Working Paper Series　No.E-12

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September, 2009

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Abstract

This paper examines theoretically the structure of optimal (Nash equilibrium) tariff rates under retaliation in a two-country economy with more than two traded goods. We provide a condition under which the equilibrium tariff rates become uniform in both countries, and explore the ranking of the equilibrium tariff rates in each country when the uniform tariff condition is not satisfied. The elasticities of compensated excess demands for goods play an important role in determining the ranking of the equilibrium tariff rates. This paper undertakes the analysis using a dual approach.

JEL Classification: F11, F13

Keywords: Optimal tariff rate; Retaliation; Uniformity; Elasticity

*This study was supported in part by a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science.

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1 Introduction

This paper examines theoretically the structure of optimal (Nash equilibrium) tariff rates under retaliation in a two-country economy with more than two traded goods. We provide a condition under which the equilibrium tariff rates are uniform in both countries, and explore the ranking of the equilibrium tariff rates in each country when the uniform tariff condition is not satisfied. In addition, we examine the sign of the aggregated tariff rates of the two countries on each traded good.

The literature on the optimal tariff problem, which arises in a large country, is vast. This field expanded rapidly from Kaldor (1940), who demonstrated the existence of the optimal tariff, and various issues within the optimal tariff framework have been discussed in many studies, such as Graaff (1949–50), Johnson (1951–52), Kemp (1967), Bhagwati and Kemp (1969), Riley (1969), Horwell and Pearce (1970), Tower (1977), Feenstra (1986), and Young (1991).[1] Recently, a few studies focused their attention on the structure of the optimal tariff rates on different traded goods by using a model with more than two traded goods. Bond (1990) established a condition under which the highest (lowest) trade tax rate is imposed on an import (export) good, and a condition under which all import tariff rates are higher than any export tax rate at the optimum in the n-good model. Bond did not analyze the structure of the optimal tariff rates within the same trade group, which may be a more important issue when there are more than two traded goods. Ogawa (2007) addressed this issue; he provided a condition under which the optimal tariff rates on the import goods are uniform, and the rules that determine the ranking of the optimal import tariff rates. However, he dealt with a two-country economy where one country engages in free trade.[2] The assumption that one country does not react to the tariff imposition of its partner country is unrealistic and, hence, it is natural to consider retaliation by tariffs.

The literature on the optimal tariff problem under retaliation is also large, with several strands and ramifications; for example, Scitovsky (1942), Johnson (1953–54), Gorman (1958), Otani (1980), Hamilton and Whalley (1983), Lockwood and Wong (2000), Syropoulos (2002a,b), and Zissimos (2009).[3] These studies, however, do not

[1] Graaff (1949–50) showed that the equilibrium tariffs for some goods can be negative in the n-good model. Johnson (1951–52) derived various formulas for the optimal tariff. Kemp (1967), Bhagwati and Kemp (1969), and Riley (1969) demonstrated that the optimal tariff can be negative if there is an inferior good in a two-good model. Horwell and Pearce (1970) derived the condition that all goods should be subject to (positive) tariffs. The relationship between the optimal tariff and the revenue-maximizing tariff is examined by Tower (1977). Feenstra (1986) examined the sign of the optimal tariff in a three-good model and Young (1991) examined the average level of the optimal tariffs in a n-good model.

[2] Bond (1990) also considered a two-country economy where one country engages in free trade.

[3] A seminal paper that considers this issue is Scitovsky (1942). Johnson (1953–54) showed the possibility that one country achieves a higher welfare level at a Nash tariff equilibrium than at a free trade equilibrium and Gorman (1958) made a further analysis of retaliative tariffs. Otani (1980) examined the existence of optimal tariffs and Hamilton and Whalley (1983) analyzed numerically the optimal tariffs. Recently, Lockwood and Wong (2000) made a welfare comparison between an ad valorem tariff and a specific tariff, and Syropoulos (2002a) showed that Johnson's (1953–54) result holds if one country is of sufficiently large size relative to its trading partner. Syropoulos (2002b)

examine the structure of the optimal tariff rates within the same trade group in each country. Although most studies in this field have not analyzed such an issue, as a notable exception, Bond and Syropoulos (1996) showed that the optimal import tariff rates of each country (trading blocks) are uniform in a multicountry economy. However, because they considered a model with symmetric structure, in which all countries are exchange economies with symmetric endowments, the representative consumers in all countries have identical constant elasticity of substitution (CES) utility functions, and all trading blocks are of the same size, the optimal (common external) tariff rates automatically become uniform in their model.[4] It is therefore necessary to clarify the optimal tariff structure under retaliation in a nonsymmetric model that allows general utility and production functions. In such a model, this paper provides the condition under which the Nash equilibrium tariff rates are uniform, and explores the ranking of the equilibrium tariff rates when the condition is not satisfied.

This paper is organized as follows. In the next section, we describe a standard international trade model with two countries trading $n+1$ goods, in which consumers and producers in each country are perfectly competitive. The government of each country behaves in a Nash manner, strategically deciding tariffs to exploit its market power in trade. Section 3 derives the condition that the best-response tariffs satisfy, which is based on all results obtained in this paper, using a dual approach. The use of the dual approach for the analysis in the model with more than two traded goods is new in this field,[5] and is useful for examining the optimal tariff structure under retaliation because it can avoid the analytical confusion created by the effects of tariff revenue return. Section 4 examines the structure of the Nash equilibrium tariff rates within the same trade group in the countries. In Section 4.1, we provide the condition under which the equilibrium tariff rates are uniform in both countries in the model with $n+1$ goods. The uniform tariff condition is evaluated using the elasticities of the compensated excess demands for goods and, hence, is independent of income effects. This is in contrast to the fact that the income effects of the goods play an important role in determining the sign of the optimal tariff, as shown by Kemp (1967), Riley (1969), and Bond (1990). Section 4.2 explores the ranking of the equilibrium tariff rates in each country when the uniform tariff condition is not satisfied. This examination is made in the models with $n+1$ goods and with three goods. In the model with $n+1$ goods, we provide the simple and indicative optimal tariff formula, which would also be useful in the analysis of various international issues, under the assumption of no cross-substitution effects between tariff-imposed goods. The case with the cross-substitution effects is treated in the three-good model. Section 5 examines the sign of the aggregated tariff rates of the two countries on each traded good. Concluding comments are provided in Section 6.

examined the ranking of the common external tariff rates that are most preferred for each member of a custom union. Zissimos (2009) analyzed the optimal tariffs in the model with a number of countries.

[4]Bond, Syropoulos and Winters (2001) considered a model consisting of two trading blocks of different size. However, because they also assumed that all countries are symmetric, the common external tariff rates inevitably become uniform.

[5]Syropoulos (2002a) and Zissimos (2009) analyzed the optimal tariff problem in a two-good framework using a dual approach.

The results obtained in this paper are suggestive and useful not only for the countries that impose tariffs independently but also for custom unions and trading blocks, which use a common external tariff. Our results can be regarded as the examination of the structure of common external tariff rates on different goods by regarding the country as an entity of a custom union. Recently, Syropoulos (2002b and 2003) examined the ranking of the common external tariff rates that is most preferred for each member of a custom union by using a two-good model in which a tariff is imposed only on one good. They, however, did not analyze the structure of the common external tariff rates on different goods.

2 The model

We consider a standard general equilibrium model of international trade. There are two countries (home and foreign countries) and $n+1$ traded goods. The home (foreign) country exports (imports) good 0 and imports (exports) goods $1, \cdots, n$.[6] In each country there is a representative consumer who has a well-behaved utility function and chooses the commodity bundle that maximizes his/her utility level, given prices and income. Producers maximize profit, taking prices as given, and the production possibility frontier of each country is well behaved. The production factors in each country, fixed in supply, are internationally immobile and are fully employed in the production sector. The markets for factors and goods are perfectly competitive. Each country imposes tariffs on traded goods in a Nash manner, and tariff revenue is returned to the consumer in a lump-sum fashion. There is no international transfer between the two countries.

Let us denote the tariff vector of the home country as $t' \equiv (t_0, t_1, \cdots, t_n)$, that of the foreign country as $t^{*\prime} \equiv (t_0^*, t_1^*, \cdots, t_n^*)$, the domestic price vector of the home country as $q' \equiv (q_0, q_1, \cdots, q_n)$, that of the foreign country as $q^{*\prime} \equiv (q_0^*, q_1^*, \cdots, q_n^*)$, and the world price vector as $p' \equiv (p_0, p_1, \cdots, p_n)$. Then

$$q = t + p, \text{ and } q^* = t^* + p. \tag{1}$$

Without loss of the generality, we assume that: $t_0 = t_0^* = 0$.[7] When the home country imposes an import tariff (subsidy) on good i, $t_i > 0$ ($t_i < 0$), and when the foreign country imposes an export tax (subsidy), $t_i^* < 0$ ($t_i^* > 0$).

The compensated excess demand functions of good i of the home and foreign countries are denoted by $z_i = z_i(q, u)$ and $z_i^* = z_i^*(q^*, u^*)$, respectively, where z_i is the excess demand of the home country for good i, z_i^* is that of the foreign country, u is the welfare of the home country, and u^* is that of the foreign country. Note that $z_i > 0$ ($z_i^* > 0$) if good i is an imported good in the home (foreign) country and $z_i < 0$ ($z_i^* < 0$) if good i is an exported good. Let $z_{ij} \equiv \partial z_i / \partial q_j$, $z_{iu} \equiv \partial z_i / \partial u$, $z_{ij}^* \equiv \partial z_i^* / \partial q_j^*$, and $z_{iu^*}^* \equiv \partial z_i^* / \partial u^*$.

[6] In our model, the export good for the home country can be interpreted as a composite good.

[7] Even after allowing for the export tax or subsidy for the home country (the import tariff or subsidy for the foreign country), the propositions obtained in this paper continue to hold.

Good i is normal in the home (foreign) country if $z_{iu} > 0$ ($z^*_{iu^*} > 0$), and good i is a substitute for good j in the home (foreign) country if $z_{ij} > 0$ ($z^*_{ij} > 0$). The function, $z_i(\cdot)$, has the following properties: (i) symmetry, $z_{ij} = z_{ji}$; (ii) homogeneity, $\sum_{j=0}^n q_j z_{ij} = 0$; and (iii) negative semidefinite, $\sum_{i=0}^n \sum_{j=0}^n k_i k_j z_{ij} = 0$ if $k = \psi q$, where ψ is a scalar and $k' \equiv (k_0, k_1, \cdots, k_n)$, and $\sum_{i=0}^n \sum_{j=0}^n k_i k_j z_{ij} < 0$ otherwise. The analogous properties hold in $z^*_i(\cdot)$.[8]

We choose good 0 as the numeraire and set $p_0 = 1$, which yields $q_0 = q^*_0 = p_0 = 1$ because $t_0 = t^*_0 = 0$. The economy is described by the following equations:

$$\sum_{i=0}^n p_i z_i(q, u) = 0, \tag{2}$$

$$\sum_{i=0}^n p_i z^*_i(q^*, u^*) = 0, \tag{3}$$

$$z_i(q, u) + z^*_i(q^*, u^*) = 0, \qquad i = 1, \cdots, n. \tag{4}$$

The first and second equations represent the trade balance constraints of the home and foreign countries, respectively. The third equation is the world market-clearing condition of good i, which for good 0 is obtained by Walras' law. In view of (1), the $n + 2$ equations of (2), (3), and (4) determine the endogenous variables $p_1, \cdots, p_n$, u and u^* when t and t^* are given exogenously.

3 The best-response tariffs

Let us define ad valorem tariff rates of the home and foreign countries on good i as, respectively

$$\tau_i \equiv \frac{t_i}{p_i}, \text{ and } \tau^*_i \equiv \frac{t^*_i}{p_i}, \tag{5}$$

and the elasticities of the compensated excess demand of the home and foreign countries for good i with respect to the price of good j as, respectively

$$\eta_{ij} \equiv \frac{q_j z_{ij}}{z_i}, \text{ and } \eta^*_{ij} \equiv \frac{q^*_j z^*_{ij}}{z^*_i}. \tag{6}$$

When good i is a nonnumeraire and is a substitute for good j in the home (foreign) country, $\eta_{ij} > 0$ ($\eta^*_{ij} < 0$).

In this economy, the following holds.

[8] The function, $z^*_i(\cdot)$, has the following properties: (i) symmetry, $z^*_{ij} = z^*_{ji}$; (ii) homogeneity, $\sum_{j=0}^n q^*_j z^*_{ij} = 0$; and (iii) negative semidefinite, $\sum_{i=0}^n \sum_{j=0}^n h_i h_j z^*_{ij} = 0$ if $h = \xi q^*$, where ξ is a scalar and $h' \equiv (h_0, h_1, \cdots, h_n)$, and $\sum_{i=0}^n \sum_{j=0}^n h_i h_j z^*_{ij} < 0$ otherwise.

Lemma 1. *The best-response tariffs of the home country satisfy*

$$1 = \sum_{i=1}^{n} \left[\frac{\tau_i^* + (\tau_i - \tau_i^*)\alpha}{1 + \tau_i^*} \right] \eta_{ji}^*, \qquad j = 1, \cdots, n, \tag{7}$$

where

$$\alpha \equiv \frac{\sum_{i=0}^{n} p_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*}, \tag{8}$$

and those of the foreign country satisfy

$$1 = \sum_{i=1}^{n} \left[\frac{\tau_i + (\tau_i^* - \tau_i)\alpha^*}{1 + \tau_i} \right] \eta_{ji}, \qquad j = 1, \cdots, n, \tag{9}$$

where

$$\alpha^* \equiv \frac{\sum_{i=0}^{n} p_i z_{iu}}{\sum_{i=0}^{n} q_i^* z_{iu}}. \tag{10}$$

Proof. Equation (7) is derived by solving the following optimization problem:

$$\begin{aligned} \max_{t_1, \cdots, t_n, p_1, \cdots, p_n, u, u^*} \quad & u \\ \text{s.t.} \quad & \sum_{i=0}^{n} p_i z_i(q, u) = 0, \\ & \sum_{i=0}^{n} p_i z_i^*(q^*, u^*) = 0, \\ & z_i(q, u) + z_i^*(q^*, u^*) = 0, \qquad i = 1, \cdots, n. \end{aligned}$$

Let us define the Lagrangian function as

$$L \equiv u - \lambda \sum_{i=0}^{n} p_i z_i(q, u) - \theta \sum_{i=0}^{n} p_i z_i^*(q^*, u^*) - \sum_{i=1}^{n} \delta_i \cdot \left[z_i(q, u) + z_i^*(q^*, u^*) \right],$$

where λ, θ and δ_i are Lagrangian multipliers. The first-order conditions are

$$L_{t_j} = \lambda \sum_{i=0}^{n} p_i z_{ij} + \sum_{i=1}^{n} \delta_i z_{ij} = 0, \qquad j = 1, \cdots, n, \tag{11}$$

$$L_{p_j} = \lambda \cdot \left(z_j + \sum_{i=0}^{n} p_i z_{ij} \right) + \theta \cdot \left(z_j^* + \sum_{i=0}^{n} p_i z_{ij}^* \right) + \sum_{i=1}^{n} \delta_i \cdot \left(z_{ij} + z_{ij}^* \right) = 0, \qquad j = 1, \cdots, n, \tag{12}$$

$$L_u = 1 - \lambda \sum_{i=0}^{n} p_i z_{iu} - \sum_{i=1}^{n} \delta_i z_{iu} = 0, \tag{13}$$

$$L_{u^*} = \theta \sum_{i=0}^{n} p_i z^*_{iu^*} + \sum_{i=1}^{n} \delta_i z^*_{iu^*} = 0, \tag{14}$$

where $L_{t_j} \equiv \partial L/\partial t_j$, $L_{p_j} \equiv \partial L/\partial p_j$, $L_u \equiv \partial L/\partial u$ and $L_{u^*} \equiv \partial L/\partial u^*$.

By using $z_{ij} = z_{ji}$ and $\sum_{i=0}^{n}(t_i + p_i)z_{ji} = 0$ and noting that $t_0 = 0$, (11) can be rewritten as

$$\begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} \delta_1 - \lambda t_1 \\ \vdots \\ \delta_n - \lambda t_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{15}$$

Because the $n \times n$ substitution matrix is nonsingular,[9] (15) shows that

$$t_i = \frac{\delta_i}{\lambda}, \qquad i = 1, \cdots, n. \tag{16}$$

Subtracting (11) from (12), and noting $z_j = -z^*_j$, we obtain

$$(\theta - \lambda) z^*_j + \theta \sum_{i=0}^{n} p_i z^*_{ij} + \sum_{i=1}^{n} \delta_i z^*_{ij} = 0, \qquad j = 1, \cdots, n. \tag{17}$$

Rearranging this and using (16) yields[10]

$$z^*_j = \sum_{i=1}^{n} \left[t^*_i + \left(\frac{\lambda}{\lambda - \theta} \right) (t_i - t^*_i) \right] z^*_{ij}, \qquad j = 1, \cdots, n. \tag{18}$$

Note that $p_i/q^*_i = 1/(1 + \tau^*_i)$ from (1) and (5). By using this, $z^*_{ij} = z^*_{ji}$, (5) and (6), (18) can be rewritten as[11]

$$1 = \sum_{i=1}^{n} \left[\frac{\tau^*_i + \left(\frac{\lambda}{\lambda-\theta}\right)(\tau_i - \tau^*_i)}{1 + \tau^*_i} \right] \eta^*_{ji}, \qquad j = 1, \cdots, n. \tag{19}$$

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[9] Property (iii) of the compensated excess demand function implies that the $n \times n$ substitution matrix in (15) has full rank. See, for example, Barten and Böhm (1982, Theorem 13.1-(v), p. 416).

[10] Eq. (18) is derived as follows. Multiplying (17) by $1/\lambda$ and using $\sum_{i=0}^{n} p_i z^*_{ij} = -\sum_{i=1}^{n} t^*_i z^*_{ij}$, we obtain

$$\left(\frac{\lambda - \theta}{\lambda} \right) z^*_j = -\left(\frac{\theta}{\lambda} \right) \sum_{i=1}^{n} t^*_i z^*_{ij} + \sum_{i=1}^{n} \left(\frac{\delta_i}{\lambda} \right) z^*_{ij}, \qquad j = 1, \cdots, n.$$

From this and (16), we have

$$z^*_j = \left(\frac{\theta}{\theta - \lambda} \right) \sum_{i=1}^{n} t^*_i z^*_{ij} + \left(\frac{\lambda}{\lambda - \theta} \right) \sum_{i=1}^{n} t_i z^*_{ij}, \qquad j = 1, \cdots, n.$$

Adding $-[\lambda/(\theta - \lambda)] \sum_{i=1}^{n} t^*_i z^*_{ij} + [\lambda/(\theta - \lambda)] \sum_{i=1}^{n} t^*_i z^*_{ij} = 0$ to this and rearranging it yields (18).

[11] It is implicitly assumed that $z_i \neq 0$ and $z^*_i \neq 0$ for all i, that is, the tariffs are nonprohibitive.

Multiplying (14) by $1/\lambda$ and using (16), we obtain

$$\left(\frac{\theta}{\lambda}\right)\sum_{i=0}^{n} p_i z^*_{iu^*} + \sum_{i=1}^{n} t_i z^*_{iu^*} = 0.$$

Adding $\sum_{i=0}^{n} p_i z^*_{iu^*} - \sum_{i=0}^{n} p_i z^*_{iu^*} = 0$ to this and using (1) and $q_0 = p_0$, we have

$$\left(\frac{\theta-\lambda}{\lambda}\right)\sum_{i=0}^{n} p_i z^*_{iu^*} + \sum_{i=0}^{n} q_i z^*_{iu^*} = 0,$$

which yields

$$\frac{\lambda}{\lambda-\theta} = \frac{\sum_{i=0}^{n} p_i z^*_{iu^*}}{\sum_{i=0}^{n} q_i z^*_{iu^*}}.$$

From this and (19) we have (7).[12] Solving the analogous welfare-maximization problem of the foreign country, we obtain (9). ■

When the foreign country engages in free trade ($\tau^*_1 = \cdots = \tau^*_n = 0$), (7) can be rewritten as $1/\alpha = \sum_{i=1}^{n} \tau_i \eta^*_{ji}$. Similarly, when the home country engages in free trade, we obtain $1/\alpha^* = \sum_{i=1}^{n} \tau^*_i \eta_{ji}$ by applying $\tau_1 = \cdots = \tau_n = 0$ to (9). These formulas are the same as that of Bond (1990) and Ogawa (2007).

4 The structure of Nash equilibrium tariffs

This section examines the structure of the Nash equilibrium tariff rates. Nash tariff equilibrium may involve multiple equilibria in this economy. However, all propositions obtained in this paper hold in any equilibria. We make the following assumption in the equilibria.[13]

Assumption 1. *There are no inferior goods in either country.*

This assumption guarantees that $\alpha > 0$ and $\alpha^* > 0$. Even if inferior goods exist, $\alpha > 0$ and $\alpha^* > 0$ are satisfied unless the inferiority is sufficiently large. Bond (1990) showed that the tariff revenue of a country is nonnegative at the optimum in an economy where its trading partner country engages in free trade if all goods are normal in the latter country, and Young (1991) stated that Bond's result holds even in an economy where the trading partner country imposes tariffs on its traded goods. Bhagwati and Kemp (1969) demonstrated that the offer curve of a tariff-imposed country may have a perversely

[12] Multiplying (13) by $1/\lambda$ and using (1) and (16), we have $\lambda = 1/\sum_{i=0}^{n} q_i z_{iu}$, which indicates that λ is the home country's marginal utility of income. From (14) and (16) we obtain $\theta = -\lambda \sum_{i=1}^{n} t_i z^*_{iu^*} / \sum_{i=0}^{n} p_i z^*_{iu^*}$. Note that $1/\sum_{i=0}^{n} p_i z^*_{iu^*}$ indicates the effect of the foreign country's receipt of international transfers on the foreign country's utility. Therefore, θ can be interpreted as the effect of international transfers on the home country's welfare through tariff revenue variation, evaluated in terms of the foreign excess demand function.

[13] We add Assumption 2 below, which is used in Propositions 4 and 5.

sloped part if there is an inferior good in that country, and using this property of the offer curve, Riley (1969) showed that when the normal-good condition is not satisfied, the equilibrium achieved by the tariff that satisfies the first order condition may be a local welfare-minimum point.

4.1 Uniform tariff

This section provides the condition under which the equilibrium tariffs are uniform in both countries. We say that the tariff rates are uniform in both countries when $\tau_1 = \cdots = \tau_n$ and $\tau_1^* = \cdots = \tau_n^*$. Using Lemma 1, we provide the following proposition.[14]

Proposition 1. *The (Nash) equilibrium tariffs satisfy the following: (i)* $\tau_1 = \cdots = \tau_n (\equiv \tau)$ *and* $\tau_1^* = \cdots = \tau_n^* (\equiv \tau^*)$ *if and only if*

$$\eta_{10} = \cdots = \eta_{n0} (\equiv \beta) \text{ and } \eta_{10}^* = \cdots = \eta_{n0}^* (\equiv \beta^*), \tag{20}$$

and (ii) the optimal uniform tariffs satisfy $\tau > 0$ *and* $\tau^* < 0$.[15]

Proof. The proof of (i): We first prove that the equilibrium tariff rates are uniform in both countries if condition (20) is satisfied. From the homogeneity condition and (20), we obtain $\beta^* + \sum_{i=1}^n \eta_{ji}^* = 0$ for $j = 1, \cdots, n$, which yields $1 = -(1/\beta^*) \sum_{i=1}^n \eta_{ji}^*$. Subtracting this from (7) yields

$$\begin{bmatrix} \eta_{11}^* & \cdots & \eta_{1n}^* \\ \vdots & & \vdots \\ \eta_{n1}^* & \cdots & \eta_{nn}^* \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{21}$$

where

$$\kappa_i \equiv \frac{\tau_i^* + (\tau_i - \tau_i^*)\alpha}{1 + \tau_i^*} + \frac{1}{\beta^*}.$$

Because the $n \times n$ matrix of the elasticities in (21) is nonsingular,[16] (21) shows that $\kappa_i = 0$ for $i = 1, \cdots, n$, i.e.,

$$\frac{\tau_i^* + (\tau_i - \tau_i^*)\alpha}{1 + \tau_i^*} = -\frac{1}{\beta^*}, \qquad i = 1, \cdots, n. \tag{22}$$

[14] Corlett and Hague (1953) showed that the optimal commodity tax rates are uniform if and only if the wage elasticities of the compensated demands for commodities are equal over all commodities. There is a clear difference between Proposition 1 and the Corlett–Hague condition; the uniform tariff condition in Proposition 1 is evaluated by the elasticities of the two countries. The model of Corlett and Hague (1953) is entirely different from that of this paper. In Corlett–Hague's model, the terms-of-trade effects never occur because of the closed economy, while the price distortion by the commodity taxes are inevitable because of a revenue constraint.

[15] Note that Proposition 1-(i) does not require Assumption 1, but (ii) requires Assumption 1.

[16] The rank of the $n \times n$ matrix of the elasticities is the same as that of the $n \times n$ substitution matrix. See footnote 9.

Similarly, using $\beta + \sum_{i=1}^{n} \eta_{ji} = 0$ $(j = 1, \cdots, n)$ and (9), we obtain

$$\frac{\tau_i + (\tau_i^* - \tau_i)\alpha^*}{1 + \tau_i} = -\frac{1}{\beta}, \qquad i = 1, \cdots, n. \tag{23}$$

From (22) and (23) we have

$$\tau_i = \frac{-\beta^* + \alpha\beta^* + \alpha^*\beta - 1}{(\beta - \alpha^*\beta + 1)(\beta^* - \alpha\beta^* + 1) - \alpha\alpha^*\beta\beta^*}, \qquad i = 1, \cdots, n,$$

which shows that $\tau_1 = \cdots = \tau_n$, and

$$\tau_i^* = \frac{-\beta + \alpha\beta^* + \alpha^*\beta - 1}{(\beta - \alpha^*\beta + 1)(\beta^* - \alpha\beta^* + 1) - \alpha\alpha^*\beta\beta^*}, \qquad i = 1, \cdots, n,$$

which shows that $\tau_1^* = \cdots = \tau_n^*$.

We next prove that condition (20) holds if the equilibrium tariff rates are uniform in both countries. When $\tau_1 = \cdots = \tau_n (\equiv \tau)$ and $\tau_1^* = \cdots = \tau_n^* (\equiv \tau^*)$, from (7) and $\eta_{j0}^* = -\sum_{i=1}^{n} \eta_{ji}^*$ we obtain

$$1 = -\left[\frac{\tau^* + (\tau - \tau^*)\alpha}{1 + \tau^*}\right] \eta_{j0}^*, \qquad j = 1, \cdots, n,$$

which shows that $\eta_{10}^* = \cdots = \eta_{n0}^*$. Similarly, using (9) and $\eta_{j0} = -\sum_{i=1}^{n} \eta_{ji}$ we can see that $\eta_{10} = \cdots = \eta_{n0}$ if $\tau_1 = \cdots = \tau_n$ and $\tau_1^* = \cdots = \tau_n^*$.

The proof of (ii): When $\tau_1 = \cdots = \tau_n (\equiv \tau)$, from (8) we obtain[17]

$$\begin{aligned} 1 - \alpha &= \frac{\sum_{i=0}^{n} q_i z_{iu^*}^* - \sum_{i=0}^{n} p_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*} \\ &= \frac{\sum_{i=1}^{n} t_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*} \qquad \text{(from (1) and } t_0 = 0) \\ &= \frac{\tau \sum_{i=1}^{n} p_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*}. \qquad \text{(from } t_i/p_i = \tau) \end{aligned} \tag{24}$$

From this and (22), we obtain[18]

$$-(1 + \tau^*) = \frac{\tau\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*}. \tag{25}$$

[17] Note that $\sum_{i=1}^{n} p_i z_{iu^*}^* / \sum_{i=0}^{n} q_i z_{iu^*}^*$ in the last equation of (24) is different from α; its numerator does not include number "0", but the numerator in α does.

[18] Eq. (25) is derived as follows. From (22) it follows that

$$\begin{aligned} -(1 + \tau^*) &= [\tau^* + (\tau - \tau^*)\alpha]\, \beta^* \\ &= \left[\frac{(1 - \alpha)\tau^*}{\tau} + \alpha\right] \tau\beta^* \\ &= \left(\frac{\tau^* \sum_{i=1}^{n} p_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*} + \frac{\sum_{i=0}^{n} p_i z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*}\right) \tau\beta^* \qquad \text{(by using (8) and (24))} \\ &= \frac{\tau\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^*}{\sum_{i=0}^{n} q_i z_{iu^*}^*}. \qquad \text{(by using } \tau_0^* = 0 \text{ and } q_i^* = (1 + \tau^*) p_i) \end{aligned}$$

Note that $\sum_{i=0}^n q_i z_{iu^*}^* > 0$ from Assumption 1, $\sum_{i=0}^n q_i^* z_{iu^*}^* > 0$,[19] and $1 + \tau^* > 0$.[20] Thus, (25) shows that $\tau > 0$ if and only if $\beta^* < 0$. From property (iii) of the compensated excess demand function, $z_{00}^* < 0$, and from properties (i) and (ii), $z_{00}^* + \sum_{i=1}^n q_i^* z_{i0}^* = 0$. These show that $\sum_{i=1}^n q_i^* z_{i0}^* > 0$. Condition (20) implies that the sign of z_{i0}^* is the same over all nonnumeraire goods. From this and $\sum_{i=1}^n q_i^* z_{i0}^* > 0$, $z_{i0}^* > 0$ for $i = 1, \cdots, n$ and hence $\beta^* < 0$. Thus, we have $\tau > 0$. Similarly, when $\tau_1^* = \cdots = \tau_n^*$, we can see that $\tau^* < 0$. ■

As is well known, under the optimal tariffs the trade indifference surface of the tariff-imposing country is tangential to the partner country's offer surface. This is observed in Proposition 1. Rearranging (25), we obtain

$$(1+\tau)\, p_i = \left[\frac{\beta^* \sum_{i=0}^n q_i^* z_{iu^*}^* - (1+\tau^*) z_{0u^*}^*}{\beta^* \sum_{i=0}^n q_i^* z_{iu^*}^* + \sum_{i=1}^n q_i^* z_{iu^*}^*} \right] p_i, \qquad i = 1, \ldots, n. \tag{26}$$

See Appendix A for the derivation of this equation. The left-hand side (LHS) is the home country's domestic price of good i in terms of the price of the numeraire, and hence it equals the home country's marginal substitution rate (MRS) between goods i and 0 on the trade indifference surface. Noting that the right-hand side (RHS) of (26) is characterized by the foreign country's parameters, we can regard it as the marginal transformation rate between goods 0 and i (hereafter simply referred to as the MTR(0-i)) on the foreign country's offer surface at the optimum, and the brackets on the RHS indicate the MTR(0-i) in terms of p_i. The MTR(0-i) in terms of p_i is the difference between the world price of good i and the MTR(0-i) and, hence, it equals the difference between the world price and the home country's domestic price at the optimum. Under condition (20), the MTR(0-i) in terms of p_i is equal over goods $1, \cdots, n$ and thus all optimal tariff rates are equal. A similar interpretation can be applied to the optimal export taxes of the foreign country.

Proposition 1 is a generalization of Ogawa (2007), which provides the condition under which the optimal tariff rates are uniform in a two-country economy where one country engages in free trade. Ogawa's condition consists of only the elasticities of the country that engages in free trade, whereas condition (20) requires those of all tariff-imposing countries. It should be noted that condition (20) is independent of income effects. This is in contrast to the fact that the sign of the optimal tariff depends on that of the income effects (for example, Kemp 1967 and Bond 1990).

An example that establishes condition (20) is that each country is an exchange economy and the representative consumer in each country has the following utility functions: $u = u(x_0, \gamma(x_1, \cdots, x_n))$ and $u^* = u^*(x_0^*, \gamma^*(x_1^*, \cdots, x_n^*))$, where x_i and x_i^* denote the home and foreign countries' demands for good i, good 0 is weakly separable for the other goods, and $\gamma(\cdot)$ and $\gamma^*(\cdot)$ are homothetic. Under this utility function, the compensated

[19] The expression $1/\sum_{i=0}^n q_i^* z_{iu^*}^*$ indicates the marginal utility of income of the foreign country.

[20] From (1), $q_i^* = t_i^* + p_i = (1+\tau_i^*)p_i$, which shows that $1+\tau_i^*$ is positive. Similarly, $1+\tau_i$ is also positive.

demand function of the home country is expressed as $x_i(q,u) = \omega(q,u)\pi_i(q_1,\cdots,q_n)$.[21] This shows that the elasticities of the compensated demands for goods with respect to the price of good 0 are equal over all nonnumeraire goods.[22] A similar condition is also satisfied in the foreign country. There are no substitution effects in production in an exchange economy. Thus, condition (20) holds. It should be noted that the form of a CES, employed by Bond and Syropoulos (1996) and Bond et al. (2001), implies homothetic separability, but the converse is not true.

4.2 The ranking of the tariff rates

This section examines the ranking of the Nash equilibrium tariff rates on different traded goods in each country in the case where the uniform tariff condition is not satisfied. In the general setting, it is difficult to clarify the equilibrium tariff structure because of market linkage between the traded goods. Therefore, we consider some special cases. First we consider the model with $n+1$ traded goods where there are no cross-substitution effects between the tariff-imposed goods and no income effects of those goods in both countries, which can remove the tangled price effects among excess demands. In this case, we provide a tractable and suggestive equilibrium tariff formula, which would also be useful for the analysis of other issues in international trade. The formula shows that the ranking of the equilibrium tariff rates depends only on the own price elasticities, which is equivalent to stating that it depends only on the elasticities of the compensated excess demands of the nonnumeraire goods with respect to the price of the numeraire good (η_{i0} for $i = 1,\cdots,n$) from property (ii) of the compensated excess demand function.

The assumption of no cross-substitution effects between the tariff-imposed goods is, however, somewhat strong, and we may miss an important relation between the cross-substitution effects and the optimal tariff structure. Next, we treat the case where the cross-substitution effects between all traded goods exist. Because this generalization creates tangled price effects among excess demands when there are a number of traded goods, we consider the three-good model to avoid analytical complexity. In this model the cross-elasticities between the numeraire good and the nonnumeraire goods (η_{i0} for $i = 1,\cdots,n$) play a critical role in determining the ranking of the equilibrium tariff rates, although the own price elasticities are not useful indicators.

Section 4.2.1 considers the model with $n+1$ traded goods where there are no cross-substitution effects between the tariff-imposed goods, and Section 4.2.2 considers the three-good traded model with the cross-substitution effects.

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[21] Note that π_i is independent of q_0. For the homothetically separable utility function and the compensated demand function, see Gorman (1975) and Blackorby et al. (1978).

[22] Let us define the elasticity of the compensated demand for good i with respect to the price of good j as $\varphi_{ij}(\equiv (q_j/x_i)(\partial x_i/\partial q_j))$. Noting that $q_0 = 1$, we obtain $(q_0/x_i)(\partial x_i/\partial q_0) = \omega_0/\omega$, where $\omega_0 \equiv \partial\omega/\partial q_0$. Thus, $\varphi_{10} = \varphi_{20} = \cdots = \varphi_{n0}$.

4.2.1 The case with $n+1$ traded goods

We consider the model with $n+1$ traded goods where the following is assumed: $z_{ij} = z_{ij}^* = z_{iu} = z_{iu^*}^* = 0$ for $i, j = 1, \cdots, n$ and $i \neq j$. An example that establishes this case is where each country is an exchange economy and the consumers in each country have the following quasilinear utility function, respectively: $u = x_0 + \sum_{i=1}^n \zeta_i(x_i)$ and $u^* = x_0^* + \sum_{i=1}^n \zeta_i^*(x_i^*)$. There are no substitution effects in production in the exchange economy and, given the specified utility function, the cross-substitution effects between the nonnumeraire goods and the income effects of the nonnumeraire goods are all zero in each country. In this case, the equilibrium tariff rate is given in the following lemma.

Lemma 2. *When $z_{ij} = z_{ij}^* = z_{iu} = z_{iu^*}^* = 0$ for $i, j = 1, \cdots, n$ and $i \neq j$, the equilibrium tariff rates are given by*

$$\tau_i = -\frac{1+\eta_{ii}}{1-\eta_{ii}\eta_{ii}^*}, \ \text{and} \ \ \tau_i^* = -\frac{1+\eta_{ii}^*}{1-\eta_{ii}\eta_{ii}^*}, \qquad i = 1, \cdots, n, \tag{27}$$

and $\tau_i > 0$ and $\tau_i^* < 0$.

Proof. Note that $\alpha = 1$ if $z_{iu^*}^* = 0$ for $i = 1, \cdots, n$, and $\alpha^* = 1$ if $z_{iu} = 0$ for $i = 1, \cdots, n$. By using these and $z_{ij} = z_{ij}^* = 0$ (i.e., $\eta_{ij} = \eta_{ij}^* = 0$) for $i, j = 1, \cdots, n$ and $i \neq j$, (7) and (9) can be rewritten as

$$1 + \tau_i^* = \tau_i \eta_{ii}^*, \ \text{and} \ \ 1 + \tau_i = \tau_i^* \eta_{ii}, \qquad i = 1, \ldots, n. \tag{28}$$

From (28) we obtain (27). Note that $\eta_{ii} < 0 < \eta_{ii}^*$ from properties (iii) of $z_i(\cdot)$ and $z_i^*(\cdot)$. The first equation in (28) shows that $\tau_i > 0$ because $1 + \tau_i^* > 0$ and $\eta_{ii}^* > 0$. Similarly, we obtain $\tau_i^* < 0$ from the second equation. ■

The formulae in (27) are very simple and suggestive. The equilibrium tariff rate for each traded good is expressed only by the own price elasticities of the corresponding good of both countries. This lemma shows that the optimal intervention for each country on each good is a trade tax, which gives the incentive that reduces international trade in each good. Using Lemma 2, we obtain the following proposition concerning the ranking of tariff rates.

Proposition 2. *The equilibrium tariff rates satisfy the following: when $z_{ij} = z_{ij}^* = 0$ and $z_{iu} = z_{iu^*}^* = 0$ for $i, j = 1, \cdots, n$ and $i \neq j$, (i) $\tau_i > \tau_j$ if $-\eta_{ii} > -\eta_{jj}$ and $\eta_{jj}^* > \eta_{ii}^*$, and (ii) $-\tau_i^* > -\tau_j^*$ if $-\eta_{jj} > -\eta_{ii}$ and $\eta_{jj}^* > \eta_{ii}^*$.*

Proof. From (27) we obtain

$$\tau_i - \tau_j = -\frac{1+\eta_{ii}}{1-\eta_{ii}^*\eta_{ii}} + \frac{1+\eta_{jj}}{1-\eta_{jj}^*\eta_{jj}}, \qquad i, j = 1, \ldots, n.$$

Adding $\eta_{ii}\eta_{jj}^* - \eta_{ii}\eta_{jj}^* = 0$ to the RHS and rearranging it yields

$$(\tau_i - \tau_j)\,\Upsilon = \left[(-\eta_{ii}) - (-\eta_{jj})\right](1 + \eta_{jj}^*) + (\eta_{jj}^* - \eta_{ii}^*)(1 + \eta_{jj})\eta_{ii}, \qquad i, j = 1, \ldots, n. \tag{29}$$

where

$$\Upsilon \equiv (1-\eta_{ii}\eta_{ii}^*)(1-\eta_{jj}\eta_{jj}^*).$$

From $\eta_{ii} < 0 < \eta_{ii}^*$, $\Upsilon > 0$. From $\tau_i > 0$ and $1-\eta_{ii}\eta_{ii}^* > 0$, (27) shows that $1+\eta_{ii} < 0$. Because $1+\eta_{jj} < 0$, $1+\eta_{jj}^* > 0$, $\eta_{ii} < 0$, and $\Upsilon > 0$, we obtain (i) from (29). We can prove (ii) in a similar way.[23] ■

This rule is similar to the Ramsey rule (the inverse elasticity rule), established in public economics and industrial organization. In contrast to the usual Ramsey rule, Proposition 2 shows that the ranking of a country's equilibrium tariff rates on the two traded goods depends on the own price elasticities of the corresponding goods of all tariff-imposing countries. This is because we consider the Nash equilibrium tariffs. Proposition 2 can be restated by using the relative size of the cross-elasticities between the numeraire good and the nonnumeraire goods. Noting that $\eta_{i0}+\eta_{ii} = 0$ and $\eta_{i0}^*+\eta_{ii}^* = 0$ $(i \neq 0)$ from the properties (ii) of $z_i(\cdot)$ and $z_i^*(\cdot)$, it follows that $\tau_i > \tau_j$ if $\eta_{i0} > \eta_{j0}$ and $-\eta_{j0}^* > -\eta_{i0}^*$, and (ii) $-\tau_i^* > -\tau_j^*$ if $\eta_{j0} > \eta_{i0}$ and $-\eta_{j0}^* > -\eta_{i0}^*$. However, this restatement is invalid when the cross-substitution effects between the nonnumeraire goods exist; the own price elasticities are not a useful indicator in determining the ranking of the equilibrium tariff rates. This is confirmed in Section 4.2.2.

It is worthwhile discussing the uniform tariffs in this model. We can see from (27) that $\tau_1 = \cdots = \tau_m$ and $\tau_1^* = \cdots = \tau_m^*$ if and only if $\eta_{11} = \cdots = \eta_{mm}$ (i.e., $\eta_{10} = \cdots = \eta_{m0}$) and $\eta_{11}^* = \cdots = \eta_{mm}^*$ (i.e., $\eta_{10}^* = \cdots = \eta_{m0}^*$) for $m \leq n$. Note that this is different from Proposition 1, which requires that the elasticities with respect to the price of the numeraire good are equal across all nonnumeraire goods; in Proposition 1, for example, when $\eta_{10} = \eta_{20}$ and $\eta_{10}^* = \eta_{20}^*$ in the case with four traded goods, $\tau_1 = \tau_2$ and $\tau_1^* = \tau_2^*$ are not guaranteed at the optimum.

4.2.2 The case with three traded goods

This section considers the three-good model where all cross-substitution effects between all traded goods exist. We assume that the home (foreign) country exports (imports) good 0 and imports (exports) goods 1 and 2. In the three-good model, the following holds.

Lemma 3. *In the three-good model, the equilibrium tariff rates are*

$$\tau_i = \frac{\alpha^*\varepsilon_{j0}^* - (1-\alpha+\varepsilon_{j0}^*)\varepsilon_{j0}}{(1-\alpha^*+\varepsilon_{j0})(1-\alpha+\varepsilon_{j0}^*) - \alpha\alpha^*}, \qquad i,j = 1,2 \text{ and } i \neq j, \tag{30}$$

$$\tau_i^* = \frac{\alpha\varepsilon_{j0} - (1-\alpha^*+\varepsilon_{j0})\varepsilon_{j0}^*}{(1-\alpha+\varepsilon_{j0}^*)(1-\alpha^*+\varepsilon_{j0}) - \alpha\alpha^*}, \qquad i,j = 1,2 \text{ and } i \neq j, \tag{31}$$

[23] From (27) we obtain

$$[(-\tau_i^*) - (-\tau_j^*)]\Upsilon = -(\eta_{jj}^* - \eta_{ii}^*)(1+\eta_{jj}) + [(-\eta_{jj}) - (-\eta_{ii})](1+\eta_{jj}^*)\eta_{ii}^*, \qquad i,j = 1,\ldots,n,$$

which proves (ii).

where

$$\varepsilon_{j0} \equiv \frac{\eta_{j0}+\eta_{21}+\eta_{12}}{\eta_{11}\eta_{22}-\eta_{12}\eta_{21}}, \ \ \textit{and} \ \ \varepsilon^*_{j0} \equiv \frac{\eta^*_{j0}+\eta^*_{21}+\eta^*_{12}}{\eta^*_{11}\eta^*_{22}-\eta^*_{12}\eta^*_{21}}, \qquad j=1,2. \tag{32}$$

Proof. In the three-good economy, (7) can be rewritten as

$$\begin{bmatrix} \eta^*_{11} & \eta^*_{12} \\ \eta^*_{21} & \eta^*_{22} \end{bmatrix} \begin{bmatrix} \frac{\tau^*_1+(\tau_1-\tau^*_1)\alpha}{1+\tau^*_1} \\ \frac{\tau^*_2+(\tau_2-\tau^*_2)\alpha}{1+\tau^*_2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

which, together with $\eta^*_{10}+\eta^*_{11}+\eta^*_{12}=0$ and $\eta^*_{20}+\eta^*_{21}+\eta^*_{22}=0$,[24] yields

$$\frac{\tau^*_i+(\tau_i-\tau^*_i)\alpha}{1+\tau^*_i} = -\varepsilon^*_{j0}, \qquad i,j=1,2 \text{ and } i \neq j. \tag{33}$$

Applying a similar procedure to (9) yields

$$\frac{\tau_i+(\tau^*_i-\tau_i)\alpha^*}{1+\tau_i} = -\varepsilon_{j0}, \qquad i,j=1,2 \text{ and } i \neq j. \tag{34}$$

From (33) and (34) we obtain (30) and (31). ■

Note that $\eta_{11}\eta_{22}-\eta_{12}\eta_{21}(=(z_{11}z_{22}-z_{12}z_{21})q_1q_2/z_1z_2)>0$ and $\eta^*_{11}\eta^*_{22}-\eta^*_{12}\eta^*_{21}(=(z^*_{11}z^*_{22}-z^*_{12}z^*_{21})q^*_1q^*_2/z^*_1z^*_2)>0$ from properties (iii) of $z_i(\cdot)$ and $z^*_i(\cdot)$. From these and (32) it follows that

$$\varepsilon_{10} \lesseqgtr \varepsilon_{20} \Longleftrightarrow \eta_{10} \lesseqgtr \eta_{20}, \text{ and } \varepsilon^*_{10} \lesseqgtr \varepsilon^*_{20} \Longleftrightarrow \eta^*_{10} \lesseqgtr \eta^*_{20}. \tag{35}$$

Using $\eta_{j0}+\eta_{jj}+\eta_{ji}=0$ ($i,j=1,2$ and $i \neq j$), (32) can be rewritten as

$$\varepsilon_{j0} = \frac{-\eta_{jj}+\eta_{ij}}{\eta_{11}\eta_{22}-\eta_{12}\eta_{21}}, \qquad i,j=1,2 \text{ and } i \neq j.$$

This shows that $0<\varepsilon_{j0}$ if good 1 is a substitute for good 2 in the home country ($\eta_{ij}>0$ for $i,j=1,2$ and $i \neq j$). Similarly, it follows that $\varepsilon^*_{j0}<0$ if good 1 is a substitute for good 2 in the foreign country.

To bring out the role of the cross-substitution effects between the nonnumeraire goods on the equilibrium tariff structure, we assume that $z_{iu}=z^*_{iu^*}=0$ for $i=1,2$, but make no restriction on the cross-substitution effects. In this case, we obtain the following.

Proposition 3. *In the three-good model, the equilibrium tariff rates are characterized by the following: when good 1 is a substitute for good 2 in both countries and* $z_{iu}=z^*_{iu^*}=0$ *for* $i=1,2$, *(i)* $\tau_i>\tau_j$ *if* $\eta_{i0}>\eta_{j0}$ *and* $-\eta^*_{j0}>-\eta^*_{i0}$, *and (ii)* $\tau^*_i>\tau^*_j$ *if* $\eta_{j0}>\eta_{i0}$ *and* $-\eta^*_{j0}>-\eta^*_{i0}$, *for* $i,j=1,2$.

[24] These are obtained from properties (ii) of $z_i(\cdot)$ and $z^*_i(\cdot)$.

Proof. When $z_{iu} = z^*_{iu^*} = 0$ for $i = 1, 2$, $\alpha = \alpha^* = 1$. From (30), we obtain

$$(\tau_1 - \tau_2)\,\Lambda = [(-\varepsilon^*_{20}) - (-\varepsilon^*_{10})]\,(1 - \varepsilon_{10}) + (\varepsilon_{10} - \varepsilon_{20})\,(1 - \varepsilon^*_{10})(-\varepsilon^*_{20}), \tag{36}$$

where

$$\Lambda \equiv (\varepsilon^*_{20}\varepsilon_{20} - 1)\,(\varepsilon^*_{10}\varepsilon_{10} - 1) > 0,$$

in which the inequality follows from $\varepsilon^*_{i0} < 0 < \varepsilon_{i0}$.

From (31), we have

$$1 + \tau^*_i = \frac{\varepsilon_{j0} - 1}{\varepsilon^*_{j0}\varepsilon_{j0} - 1}, \qquad i, j = 1, 2 \text{ and } i \neq j.$$

Because $1 + \tau^*_1 > 0$ and $\varepsilon^*_{j0}\varepsilon_{j0} - 1 < 0$, this shows that $0 < 1 - \varepsilon_{j0}$. From this, $\varepsilon^*_{i0} < 0$, $\Lambda > 0$, (35) and (36), we obtain (i). We can prove (ii) in a similar way. ■

As mentioned above, Proposition 2 can be restated by the cross-elasticities between the numeraire good and the nonnumeraire goods, whereas Proposition 3 cannot be restated by the own price elasticities as $\eta_{10} - \eta_{20} \neq (-\eta_{11}) - (-\eta_{22})$ because of the existence of the cross-elasticities between the nonnumeraire goods. From property (ii) of $z_i(\cdot)$, we obtain $\eta_{i0} + \eta_{ii} + \eta_{ij} = 0$ for $i, j = 1, 2$ and $i \neq j$. This yields $\eta_{10} - \eta_{20} = (-\eta_{11}) - (-\eta_{22}) - \eta_{12} + \eta_{21}$, which shows that the sign of $\eta_{10} - \eta_{20}$ is not always the same as that of $(-\eta_{11}) - (-\eta_{22})$. This implies that the own price elasticities do not correctly characterize the equilibrium tariff structure. Proposition 3 is valid even if one of the nonnumeraire goods is complementary to good 0 (i.e., $z_{i0} < 0$ and hence $\eta_{i0} < 0$) as long as $\eta_{i0} + \eta_{21} + \eta_{12} > 0$ (i.e., $\varepsilon_{i0} > 0$) for $i = 1, 2$.[25] A similar argument can be applied to the tariffs in the foreign country.

Next, we consider the case where one of the conditions in (20) is not satisfied, but there are no restrictions on the income effects. We add the following assumption in Nash equilibria.

Assumption 2. *All goods are substitutes for each other in both countries.*

We obtain the following proposition under Assumptions 1 and 2.

Proposition 4. *In the three-good model, the equilibrium tariff rates are characterized by the following: (i) when $\eta_{10} = \eta_{20}$, $\tau_i > \tau_j$ if and only if $-\eta^*_{j0} > -\eta^*_{i0}$ and (ii) when $\eta^*_{10} = \eta^*_{20}$, $\tau^*_i > \tau^*_j$ if and only if $\eta_{j0} > \eta_{i0}$, for $i, j = 1, 2$.*

Proof. From (35), $\varepsilon_{10} = \varepsilon_{20} (\equiv \varepsilon)$ if and only if $\eta_{10} = \eta_{20}$. Using (30), we obtain

$$(\tau_1 - \tau_2)\Omega = -[(-\varepsilon^*_{20}) - (-\varepsilon^*_{10})](1 - \alpha^* - \alpha + \varepsilon)\alpha^*, \tag{37}$$

[25] Note that at least either of goods 1 and 2 is a substitute for good 0. It follows that $z_{00} + z_{10} + z_{20} = 0$ from properties (i) and (ii) and $z_{00} < 0$ from property (iii). Suppose that $z_{10} \leq 0$ and $z_{20} \leq 0$. This is contradictory to $z_{00} + z_{10} + z_{20} = 0$.

where

$$\Omega \equiv [(1-\alpha+\varepsilon_{20}^*)(1-\alpha^*+\varepsilon)-\alpha\alpha^*]\,[(1-\alpha+\varepsilon_{10}^*)(1-\alpha^*+\varepsilon)-\alpha\alpha^*]\,.$$

From (30), we have

$$1+\tau_i = \frac{1-\alpha^*-\alpha+\varepsilon_{j0}^*}{(1-\alpha^*+\varepsilon_{j0})(1-\alpha+\varepsilon_{j0}^*)-\alpha\alpha^*}, \qquad i,j=1,2 \text{ and } i \neq j. \tag{38}$$

It follows that $1-\alpha-\alpha^* < 0$ at the optimum, which is proved in Appendix B. From $\varepsilon_{j0}^* < 0$, $1-\alpha^*-\alpha < 0$ and $1+\tau_i > 0$, (38) shows that $(1-\alpha^*+\varepsilon_{j0})(1-\alpha+\varepsilon_{j0}^*)-\alpha\alpha^* < 0$. This shows that $\Omega > 0$.

From (31), we obtain

$$1+\tau_i^* = \frac{1-\alpha-\alpha^*+\varepsilon}{(1-\alpha+\varepsilon_{j0}^*)(1-\alpha^*+\varepsilon)-\alpha\alpha^*}, \qquad i,j=1,2 \text{ and } i \neq j,$$

which shows that $1-\alpha-\alpha^*+\varepsilon < 0$ because $(1-\alpha^*+\varepsilon_{j0})(1-\alpha+\varepsilon_{j0}^*)-\alpha\alpha^* < 0$ and $1+\tau_i^* > 0$. By using this and $\Omega > 0$, (37) shows that the sign of $\tau_1 - \tau_2$ is the same as that of $(-\varepsilon_{20}^*) - (-\varepsilon_{10}^*)$. From this and (35) we obtain (i). We can prove (ii) in a similar way. ■

This shows that when $\eta_{10} = \eta_{20}$, the ranking of τ_1 and τ_2 is determined by the relative size of η_{10}^* and η_{20}^*. From (34), we obtain $(\varepsilon+1-\alpha^*)(\tau_1-\tau_2) = \alpha^*(\tau_2^*-\tau_1^*)$, which shows that the equilibrium tariff rates in the foreign country are also nonuniform as long as $\varepsilon+1-\alpha^* \neq 0$. Suppose that $\eta_{10} = \eta_{20}$ and $-\eta_{20}^* > -\eta_{10}^*$. Then, the sign of $\tau_2^* - \tau_1^*$ is the same as that of $\varepsilon+1-\alpha^*$. When $z_{1u} = z_{2u} = 0$, $\alpha^* = 1$ from (10) and hence $\varepsilon+1-\alpha^* = \varepsilon > 0$. In this case, $\tau_i > \tau_j$ and $\tau_j^* > \tau_i^*$ if and only if $-\eta_{j0}^* > -\eta_{i0}^*$. A similar argument can be applied to the case of $\eta_{10}^* = \eta_{20}^*$. These show that if one of the conditions in (20) is not satisfied, the equilibrium tariff rates are generally not uniform in both countries. Proposition 4 is not generally valid in the model with $n+1$ traded goods. However, making two composite goods from the nonnumeraire goods, the intuition behind Proposition 4 can be applied to the tariff rates on the two composite goods, even in such a model.

5 The aggregated tariff rates on each traded good

This section examines the structure of the Nash equilibrium tariff rates on each traded good imposed by the two countries in the model with $n+1$ traded goods. Let us first provide the following lemma.

Lemma 4. *Suppose that Assumption 2 is satisfied. Then, the best-response tariffs of the home country satisfy*

$$\frac{\tau_i^* + (\tau_i - \tau_i^*)\alpha}{1+\tau_i^*} = v_i^* > 0, \qquad i = 1, \cdots, n, \tag{39}$$

for a positive scalar υ_i^**, and those of the foreign country satisfy*

$$\frac{\tau_i + (\tau_i^* - \tau_i)\alpha^*}{1+\tau_i} = \upsilon_i < 0, \qquad i = 1, \cdots, n, \tag{40}$$

for a negative scalar υ_i*.*

Proof. Applying Cramel's rule to (7) yields

$$\frac{\tau_i^* + (\tau_i - \tau_i^*)\alpha}{1+\tau_i^*} = \frac{|\Phi_i^*|}{|\Phi^*|}, \qquad i = 1, \cdots, n, \tag{41}$$

where Φ^* is the $n \times n$ matrix of the elasticities of the foreign country and Φ_i^* is obtained by changing all elements of the i-th column of Φ^* to "1":

$$\Phi^* \equiv \begin{bmatrix} \eta_{11}^* & \cdots & \eta_{1n}^* \\ \vdots & & \vdots \\ \eta_{n1}^* & \cdots & \eta_{nn}^* \end{bmatrix}, \text{ and } \Phi_i^* \equiv \begin{bmatrix} \eta_{11}^* & \cdots & \eta_{1(i-1)}^* & 1 & \eta_{1(i+1)}^* & \cdots & \eta_{1n}^* \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \eta_{n1}^* & \cdots & \eta_{n(i-1)}^* & 1 & \eta_{n(i+1)}^* & \cdots & \eta_{nn}^* \end{bmatrix}.$$

The determinant of Φ_i^* is

$$|\Phi_i^*| = \begin{vmatrix} 1 & \phi^{*T} \\ \sigma & \Phi_{ii}^* \end{vmatrix}, \qquad i = 1, \cdots, n, \tag{42}$$

where σ is the $n-1$ dimensional vector whose elements are all "1", ϕ^* is the $n-1$ dimensional vector of the elasticities of good i but does not include η_{ii}^*, and Φ_{ii}^* is the $(n-1)\times(n-1)$ matrix obtained by deleting the i-th column and row of Φ^*:[26]

$$\phi^* \equiv \begin{bmatrix} \eta_{i1}^* \\ \vdots \\ \eta_{i(i-1)}^* \\ \eta_{i(i+1)}^* \\ \vdots \\ \eta_{in}^* \end{bmatrix}, \text{ and } \Phi_{ii}^* \equiv \begin{bmatrix} \eta_{11}^* & \cdots & \eta_{1(i-1)}^* & \eta_{1(i+1)}^* & \cdots & \eta_{1n}^* \\ \vdots & & \vdots & \vdots & & \vdots \\ \eta_{(i-1)1}^* & \cdots & \eta_{(i-1)(i-1)}^* & \eta_{(i-1)(i+1)}^* & \cdots & \eta_{(i-1)n}^* \\ \eta_{(i+1)1}^* & \cdots & \eta_{(i+1)(i-1)}^* & \eta_{(i+1)(i+1)}^* & \cdots & \eta_{(i+1)n}^* \\ \vdots & & \vdots & \vdots & & \vdots \\ \eta_{n1}^* & \cdots & \eta_{n(i-1)}^* & \eta_{n(i+1)}^* & \cdots & \eta_{nn}^* \end{bmatrix}.$$

From (42), we obtain[27]

$$\frac{|\Phi_i^*|}{|\Phi^*|} = \left(1 - \phi^{*T}\Phi_{ii}^{*-1}\sigma\right)\frac{|\Phi_{ii}^*|}{|\Phi^*|}, \qquad i = 1, \cdots, n.$$

[26] Note that ϕ^{*T} is the transpose of ϕ^*.

[27] The determinant of the partitioned matrix can be rewritten as

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \left|A - BD^{-1}C\right| |D|,$$

where A and D are square and D is nonsingular. See Johnston (1972, p. 95).

From property (iii) of $z_i^*(\cdot)$, $|\Phi_{ii}^*| / |\Phi^*| > 0$.[28] Note that $\phi^* < 0$ and $\Phi_{ii}^{*-1} > 0$ from Assumption 2.[29] This shows that $1 - \phi^{*T}\Phi_{ii}^{*-1}\sigma > 0$. Thus, $|\Phi_i^*| / |\Phi^*| > 0$. Defining $\upsilon_i^* \equiv |\Phi_i^*| / |\Phi^*|$, from (41) we obtain (39). In a similar way, we obtain (40). ■

When the foreign country engages in free trade (i.e., $\tau_i^* = 0$ for $i = 1, \cdots, n$), (39) is reduced to $\alpha\tau_i = \upsilon_i^* > 0$ for $j = 1, \cdots, n$, which shows that all optimal import tariffs in the home country are positive (Horwell and Pearce 1970). When $\tau_i = 0$ for $i = 1, \cdots, n$, we obtain $\alpha^*\tau_i^* = \upsilon_i < 0$ from (40), which shows that the imposition of the export tax on all export goods is optimal for the foreign country.

Using Lemma 4, we obtain the following proposition.[30]

Proposition 5. *The equilibrium tariffs satisfy* $0 < \tau_i - \tau_i^*$ *for* $i = 1, \cdots, n$.

Proof. From (39) and (40), we obtain

$$(1 - \alpha - \alpha^*)(\tau_i^* - \tau_i) = (1 + \tau_i)\upsilon_i^* - (1 + \tau_i^*)\upsilon_i, \qquad i = 1, \cdots, n.$$

Note that $\upsilon_i < 0 < \upsilon_i^*$, $1 + \tau_i > 0$ and $1 + \tau_i^* > 0$. From Appendix B, $1 - \alpha - \alpha^* < 0$. Thus, $0 < \tau_i - \tau_i^*$ for $i = 1, \cdots, n$. ■

Proposition 5 shows that the aggregate of the equilibrium tariff rates on each traded good is positive. If the optimal intervention on good i by the home country is an import subsidy ($\tau_i < 0$), that of the foreign country must be an export tax ($\tau_i^* < 0$), and the subsidy rate by the home country is never higher than the export tax rate by the foreign country. If the optimal intervention on good i by the foreign country is an export subsidy ($\tau_i^* > 0$), that of the home country must be an import tariff ($\tau_i > 0$), and the subsidy rate by the foreign country is never higher than the import tariff rate by the home country. Under the Nash equilibrium tariffs, the totality of trade interventions on each traded good by the two countries is a tax, which has the incentive to reduce international trade.

6 Conclusion

This paper examined the structure of Nash equilibrium tariffs using a two-country economy with more than two traded goods. We proved that in a model with $n + 1$ traded goods, the equilibrium tariff rates are uniform in both countries if and only if the elasticities with respect to the price of the numeraire good are equal across all nonnumeraire goods in both countries. Then, when the uniform condition is not

[28] Let us denote the $n \times n$ foreign country's substitution matrix of nonnumeraire goods as Z^* and the $(n-1) \times (n-1)$ matrix obtained by deleting the i-th column and row of Z^* as Z_{ii}^*. Then we have $|\Phi_{ii}^*| / |\Phi^*| = (z_i^*/q_i^*)\,(|Z_{ii}^*| / |Z^*|)$. From property (iii) of $z_i^*(\cdot)$, $|Z_{ii}^*| / |Z^*| < 0$, which, together with $z_i^* < 0$ $(i = 1, \cdots, n)$, yields $|\Phi_{ii}^*| / |\Phi^*| > 0$.

[29] If (i) all of the diagonal elements of an $n \times n$ matrix A are positive and all of the off-diagonal elements are negative, and (ii) $Ae > 0$ for some $n \times 1$ positive vector e, then $A^{-1} > 0$.

[30] Note that Assumptions 1 and 2 are satisfied in the following proposition.

satisfied, we examined the ranking of the tariff rates in the model with $n+1$ goods where there are no cross-substitution effects between the tariff-imposed goods and in the three-good model with the cross-substitution effects. In addition, we explored the sign of the aggregated equilibrium tariff rates of the two countries on each traded good in the model with $n+1$ traded goods. The uniform tariff condition (condition (20)) is evaluated by the elasticities of the compensated excess demands for goods and is independent of the income effects. This is in contrast to the fact that the income effects of the goods play a critical role in determining the sign of the optimal tariff (Kemp 1967 and Bond 1990). The formulae (30) and (31) in this paper also showed that the sign of the equilibrium tariff rate depends on the sign of the income effects. However, Propositions 1-(ii), 4 and 5 are unaffected even if some goods are inferior, as long as there are no sufficiently strong inferior goods.[31] This may be a practical advantage for tariff-imposing active countries, and custom unions that impose common external tariffs.

Finally, we showed that the model in this paper may be generalized in at least two ways. First, there is scope to extend the model to more than two countries. Second, it is worth examining the ranking condition for the optimal tariff rates in the model with more than three traded goods under full substitutability. This analysis, however, would be complex because of market linkages between the goods.

Appendix A: The derivation of (26)

From (25), we have

$$\tau\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^* = -(1+\tau^*) \sum_{i=0}^{n} q_i z_{iu^*}^*.$$

By noting that $q_i = (1+\tau)p_i$ and $q_i^* = (1+\tau^*)p_i$ for $i = 1, \cdots, n$, this can be rewritten as

$$\begin{aligned} \tau\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^* &= -(1+\tau^*)\left(\sum_{i=0}^{n} p_i z_{iu^*}^* + \tau \sum_{i=1}^{n} p_i z_{iu^*}^*\right) \\ &= -(1+\tau^*)\sum_{i=0}^{n} p_i z_{iu^*}^* - \tau \sum_{i=1}^{n} q_i^* z_{iu^*}^*, \end{aligned}$$

which yields

$$\begin{aligned} \tau &= \frac{-(1+\tau^*)\sum_{i=0}^{n} p_i z_{iu^*}^*}{\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^* + \sum_{i=1}^{n} q_i^* z_{iu^*}^*} \\ &= \frac{-(1+\tau^*) z_{0u^*}^* - \sum_{i=1}^{n} q_i^* z_{iu^*}^*}{\beta^* \sum_{i=0}^{n} q_i^* z_{iu^*}^* + \sum_{i=1}^{n} q_i^* z_{iu^*}^*}. \end{aligned}$$

[31] Note that it was assumed in Propositions 2 and 3 that there are no income effects of the tariff-imposed goods.

From this, we obtain

$$1+\tau = \frac{\beta^* \sum_{i=0}^n q_i^* z_{iu^*}^* - (1+\tau^*) z_{0u^*}^*}{\beta^* \sum_{i=0}^n q_i^* z_{iu^*}^* + \sum_{i=1}^n q_i^* z_{iu^*}^*},$$

which yields (26).

Appendix B: The proof of $1-\alpha-\alpha^* < 0$

This proof is made in the model with n nonnumeraire goods and is also used in the proof of Lemma 4. From (39) and (40), we obtain

$$(1-\alpha-\alpha^*)(\tau_i^* - \tau_i) = \epsilon_i, \qquad i = 1, \cdots, n, \tag{B-1}$$

where

$$\epsilon_i \equiv (1+\tau_i)\upsilon_i^* - (1+\tau_i^*)\upsilon_i > 0, \qquad i = 1, \cdots, n,$$

in which the inequality follows from $\upsilon_i < 0 < \upsilon_i^*$,[32] $1+\tau_i > 0$ and $1+\tau_i^* > 0$ for $i = 1, \cdots, n$.

From (40),

$$\tau_i^* - \tau_i = \frac{(1+\tau_i)\,\upsilon_i - \tau_i}{\alpha^*}, \qquad i = 1, \cdots, n.$$

Substituting this for $(\tau_i^* - \tau_i)$ in (B-1) yields

$$\tau_i = -\left(\frac{\alpha^*}{1-\alpha-\alpha^*}\right)\epsilon_i + (1+\tau_i)\,\upsilon_i, \qquad i = 1, \cdots, n. \tag{B-2}$$

From (8) and (10), we obtain

$$\begin{aligned} 1-\alpha-\alpha^* &= 1 - \frac{\sum_{i=0}^n p_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*} - \frac{\sum_{i=0}^n p_i z_{iu}}{\sum_{i=0}^n q_i^* z_{iu}} \\ &= \frac{\sum_{i=1}^n \tau_i p_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*} - \frac{\sum_{i=0}^n p_i z_{iu}}{\sum_{i=0}^n q_i^* z_{iu}}. \end{aligned}$$

Substituting (B-2) for τ_i in this equation and using (1), we have

$$1-\alpha-\alpha^* = -\left(\frac{\alpha^*}{1-\alpha-\alpha^*}\right)\left(\frac{\sum_{i=1}^n \epsilon_i p_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*}\right) + \frac{\sum_{i=1}^n \upsilon_i q_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*} - \frac{\sum_{i=0}^n p_i z_{iu}}{\sum_{i=0}^n q_i^* z_{iu}}.$$

After some manipulation, we obtain

$$(1-\alpha-\alpha^*)^2 + \frac{\alpha^* \sum_{i=1}^n \epsilon_i p_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*} = (1-\alpha-\alpha^*)\left(\frac{\sum_{i=1}^n \upsilon_i q_i z_{iu^*}^*}{\sum_{i=0}^n q_i z_{iu^*}^*} - \frac{\sum_{i=0}^n p_i z_{iu}}{\sum_{i=0}^n q_i^* z_{iu}}\right).$$

The LHS is positive, and the expression in brackets on the RHS is negative. Hence, $1-\alpha-\alpha^* < 0$.

[32] From Assumption 2, $\upsilon_i < 0 < \upsilon_i^*$ for $i = 1, \cdots, n$.

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