

Working Paper Series No.E-14

Inflation, Fiscal Deficits, and Multiple Steady States in a  
Cash-in-Advance Model

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October, 2010

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# Inflation, Fiscal Deficits, and Multiple Steady States in a Cash-in-Advance Model

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**Abstract:** Using a model where a cash-in-advance constraint is imposed on both consumption and investment and the central bank is compelled to finance a fiscal deficit through money creation, this paper shows that there are two or three steady states. If two steady states exist, a high-inflation trap can appear, and an economy will most probably converge to a high-inflation steady state. If three steady states exist, a poverty trap can occur, and an economy where the initial capital stock is less than a threshold level reaches a high-inflation and low-capital steady state.

**JEL Classification Codes:** E31; E52; E62

**Keywords:** inflation; fiscal deficit; multiple steady states

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# 1 Introduction

There are many studies that analyze inflation caused by fiscal deficits. For example, Sargent and Wallace (1981, 1987) show that fiscal deficits financed by seigniorage generate two steady states and that under rational expectations, there is a continuum of paths that converge to a high-inflation steady state, whereas a low-inflation steady state is unstable. As shown in Bruno and Fischer (1990), a reduction in fiscal deficits aggravates inflation in such a high-inflation steady state. Thus, it can be concluded that under rational expectations, an economy converges to a high-inflation steady state in which the results of comparative statics are counterintuitive. This is often regarded as a shortcoming of considering inflation caused by fiscal deficits in rational expectations models.

Several authors therefore examine such inflation in models with learning processes instead of with rational expectations. For example, Marcet and Sargent (1989), in contrast to Sargent and Wallace (1987), show that under least squares learning, a low-inflation steady state is stable and a high-inflation steady state is unstable. Evans et al. (2001) construct two-period overlapping generations models where various patterns of inflation dynamics are possible. Marcet and Nicolini (2003) develop a model that accounts for some observations that occurred during the hyperinflations of the 1980s. However, in recent years, there have been studies that still use models with rational expectations (perfect foresight). For example, Gutiérrez and Vázquez (2004) analyze the existence of an inflation-tax Laffer curve in both a cash-in-advance (CIA) model and a money-in-the-utility-function model.

Barbosa et al. (2006) present a model that explains several stylized facts of hyperinflation.

The present paper reexamines inflation caused by fiscal deficits under rational expectations (perfect foresight) in the following dynamic general equilibrium model. As in the papers cited above, the fiscal authority runs a deficit and compels the monetary authority to finance the fiscal deficit through money creation. In contrast to them, however, capital accumulation is introduced into the present model, and a CIA constraint is imposed on both consumption and investment à la Stockman (1981), which creates a negative effect of inflation on investment.<sup>1</sup>

In this setting, an increase in the capital stock results in a decrease in the marginal productivity of capital, which negatively affects the marginal benefit of holding capital. On the other hand, it expands real money balances (the inflation tax base) and hence lowers the inflation rate (the inflation tax rate). This decline in the inflation rate positively affects the marginal benefit under the CIA constraint. Because of these opposing effects, the steady-state marginal benefit is given as nonmonotonic functions of the capital stock. Consequently, there are two or three steady states.

If two steady states exist, a high inflation trap similar to those of Sargent and Wallace (1987) and Bruno and Fischer (1990) can occur. There is a continuum of paths that converge to a high-inflation steady state, whereas there is only one path that converges to a low-inflation steady state. Therefore, the economy will most probably converge to the high-inflation steady state,

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<sup>1</sup>A negative association between inflation and investment is empirically found (see, e.g., Fischer, 1993).

where a reduction in the fiscal deficit leads to a rise in the inflation rate.

If three steady states exist, a poverty trap can appear, and the initial capital stock determines which steady state the economy reaches. If it is less than a threshold, the economy converges to a high-inflation, low-capital steady state. In the high-inflation, low-capital steady state, in contrast to the case of two steady states, a reduction in the fiscal deficit lowers the inflation rate; namely, the comparative static result is not counterintuitive. If it is greater than a threshold, the economy converges to a low-inflation, high-capital steady state. However, if the fiscal deficit is sufficiently reduced, this poverty trap disappears and the economy reaches the low-inflation, high-capital steady state independently of the initial capital stock.

The remainder of this paper is organized as follows. Section 2 presents the model, Section 3 shows that there are two or three steady states, and Section 4 concludes. The Appendix investigates the dynamic stability of the steady states.

## 2 The Model

We consider an economy consisting of a representative firm, a representative household, and the government. Given the real capital rent  $r_t$  and the real wage  $w_t$ , the firm employs capital  $k_t$  and labor, and produces a commodity in order to maximize its profits. Let  $f(k_t)$ , which satisfies  $f'(\cdot) > 0$  and  $f''(\cdot) < 0$ , denote the per capita production function of the firm. As usual,

the firm's profit maximization yields

$$r_t = f'(k_t), \quad (1)$$

$$w_t = f(k_t) - f'(k_t)k_t. \quad (2)$$

The fiscal authority always runs a constant fiscal deficit  $d$  as follows:

$$d = g - \tau > 0, \quad (3)$$

where  $g$  is government spending and  $\tau$  is a lump sum tax (or a lump sum transfer). The monetary authority is compelled to finance the deficit through money creation:

$$d = \frac{M_{t+1} - M_t}{P_t} = (1 + \pi_{t+1})m_{t+1} - m_t, \quad (4)$$

where  $M_t$  is the nominal money supply,  $P_t$  is the nominal commodity price,  $\pi_{t+1} (\equiv (P_{t+1} - P_t)/P_t)$  is the inflation rate of the price, and  $m_t (\equiv M_t/P_t)$  is real money balances.

The household maximizes its lifetime utility  $U$ :

$$U = \sum_{t=0}^{\infty} \frac{u(c_t)}{(1 + \rho)^t}, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad \rho > 0,$$

where  $c_t$  denotes consumption. The flow budget constraint is

$$k_{t+1} - k_t + \frac{M_{t+1} - M_t}{P_t} = r_t k_t + w_t - c_t - \tau, \quad (5)$$

where the initial capital stock  $k_0$  and the initial nominal money stock  $M_0$  are historically given. Note that the household inelastically supplies its labor endowment normalized to unity. Following Stockman (1981), we assume that the household faces the CIA constraint imposed on both consumption and investment:

$$c_t + k_{t+1} - k_t \leq \frac{M_t}{P_t} - \tau. \quad (6)$$

The first-order conditions with respect to  $c_t$ ,  $k_{t+1}$ , and  $M_{t+1}$  are

$$u'(c_t) = \lambda_t + \gamma_t, \quad (7)$$

$$-(\lambda_t + \gamma_t) + \frac{\lambda_{t+1}r_{t+1} + \lambda_{t+1} + \gamma_{t+1}}{1 + \rho} = 0, \quad (8)$$

$$-\frac{\lambda_t}{P_t} + \frac{\lambda_{t+1} + \gamma_{t+1}}{P_{t+1}(1 + \rho)} = 0, \quad (9)$$

$$\gamma_t \geq 0, \quad \frac{M_t}{P_t} - \tau \geq c_t + k_{t+1} - k_t, \quad \gamma_t \left( \frac{M_t}{P_t} - \tau - c_t - k_{t+1} + k_t \right) = 0,$$

where  $\lambda_t$  and  $\gamma_t$  are the Lagrange multipliers associated with (5) and (6) respectively. Throughout the present paper, we focus on the case where the CIA constraint (6) is binding (i.e.,  $\gamma_t > 0$ ). From (7)–(9), we therefore obtain

$$u'(c_t) = \frac{u'(c_{t+1})}{1 + \rho} + \frac{r_{t+1}}{1 + \pi_{t+2}} \frac{u'(c_{t+2})}{(1 + \rho)^2}, \quad (10)$$

where the left-hand side represents the marginal cost of holding capital (giving up consumption) in period  $t$ , and the right-hand side represents the marginal benefit of holding capital in period  $t$ , and where an increase in the inflation rate  $\pi_{t+2}$  negatively affects the marginal benefit because of the CIA constraint on consumption and investment.<sup>2</sup>

Note that from (3), (4), and (6), aggregate expenditure in period  $t$  is financed by the sum of money held at the beginning of period  $t$  and money created in period  $t$ :

$$c_t + k_{t+1} - k_t + g = \frac{M_t}{P_t} + \frac{M_{t+1} - M_t}{P_t}. \quad (11)$$

From (1)–(5), the commodity market equilibrium is

$$c_t + k_{t+1} - k_t + g = f(k_t). \quad (12)$$

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<sup>2</sup>See Stockman (1981) and Abel (1985) for the equation and the effect of inflation in detail.

Substituting (4) and (12) into (11) gives real money balances  $m_t$  as an increasing function of  $k_t$ :

$$m_t = f(k_t) - d \equiv m(k_t), \quad m'(k_t) > 0. \quad (13)$$

### 3 Steady States

From (4) and (13), the steady-state inflation rate  $\pi$  is shown as a decreasing function of  $k$ :

$$\pi = \frac{d}{m} = \frac{d}{f(k) - d} \equiv \pi(k), \quad \pi'(k) < 0, \quad (14)$$

where an increase in  $k$  expands the inflation tax base  $m$  and thus lowers the inflation tax rate  $\pi$ . Because we naturally consider the case where  $m > 0$ , from (14), we have  $\pi > 0$ , which implies that from (9), the condition for  $\gamma$  to be positive in a steady state,  $\pi > -\rho/(1 + \rho)$ , is satisfied. Hence, the CIA constraint (6) is binding in a steady state. From (1), (10), and (14), we obtain

$$1 = \frac{1}{1 + \rho} + \frac{f'(k)}{[1 + \pi(k)](1 + \rho)^2} = \frac{1}{1 + \rho} + \frac{f'(k)[f(k) - d]}{(1 + \rho)^2 f(k)} \equiv h(k). \quad (15)$$

Once  $k$  is determined by (15), all steady-state variables are obtained. However, the marginal benefit of holding capital,  $h(k)$ , is not necessarily a monotonic function of  $k$ , because both the marginal productivity of capital  $f'(k)$  and the inflation rate  $\pi(k)$  are decreasing in  $k$ . Therefore, multiple steady states may arise, depending on the shape of  $h(k)$ .

In what follows, we examine the shape of  $h(k)$  and show that there can be a unique steady state, two steady states, or three steady states by assuming the production function  $f(k)$  as the following constant elasticity of



substitution function:

$$f(k) = A(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}}, \quad 0 < \alpha < 1, \quad -\infty < \epsilon < 1, \quad (16)$$

where the elasticity of substitution is  $1/(1 - \epsilon)$  and  $f(\cdot)$  satisfies

$$f(0) = 0, \quad f(\infty) = A(1 - \alpha)^{\frac{1}{\epsilon}} \quad \text{if } -\infty < \epsilon \leq 0, \quad (17)$$

$$f(0) = A(1 - \alpha)^{\frac{1}{\epsilon}}, \quad f(\infty) = \infty \quad \text{if } 0 < \epsilon < 1. \quad (18)$$

Differentiating (16) yields

$$f'(k) = A\alpha(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}-1} k^{\epsilon-1} = A\alpha(\alpha + (1 - \alpha)k^{-\epsilon})^{\frac{1-\epsilon}{\epsilon}} > 0, \quad (19)$$

$$f''(k) = -A\alpha(1 - \alpha)(1 - \epsilon)(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}-2} k^{\epsilon-2} < 0. \quad (20)$$

We here assume

$$A(1 - \alpha)^{\frac{1}{\epsilon}} - d > 0. \quad (21)$$

From (13), (17), (18), and (21), we obtain  $m'(k) > 0$  and

$$m(0) = -d < 0, \quad m(\infty) = A(1 - \alpha)^{\frac{1}{\epsilon}} - d > 0 \quad \text{if } -\infty < \epsilon \leq 0,$$

$$m(0) = A(1 - \alpha)^{\frac{1}{\epsilon}} - d > 0, \quad m(\infty) = \infty > 0 \quad \text{if } 0 < \epsilon < 1.$$

Thus, we find

$$m > 0 \quad \text{for } k > \underline{k} \quad \text{if } -\infty < \epsilon \leq 0,$$

$$m > 0 \quad \text{for } k > 0 \quad \text{if } 0 < \epsilon < 1,$$

where  $\underline{k}$  is a unique value satisfying  $m = 0$  as follows:

$$m(\underline{k}) = f(\underline{k}) - d = A(\alpha \underline{k}^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}} - d = 0. \quad (22)$$

If  $-\infty < \epsilon \leq 0$ , therefore, we must treat only the case where  $k > \underline{k}$ .

Because from (19) we have

$$f'(\underline{k}) = A\alpha(\alpha + (1 - \alpha)\underline{k}^{-\epsilon})^{\frac{1-\epsilon}{\epsilon}}, \quad f'(\infty) = 0 \quad \text{if } -\infty < \epsilon \leq 0,$$

$$f'(0) = \infty, \quad f'(\infty) = A\alpha^{\frac{1}{\epsilon}} \quad \text{if } 0 < \epsilon < 1,$$

from (15), (17), (18), and (22),  $h(\cdot)$  satisfies

$$h(\underline{k}) = (1 + \rho)^{-1} < 1, \quad h(\infty) = (1 + \rho)^{-1} < 1 \quad \text{if } -\infty < \epsilon \leq 0, \quad (23)$$

$$h(0) = \infty, \quad h(\infty) = \frac{1}{1 + \rho} + \frac{A\alpha^{\frac{1}{\epsilon}}}{(1 + \rho)^2} \quad \text{if } 0 < \epsilon < 1. \quad (24)$$

By differentiating  $h(k)$  in (15) and taking (16), (19), and (20) into account, we derive

$$h'(k) = \frac{1}{(1 + \rho)^2} \left[ f''(k) - d \frac{f''(k)}{f(k)} + d \left( \frac{f'(k)}{f(k)} \right)^2 \right] \quad (25)$$

$$= \frac{\alpha(1 - \epsilon)(1 - \alpha)k^{\epsilon-2}}{(1 + \rho)^2(\alpha k^\epsilon + 1 - \alpha)^2} \left[ -A(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}} + d + \frac{\alpha dk^\epsilon}{(1 - \epsilon)(1 - \alpha)} \right],$$

which implies

$$(\psi(k) \equiv) -A(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}} + d + \frac{\alpha dk^\epsilon}{(1 - \epsilon)(1 - \alpha)} \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff h'(k) \begin{matrix} \geq \\ \leq \end{matrix} 0. \quad (26)$$

From (26), the differential of  $\psi(k)$  is

$$\psi'(k) = \frac{\alpha k^{\epsilon-1}}{1 - \alpha} \left[ -A(1 - \alpha)(\alpha k^\epsilon + 1 - \alpha)^{\frac{1}{\epsilon}-1} + \frac{\epsilon d}{1 - \epsilon} \right]. \quad (27)$$

### 3.1 Two Steady States

We first show that there can be two steady states if  $-\infty < \epsilon \leq 0$  (i.e., the elasticity of substitution is unity or less). From (21), (22), (26), and (27), we obtain

$$\psi(\underline{k}) = \frac{\alpha d \underline{k}^\epsilon}{(1 - \epsilon)(1 - \alpha)} > 0, \quad \psi(\infty) = -A(1 - \alpha)^{\frac{1}{\epsilon}} + d < 0, \quad \psi'(k) < 0,$$

which implies that, as  $k$  increases,  $\psi(k)$  changes from positive to negative. Because from (26)  $h'(k)$  also changes from positive to negative,  $h(k)$  has the maximum. Taking (23) into account, we find that if the maximum of  $h(\cdot)$  is larger than one, there are two values of  $k$  satisfying (15). This is shown by Figure 1, where  $k^l$  and  $k^h$  denote the two values and satisfy  $k^l < k^h$ . Note that from (14),  $\pi(k^l) > \pi(k^h)$ .

**Proposition 1.** *If  $-\infty < \epsilon \leq 0$ , there can be two steady states ( $k = k^l, k^h$ ). The low-inflation steady state where  $k = k^h$  is saddle path stable, and there is a unique path that converges to it. However, the high-inflation steady state where  $k = k^l$  may be unstable or stable. If it is stable, there is a continuum of paths that converge to it.*

*Proof.* See the Appendix for the stability of the two steady states. □

If the high-inflation steady state is stable, the economy will most probably converge to it, because there is a continuum of paths that converge to it whereas there is only one path that converges to the low-inflation steady state. That is, a high-inflation trap occurs. In contrast, if it is unstable, the dynamic behavior of the economy hinges upon the initial capital stock  $k_0$ . If  $k_0$  is larger than  $k^l$ , the economy converges to the low-inflation steady state where  $k = k^h$ . If  $k_0$  is smaller than  $k^l$ , the capital stock decreases over time and becomes less than  $\underline{k}$ . Hence, such a path will be infeasible.

From (15), a reduction in the fiscal deficit  $d$  increases the marginal benefit of holding capital (it shifts  $h(k)$  upward from the solid line to the dashed line in Figure 1). Thus, in the high-inflation steady state where  $k = k^l$ , it decreases  $k$  and increases  $\pi$ . This unintuitive result of comparative statics is

similar to Sargent and Wallace (1987) and Bruno and Fischer (1990).

### 3.2 A Unique Steady State

We next consider the case where the elasticity of substitution  $1/(1 - \epsilon)$  is greater than unity and the fiscal deficit  $d$  is small enough to satisfy the following second property:

$$0 < \epsilon < 1, \quad -A(1 - \alpha)^{\frac{1}{\epsilon}} + \frac{\epsilon d}{1 - \epsilon} \leq 0, \quad (28)$$

which implies that from (27)  $\psi'(0) \leq 0$ . Because the first term in square brackets in (27) decreases monotonically from  $-A(1 - \alpha)^{1/\epsilon}$  to  $-\infty$  as  $k$  increases, we find that under (28), we have

$$\psi'(k) < 0 \quad \text{for } k > 0.$$

From (21) and (26), we derive

$$\psi(0) = -A(1 - \alpha)^{\frac{1}{\epsilon}} + d < 0.$$

Hence, we have  $\psi(k) < 0$  for  $k > 0$ , which implies that from (26),  $h'(k) < 0$  for  $k > 0$ . From (24), where  $h(0) > 1$ , if

$$h(\infty) = \frac{1}{1 + \rho} + \frac{A\alpha^{\frac{1}{\epsilon}}}{(1 + \rho)^2} < 1, \quad (29)$$

then there is a unique value of  $k$  that satisfies (15).<sup>3</sup>

**Proposition 2.** *If (28) and (29) are valid, there exists a unique steady state, which is saddle path stable.*

*Proof.* See the Appendix for the stability of this steady state.  $\square$

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<sup>3</sup>If (29) is invalid, capital continues to accumulate, and thus the economy will permanently grow.

### 3.3 Three Steady States

Finally, we consider the case where the first property of (28) is valid but the fiscal deficit is so large that the second one is invalid:

$$0 < \epsilon < 1, \quad -A(1 - \alpha)^{\frac{1}{\epsilon}} + \frac{\epsilon d}{1 - \epsilon} > 0. \quad (30)$$

Because from (27) and (30)  $\psi'(0) > 0$ ,  $\psi'(\infty) < 0$ , and there is a unique value of  $k$  that satisfies  $\psi'(k) = 0$ ,  $\psi(k)$  is maximized at the unique value. From (21) and (26), we obtain<sup>4</sup>

$$\psi(0) = -A(1 - \alpha)^{\frac{1}{\epsilon}} + d < 0, \quad \psi(\infty) = -\infty < 0.$$

Therefore, if the maximum of  $\psi(\cdot)$  is positive, there are two values of  $k$  satisfying  $\psi(k) = 0$ . Moreover, as  $k$  increases,  $\psi(k)$  changes from negative to positive and then goes back to negative. Because from (26)  $h'(k)$  also changes from negative to positive and goes back to negative and from (24) and (29) we have  $h(0) > 1$  and  $h(\infty) < 1$ , there can be three values of  $k$  satisfying (15), as illustrated by Figure 2, where  $k^L$ ,  $k^M$ , and  $k^H$ , satisfying  $k^L < k^M < k^H$ , denote the three values. Note that from (14),  $\pi(k^L) > \pi(k^M) > \pi(k^H)$ .

**Proposition 3.** *If (29) and (30) are valid, there can be three steady states ( $k = k^L, k^M, k^H$ ). The high-inflation steady state where  $k = k^L$  and the*

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<sup>4</sup>By arranging  $\psi(k)$  as follows:

$$\psi(k) = k \left[ -A(\alpha + (1 - \alpha)k^{-\epsilon})^{\frac{1}{\epsilon}} + dk^{-1} + \frac{\alpha dk^{\epsilon-1}}{(1 - \epsilon)(1 - \alpha)} \right],$$

we easily find the second property:

$$\psi(\infty) = \infty \cdot (-A\alpha^{\frac{1}{\epsilon}}) = -\infty < 0.$$

low-inflation steady state where  $k = k^H$  are saddle path stable, and there are respective unique paths that converge to them. However, the moderate-inflation steady state where  $k = k^M$  may be unstable or stable. If it is stable, there is a continuum of paths that converge to it.

*Proof.* See the Appendix for the stability of the three steady states.  $\square$

If the steady state where  $k = k^M$  is stable, the economy will most probably converge to it. However, if it is unstable, a poverty trap appears, and the initial capital stock  $k_0$  determines which steady state the economy reaches. If  $k_0$  is smaller than a threshold  $k^M$ , the economy converges to the high-inflation steady state where  $k = k^L$ . If  $k_0$  is larger than  $k^M$ , the economy converges to the low-inflation steady state where  $k = k^H$ . Note that this poverty trap occurs without increasing returns to scale, which are often regarded as a cause of poverty traps.

Because from (15) a reduction in the fiscal deficit  $d$  increases the marginal benefit of holding capital (it shifts  $h(k)$  upward from the solid line to the dashed line in Figure 2), it increases  $k$  and decreases  $\pi$  in the high-inflation steady state where  $k = k^L$ . This comparative static result is in contrast with the case of two steady states. If  $d$  is further reduced and  $h(k)$  shifts sufficiently upward, as implied by Figure 2, the high-inflation steady state where  $k = k^L$  and the moderate-inflation steady state where  $k = k^M$  disappear, and only the low-inflation steady state where  $k = k^H$  can exist.<sup>5,6</sup> That

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<sup>5</sup>If  $d$  is reduced enough to violate the second condition in (30) and satisfy the second one in (28), as mentioned in Subsection 3.2,  $h(k)$  is a monotonically decreasing function of  $k$  and thus there exists a unique steady state.

<sup>6</sup>Using a two-period overlapping generations model with an adaptive learning rule, Evans et al. (2001) also show that a tight fiscal policy causes only a steady state with low inflation to exist.

is, the poverty trap disappears. Conversely, if  $d$  is so large that  $h(k)$  shifts downward sufficiently, the low-inflation steady state where  $k = k^H$  and the moderate-inflation steady state where  $k = k^M$  will disappear, and there can exist only the high-inflation steady state where  $k = k^L$ . Consequently, the economy converges to the high-inflation steady state independently of the initial capital stock.

The present analysis suggests that when the economy is on its way to reaching the high-inflation, low-capital steady state, a reduction in the fiscal deficit enables the economy to turn around and to advance toward the low-inflation, high-capital steady state. This may be somewhat similar to the case of Bolivia. In September 1985, a fiscal reform, including a reduction in fiscal deficits, stopped hyperinflation in Bolivia. Afterwards, the rate of real GDP growth in Bolivia changed from negative to positive.<sup>7</sup>

Finally, note that both the high-inflation trap and the poverty trap result from the combination of the fiscal deficit and the dependent monetary authority. These traps are eliminated if there is no fiscal deficit ( $d = 0$ ) or if the monetary authority sets independently the constant money growth rate  $\mu$  ( $\equiv (M_{t+1} - M_t)/M_t$ ), which implies that the steady-state inflation rate equals the money growth rate ( $\pi = \mu$ ). This is because if  $d = 0$  or  $\pi = \mu$ , from (15), the marginal benefit of holding capital  $h(k)$  is decreasing monotonically in  $k$  and there exists a unique steady state. This result is similar to Bruno and Fischer (1990), who show that there exists a unique steady state if the money growth rate is constant.

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<sup>7</sup>See, e.g., Sachs (2005, Chapter 5) for details of the case of Bolivia. Sargent et al. (2009) examine Latin American hyperinflations and conclude that in Bolivia, fiscal reforms ended hyperinflation.

## 4 Concluding Remarks

We construct a dynamic general equilibrium model where a CIA constraint is imposed on both consumption and investment, the fiscal authority runs a deficit, and the monetary authority is compelled to finance the fiscal deficit through money creation. In the model, not only does a high inflation trap arise, as in existing studies, but also a poverty trap occurs.

If there are two steady states, the high-inflation trap can appear and the economy will most probably reach a high-inflation steady state. If there are three steady states, the poverty trap can appear and the initial capital stock determines which steady state the economy reaches. If it is smaller than a threshold, the economy converges to a high-inflation, low-capital steady state. However, if the fiscal deficit is sufficiently reduced, the economy escapes from the poverty trap and reaches a low-inflation, high-capital steady state independently of it.

Because the combination of the fiscal deficit and the dependent monetary authority creates both these traps, they disappear if the monetary authority is independent or if there is no fiscal deficit. This result suggests that reducing fiscal deficits and enhancing the independence of the central bank will be important in restraining inflation, stabilizing economies, and enhancing economic growth.



## Appendix: The Stability of the Steady States

We show the dynamic behavior of the economy by a dynamic system composed of  $k_t$ ,  $c_t$ , and  $\lambda_t$ , and analyze the stability of the steady states.<sup>8</sup> Note that  $k_t$  is a state variable, and  $c_t$  and  $\lambda_t$  are jump variables. From (12),  $k_{t+1}$  is a function of  $k_t$  and  $c_t$ :

$$k_{t+1} = k_t + f(k_t) - c_t - g \equiv k(k_t, c_t). \quad (\text{A1})$$

From (4), (13), and (A1),  $\pi_{t+1}$  is a function of  $k_t$  and  $c_t$ :

$$\pi_{t+1} = \frac{f(k_t)}{f(k(k_t, c_t)) - d} - 1 \equiv \pi(k_t, c_t). \quad (\text{A2})$$

From (7), (9), and (A2), we find

$$u'(c_{t+1}) = (1 + \rho) [1 + \pi(k_t, c_t)] \lambda_t, \quad (\text{A3})$$

which implies that  $c_{t+1}$  is a function of  $k_t$ ,  $c_t$ , and  $\lambda_t$ :

$$c_{t+1} \equiv c(k_t, c_t, \lambda_t). \quad (\text{A4})$$

From (1), (7), (8), (A1), and (A4),  $\lambda_{t+1}$  is a function of  $k_t$ ,  $c_t$ , and  $\lambda_t$  as follows:

$$\lambda_{t+1} = \frac{(1 + \rho)u'(c_t) - u'(c(k_t, c_t, \lambda_t))}{f'(k(k_t, c_t))} \equiv \lambda(k_t, c_t, \lambda_t). \quad (\text{A5})$$

In a steady state, from (A2), (A3), and (A5), we have

$$1 + \pi = \frac{f}{f - d}, \quad u' = (1 + \rho)(1 + \pi)\lambda, \quad \lambda = \frac{(1 + \rho)u' - u'}{f'}. \quad (\text{A6})$$

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<sup>8</sup>Alternatively, as in Abel (1985), we can examine the stability by a third-order difference equation of the capital stock.

Using (A1), (A2), (A3), (A5), and (A6), we find partial differentials of  $k_{t+1}$ ,  $\pi_{t+1}$ ,  $c_{t+1}$ , and  $\lambda_{t+1}$  with respect to  $k_t$ ,  $c_t$ , and  $\lambda_t$ , evaluated in a steady state as follows:

$$\begin{aligned}
k_k &= 1 + f' > 0, \quad k_c = -1 < 0; \\
\pi_k &= -\frac{f'(ff' + d)}{(f-d)^2} < 0, \quad \pi_c = \frac{ff'}{(f-d)^2} > 0; \\
c_k &= \frac{(1+\rho)\pi_k\lambda}{u''} = -\frac{f'u'}{(f-d)u''} \left( f' + \frac{d}{f} \right) > 0, \\
c_c &= \frac{(1+\rho)\pi_c\lambda}{u''} = \frac{f'u'}{(f-d)u''} < 0, \quad c_\lambda = \frac{(1+\rho)(1+\pi)}{u''} = \frac{(1+\rho)f}{(f-d)u''} < 0; \\
\lambda_k &= -\frac{u''c_k}{f'} - \frac{(1+f')(f-d)f''u'}{(1+\rho)ff'} > 0, \\
\lambda_c &= \frac{(1+\rho)u'' - u''c_c}{f'} + \frac{(f-d)f''u'}{(1+\rho)ff'} < 0, \quad \lambda_\lambda = -\frac{u''c_\lambda}{f'} < 0.
\end{aligned}$$

By linearizing (A1), (A4), and (A5) in the neighborhood of a steady state, we therefore obtain the following characteristic equation:

$$\begin{vmatrix}
1 + f' - z & -1 & 0 \\
c_k & c_c - z & c_\lambda \\
\lambda_k & \lambda_c & \lambda_\lambda - z
\end{vmatrix} = 0,$$

where  $z$  is a characteristic root. It leads to

$$n(z) \equiv z^3 + \theta_2 z^2 + \theta_1 z + \theta_0 = 0, \quad (\text{A7})$$

where

$$\theta_2 \equiv -(1+f') - c_c - \lambda_\lambda = -(1+f') - \frac{f'u'}{(f-d)u''} + \frac{(1+\rho)f}{(f-d)f'}, \quad (\text{A8})$$

$$\begin{aligned}
\theta_1 &\equiv (1+f')(c_c + \lambda_\lambda) + c_k + c_c\lambda_\lambda - c_\lambda\lambda_c \\
&= -\frac{(1+\rho)(1+f')f}{(f-d)f'} - \frac{(1+\rho)^2 f}{(f-d)f'} + \frac{u'}{u''} \left( \frac{f'}{f} - \frac{f''}{f'} \right) < 0, \quad (\text{A9})
\end{aligned}$$

$$\theta_0 \equiv -(1+f')(c_c\lambda_\lambda - c_\lambda\lambda_c) + c_\lambda\lambda_k - c_k\lambda_\lambda = \frac{(1+\rho)^2(1+f')f}{(f-d)f'} > 0. \quad (\text{A10})$$

From (15), (25), (A7), (A8), (A9), and (A10),  $n(\cdot)$  satisfies

$$n(0) = \theta_0 > 0, \quad (\text{A11})$$

$$n(1) = 1 + \theta_2 + \theta_1 + \theta_0 = -\frac{(1 + \rho)^2 f u'}{(f - d) f' u''} h'(k), \quad (\text{A12})$$

$$\begin{aligned} n(-1) &= -1 + \theta_2 - \theta_1 + \theta_0 \\ &= \frac{2(1 + \rho)(2 + f')f}{(f - d)f'} - \frac{u'}{u''} \left( \frac{f'}{f - d} + \frac{f'}{f} - \frac{f''}{f'} \right) > 0. \end{aligned} \quad (\text{A13})$$

We first examine the stability of such steady states where  $h'(\cdot) < 0$  as the steady state where  $k = k^h$  in Proposition 1, the steady state in Proposition 2, and the steady state where  $k = k^L$  and the steady state where  $k = k^H$  in Proposition 3. See Figures 1 and 2 and Subsection 3.2 for the property that  $h'(\cdot) < 0$  in these steady states. From (A12), we find

$$n(1) < 0 \text{ if } h' < 0. \quad (\text{A14})$$

Let  $z_1, z_2$ , and  $z_3$ , satisfying  $z_1 \leq z_2 \leq z_3$ , denote roots of (A7). From (A11), (A13), and (A14), we have

$$z_1 < -1, \quad 0 < z_2 < 1, \quad 1 < z_3.$$

Because  $k_t$  is a state variable and  $c_t$  and  $\lambda_t$  are jump variables, these steady states are saddle path stable and there are respective unique paths that converge to them.

We next investigate the stability of such steady states where  $h'(\cdot) > 0$  as the steady state where  $k = k^l$  in Proposition 1 and the steady state where  $k = k^M$  in Proposition 3. Figures 1 and 2 imply the property that  $h'(\cdot) > 0$  in the two steady states. From (A12), we find

$$n(1) > 0 \text{ if } h' > 0. \quad (\text{A15})$$

Because (A11), (A13), and (A15) alone are insufficient to discriminate values of the roots of (A7), we explore further properties of  $n(\cdot)$ . Differentiating  $n(z)$  in (A7) yields

$$n'(z) = 3z^2 + 2\theta_2 z + \theta_1. \quad (\text{A16})$$

From (15), (25), (A8), (A9), and (A16), we obtain

$$n'(0) = \theta_1 < 0, \quad (\text{A17})$$

$$\begin{aligned} n'(1) &= 3 + 2\theta_2 + \theta_1 \\ &= -2f' - \frac{(1+\rho)f}{f-d} - \frac{fu'}{(f-d)f'u''} \left[ (1+\rho)^2 h'(k) + \frac{(f')^2}{f} \right]. \end{aligned} \quad (\text{A18})$$

If  $h'(\cdot) > 0$ , the third term in (A18) is positive whereas the first and second terms are negative. Thus, whether  $n'(1)$  is positive or negative depends on the shape of the instantaneous utility function.

From (A11), (A13), (A15), and (A17), we find

$$\begin{aligned} z_1 &< -1, \quad 1 < z_2, \quad 1 < z_3 \quad \text{if } n'(1) \leq 0, \\ z_1 &< -1, \quad 0 < z_2 < 1, \quad 0 < z_3 < 1 \quad \text{if } n'(1) > 0. \end{aligned}$$

Because  $k_t$  is a state variable and  $c_t$  and  $\lambda_t$  are jump variables, those steady states are unstable if  $n'(1) \leq 0$ . In contrast, if  $n'(1) > 0$ , they are stable and there is a continuum of paths that converge to them.

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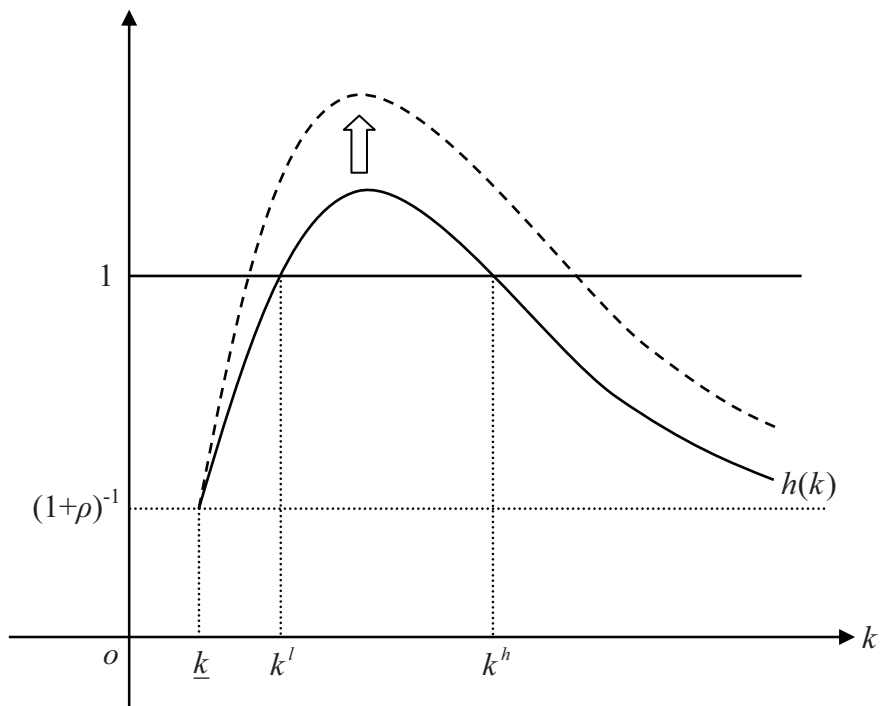


Figure 1: The existence of two steady states and the effect of a reduction in the fiscal deficit

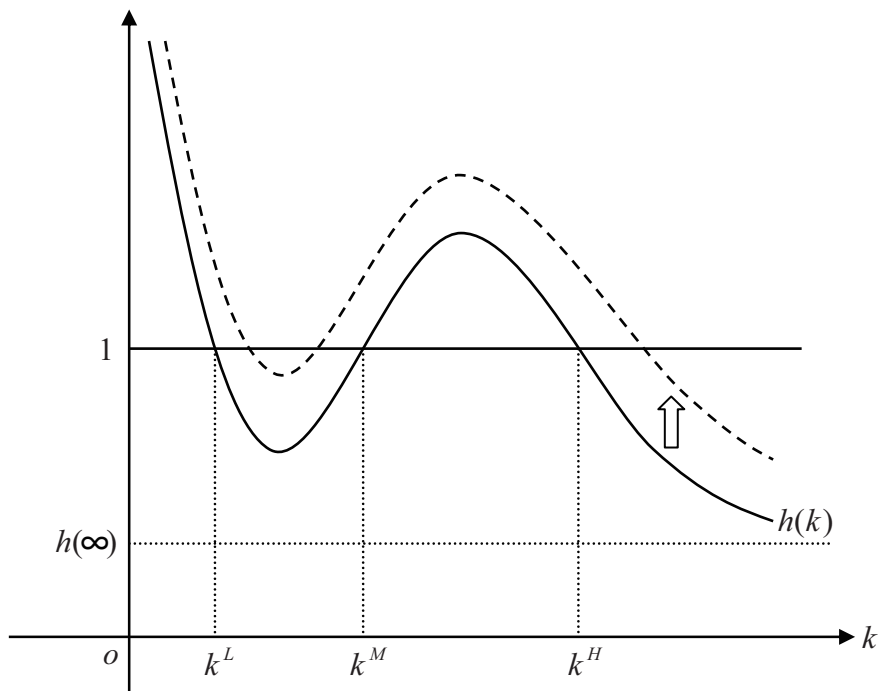


Figure 2: The existence of three steady states and the effect of a reduction in the fiscal deficit