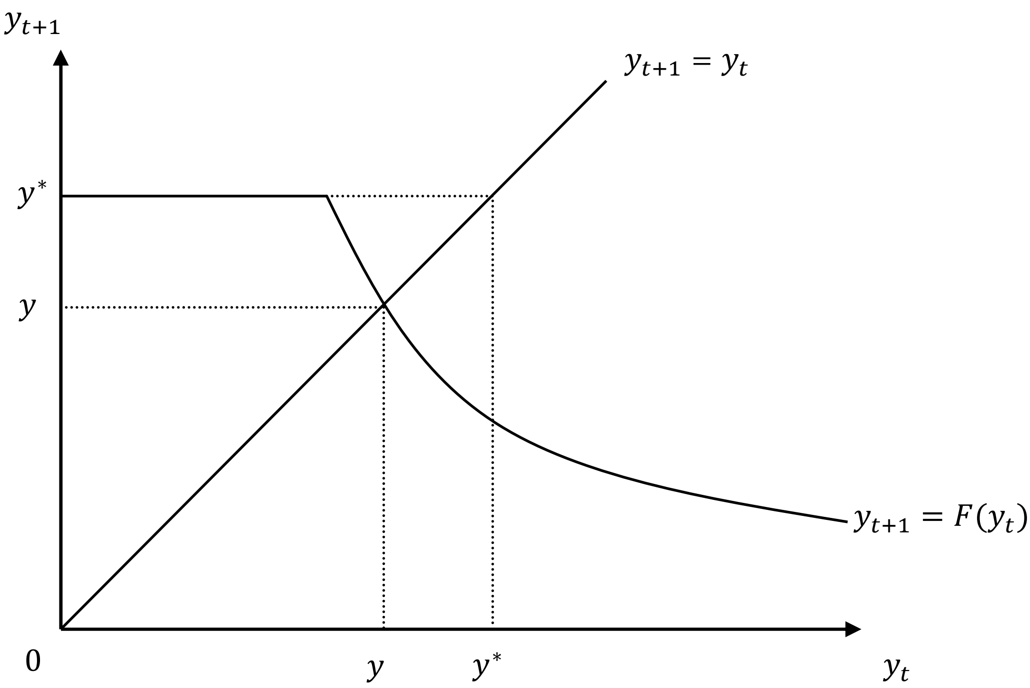

Figure 2b: Endogenous business cycle