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by

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Abstract: This paper shows that multiple steady states occur if a central bank is not independent and is compelled to finance a large fiscal deficit through seigniorage. If the initial capital stock is less than a certain threshold, the economy converges to a steady state where the capital stock is low and the inflation rate is high. Tight fiscal policies bring the economy out of the poverty trap. Moreover, if the central bank is independent, the poverty trap is removed independently of the fiscal position.

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1 Introduction

Sargent and Wallace (1981) and Sargent (1986) present the view that sustained fiscal deficits generate long-run inflation because they must eventually be financed by money creation. Several studies find evidence consistent with this relationship between fiscal deficits, seigniorage, and inflation. Using data on 94 market economies, Fischer et al. (2002) obtain a relationship between fiscal deficits and seigniorage such that a 10 percent deterioration in fiscal balances leads to a 1.5 percent increase in seigniorage revenue. Moser (1995) reports that monetary expansions, driven mainly by expansionary fiscal policies, largely explain inflation in Nigeria. Using data from 107 countries over the period 1960–2001, Catão and Terrones (2005) find a strong positive relationship between fiscal deficits and inflation in groups of high-inflation developing countries. However, they find no comparable relationship for groups of low-inflation developed countries.

Moreover, seigniorage is related to economic development. Generally, developing countries rely more on seigniorage than developed countries. Easterly and Schmidt-Hebbel (1993) report that seigniorage revenue in developing countries on average exceeds 2 percent of gross domestic product (GDP) over the period 1965–1989, whereas in industrial countries it averages only 1 percent. Similarly, Aisen and Veiga (2008) find that seigniorage revenue is five times higher in developing countries than developed countries over the period 1960–1999. In addition, they find that in the 1990s average seigniorage revenue is 14.65 percent of total government revenue in developing countries and only 1.64 percent in industrial countries. In explanation, Cukierman et al. (1992), Berument (1998), Click (1998), and Aisen and Veiga (2008) argue that low central bank independence and unstable political situations cause governments in developing countries to rely excessively on seigniorage.

From the studies cited above, there appears to be a relationship between fiscal deficits, seigniorage, inflation, and economic development. The present paper theoretically analyzes what causal relationship exists between them and shows that loose fiscal policies create a poverty trap if a central bank is not independent, using the following dynamic general equilibrium model. A Household faces a cash-in-advance (CIA) constraint when it purchases goods. Similar to Evans and Yarrow (1981), Sargent and Wallace (1981), Weil (1987), Bruno and Fischer (1990), and Evans (1995), a fiscal authority runs a deficit and compels a monetary authority to finance the fiscal deficit through money creation. However, unlike these papers, the present model includes reserve requirements and capital accumulation. As in Walsh (1984) and Romer (1985), a commercial bank is required to hold money as reserves.[1] As in Haslag (1998), all capital is intermediated by the commercial bank.[2]

If the ratio of government spending to GDP is high and the fiscal deficit is large, there are multiple steady states and the initial capital stock determines which steady state is reached. If the initial capital stock is less than a certain threshold, the demand for cash and reserves is low because production and deposits are low. The low demand reduces the inflation tax base (real money balances), so a high inflation tax rate, i.e. a high inflation rate, is needed to finance the fiscal deficit. Under the reserve requirement, the high inflation rate causes capital decumulation by lowering the real rate of interest on deposits. The capital decumulation further reduces the inflation tax base, raises the inflation rate, and lowers the real rate of interest on deposits, again

[1] See also Brock (1989), Chari et al. (1995), Haslag (1998), Freeman and Kydland (2000), and Basu (2001) for reserve requirements.

[2] This assumption is plausible because according to Fry (1995), financial systems in developing economies are dominated by commercial banks.

decumulating capital. Because of this mechanism, the economy converges to a steady state where the capital stock is low and the inflation rate is high.

Alternatively, if the initial capital stock is more than the threshold, the economy converges to a steady state where the capital stock is high and the inflation rate is low. Both of the steady states are saddle-path stable and the paths converging to each steady state are uniquely determined. This is the difference between the present paper and other studies of inflation caused by fiscal deficits (Evans and Yarrow, 1981; Sargent and Wallace, 1981; Weil, 1987; Bruno and Fischer, 1990; Evans, 1995). In these studies, there are two steady states, a high-inflation steady state and a low-inflation steady state. However, the equilibrium paths are indeterminate and do not depend on the initial capital stock.

The present paper explores what policies are helpful for the economy to escape from the poverty trap. Decreases in the fiscal deficit and the government's propensity to spend reduce the inflation rate by causing the government to rely less on seigniorage or expanding the inflation tax base. The reduction in the inflation rate increases the real rate of interest on deposits and consequently brings the economy out of the poverty trap. Furthermore, if the monetary authority is independent, the poverty trap is eliminated, that is, a unique steady state obtains. It is because the steady-state inflation rate equals the money growth rate set by the monetary authority independently of the fiscal position. This is the same as the elimination of the high inflation trap discussed in Bruno and Fischer (1990).

The remainder of the present paper is organized as follows. Sections 2 and 3 describe the structure and the dynamics of the model economy respectively. Section 4 shows that under a dependent monetary authority, a poverty trap arises. This section also argues that tight fiscal policies enable the economy to escape the poverty trap, and that an independent monetary authority removes the poverty trap. Section 5 concludes. The appendices examine the existence and the dynamic stability of the steady states.

2 The Model

We consider an economy where there are a private sector composed of a representative firm, a representative household, and a representative commercial bank, and a public sector composed of a fiscal and a monetary authority. There are two assets, money and capital. Whereas both the household and the commercial bank hold money, the commercial bank directly holds all of capital.

2.1 The Representative Firm

The representative firm produces goods using labor supplied inelastically by the representative household and capital k_t. As in Haslag (1998), we assume that all capital is intermediated by the representative commercial bank.

Let $f(k_t)$, which satisfies $f'(\cdot) > 0$, $f''(\cdot) < 0$, and the Inada condition, denote the per capita production function of the firm. As usual, the firm's profit-maximization problem yields $w_t = f(k_t) - f'(k_t)k_t$ and

$$r_t = f'(k_t), \tag{1}$$

where w_t and r_t are the real wage and the real capital rent respectively.

2.2 The Representative Household

The representative household maximizes its lifetime utility:

$$\int_0^\infty u(c_t)\exp(-\rho t)\mathrm{d}t, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0,$$

where c_t is consumption and ρ (> 0) is the subjective discount rate. The household faces the CIA constraint:

$$c_t \leq m_t^H, \tag{2}$$

where m_t^H represents real cash holdings, and the flow budget constraint:

$$\dot{a}_t = r_t^D d_t - \pi_t m_t^H + w_t - c_t - \tau_t, \tag{3}$$

where a_t denotes real total assets, consisting of real deposit holdings d_t and m_t^H:

$$a_t = d_t + m_t^H, \tag{4}$$

and r_t^D is the real rate of interest on deposits, π_t ($\equiv \dot{P}_t / P_t$) is the inflation rate of the price of goods P_t, and τ_t is a lump-sum tax.

The current-value Hamiltonian function for the utility-maximization problem is given by

$$\begin{aligned}\mathcal{H}_t = u(c_t) + \lambda_t(r_t^D d_t - \pi_t m_t^H + w_t - c_t - \tau_t) + \gamma_t(a_t - d_t - m_t^H) \\ + \eta_t(m_t^H - c_t),\end{aligned}$$

where λ_t is the co-state variable associated with (3) and γ_t and η_t are the Lagrange multipliers associated with (4) and (2) respectively. We consider only the case where (2) is binding ($\eta_t > 0$).[3] The first-order conditions are then

$$u'(c_t) = \lambda_t + \eta_t, \tag{5}$$

$$r_t^D \lambda_t = \gamma_t, \tag{6}$$

$$-\pi_t \lambda_t + \eta_t = \gamma_t, \tag{7}$$

$$\dot{\lambda}_t - \rho \lambda_t = -\gamma_t, \tag{8}$$

[3] From (6) and (7), $\eta_t > 0$ if $r_t^D > -\pi_t$ (the nominal rate of interest on deposits is positive).

and the transversality condition: $\lim_{t\to\infty} \lambda_t a_t \exp(-\rho t) = 0$.

2.3 The Representative Commercial Bank

The representative commercial bank collects deposits from the household and rents capital to the firm in order to maximize its profit:

$$r_t k_t - \pi_t m_t^B - r_t^D d_t,$$

where m_t^B denotes real reserve holdings, subject to the balance sheet:

$$k_t + m_t^B = d_t \tag{9}$$

and the reserve requirement:

$$m_t^B \geq \epsilon d_t, \quad 0 < \epsilon < 1, \tag{10}$$

where ϵ is the reserve requirement ratio. The reserve requirement is the same as those of Walsh (1984), Romer (1985), Chari et al. (1995), Haslag (1998), Freeman and Kydland (2000), and Basu (2001).

As in these papers, we consider only the case where (10) is binding. Note that if $r_t > -\pi_t$, which implies that renting capital is more profitable than holding reserves, then the bank reduces reserve holdings to the minimum level and therefore it is binding. In this case, the bank's profit-maximization problem gives

$$r_t^D = (1 - \epsilon) r_t - \epsilon \pi_t, \tag{11}$$

representing the equality between the marginal cost and the marginal revenue of collecting deposits. From (11), increases in the reserve requirement ratio and the inflation rate reduce the marginal revenue by worsening the distortion caused by the reserve requirement or making holding reserves more costly, and hence they lead to a fall in the real rate of interest on deposits:[4]

$$\frac{\partial r_t^D}{\partial \epsilon} = -(r_t + \pi_t) < 0, \quad \frac{\partial r_t^D}{\partial \pi_t} = -\epsilon < 0. \tag{12}$$

From (9) and (10), we obtain $d_t = k_t/(1-\epsilon)$ and

$$m_t^B = \frac{\epsilon}{1-\epsilon} k_t. \tag{13}$$

2.4 The Fiscal and Monetary Authorities

Following Evans and Yarrow (1981), Bruno and Fischer (1990, Section 1), and Evans (1995), we consider the case where a fiscal deficit is only financed by seigniorage.[5] As Bruno and Fischer (1990) point out, this case is consistent with the situation where government bonds are not absorbed in the domestic and international financial markets.[6]

The fiscal authority spends a constant ratio of real GDP:

$$g_t = \theta f(k_t), \quad 0 < \theta < 1, \tag{14}$$

where g_t is government spending and θ indicates the government's propensity to spend. It always runs a constant fiscal deficit ϕ as follows:

$$g_t - \tau_t = \phi, \quad \phi > 0.$$

The monetary authority is not independent and is compelled to finance the fiscal deficit through money creation:

$$\phi = \dot{m}_t + \pi_t m_t \ (= \dot{M}_t / P_t), \tag{15}$$

where m_t ($\equiv M_t/P_t$) and M_t denote real money balances and the nominal money supply respectively.

[4]The first inequality holds in the case where $r_t > -\pi_t$ and (10) is binding.

[5]Sargent and Wallace (1981), Weil (1987), and Bruno and Fischer (1990, Section 2) include both money and bond financing.

[6]Click (1998) and Aisen and Veiga (2008) find that the less available external borrowing becomes, the more a government relies on seigniorage.

3 The Dynamics

The money market equilibrium is

$$m_t = m_t^H + m_t^B. \tag{16}$$

From (2), (13), and (16), c_t is given as a function of k_t and m_t:

$$c_t = m_t - \frac{\epsilon}{1-\epsilon} k_t \equiv c(k_t, m_t). \tag{17}$$

From (1), (5)-(7), (11), and (17), the inflation rate is shown as a function of λ_t, k_t, and m_t:

$$\pi_t = \frac{1}{1-\epsilon}\left[\frac{u'(c(k_t, m_t))}{\lambda_t} - 1\right] - f'(k_t) \equiv \pi(\lambda_t, k_t, m_t). \tag{18}$$

From (1), (6), (8), (11), and (18), the law of motion of λ_t is

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - (1-\epsilon) f'(k_t) + \epsilon \pi(\lambda_t, k_t, m_t). \tag{19}$$

The goods market equilibrium is

$$c_t + \dot{k}_t + g_t = f(k_t). \tag{20}$$

Substituting (14) and (17) into (20) gives the dynamic equation of k_t:

$$\dot{k}_t = (1-\theta) f(k_t) + \frac{\epsilon}{1-\epsilon} k_t - m_t. \tag{21}$$

From (15) and (18), m_t moves according to

$$\dot{m}_t = \phi - \pi(\lambda_t, k_t, m_t) m_t. \tag{22}$$

(19), (21), and (22) formulate an autonomous dynamic system of λ_t, k_t, and m_t.

4 Multiple Steady States

From (19) where $\dot{\lambda} = 0$, the time preference rate equals the real rate of interest on deposits:

$$\rho = (1-\epsilon)f'(k) - \epsilon\pi \ (= r^D). \tag{23}$$

From (21) where $\dot{k} = 0$, steady-state real money balances are given by

$$m = (1-\theta)f(k) + \frac{\epsilon}{1-\epsilon}k, \tag{24}$$

which also represents the steady-state money market equilibrium. Note that from (2), (13), (14), and (20) where $\dot{k} = 0$, the first and second terms on the right-hand side of (24) are the demand for cash and that for reserves respectively. From (24), increases in k and ϵ and a decrease in θ increase the demand for cash and reserves and hence

$$\frac{\mathrm{d}m}{\mathrm{d}k} > 0, \quad \frac{\partial m}{\partial \epsilon} > 0, \quad \frac{\partial m}{\partial \theta} < 0. \tag{25}$$

Substituting (24) into (22) where $\dot{m} = 0$ gives the steady-state inflation rate as a function of k:[7]

$$\pi = \frac{\phi}{m} = \frac{\phi}{(1-\theta)f(k) + \frac{\epsilon}{1-\epsilon}k} \equiv \pi(k) > 0, \tag{26}$$

which implies

$$\frac{\mathrm{d}\pi}{\mathrm{d}k} < 0, \quad \frac{\partial \pi}{\partial \epsilon} < 0, \quad \frac{\partial \pi}{\partial \theta} > 0, \quad \frac{\partial \pi}{\partial \phi} > 0. \tag{27}$$

Increases in k and ϵ and a decrease in θ lower π because from (25) they expand the inflation tax base. An increase in ϕ raises π because a higher inflation tax rate is required to finance the deteriorating fiscal deficit.

[7]Since $\pi > 0$, we find $r^D > -\pi$ and $r > -\pi$, implying that both the CIA constraint (2) and the reserve requirement (10) are binding.

Substituting (26) into (23) yields

$$\rho = (1-\epsilon)f'(k) - \frac{\epsilon\phi}{(1-\theta)f(k) + \dfrac{\epsilon}{1-\epsilon}k} \equiv \psi(k), \tag{28}$$

where $\psi(k)$ represents the real rate of interest on deposits. Once k is determined by (28), all steady-state endogenous variables are obtained. However, $\psi(0)$ $(= \infty - \infty)$ is an indeterminate form whereas $\psi(\infty) = 0 - 0 = 0 < \rho$. Thus, if $\psi(0) < \rho$, there may be no value of k satisfying (28) and no steady state.

To ensure the existence of the steady state, the per capita production function is specified as follows:[8]

$$f(k) = Ak^{\alpha}, \quad A > 0, \quad 0 < \alpha < 1/2.$$

Then $\psi(k)$ reduces to

$$\psi(k) = \alpha(1-\epsilon)Ak^{\alpha-1} - \frac{\epsilon\phi}{(1-\theta)Ak^{\alpha} + \dfrac{\epsilon}{1-\epsilon}k}. \tag{29}$$

By arranging $\psi(k)$ as follows:

$$\psi(k) = k^{\alpha-1}\left[\alpha(1-\epsilon)A - \frac{\epsilon\phi}{(1-\theta)Ak^{2\alpha-1} + \dfrac{\epsilon}{1-\epsilon}k^{\alpha}}\right],$$

we find $\psi(0) = \infty \cdot \alpha(1-\epsilon)A = \infty > \rho$. Hence, there is at least a value of k satisfying (28) and a steady state.

While the first term on the right-hand side of (29) is decreasing in k, the second term is increasing in k because of the first property of (27). Thus, $\psi(k)$ can be a nonmonotonic function of k, depending on the values of the

[8]The assumption that $0 < \alpha < 1/2$ is plausible because the capital elasticity of output α is usually accepted as being about 0.3.

parameters. If θ and ϕ are large, $\psi(k)$ is indeed nonmonotonic (see Appendix A). Intuitively this is because the second term influences $\psi(k)$ more significantly as θ and ϕ become larger. Because of the nonmonotonicity of $\psi(k)$, there are three values of k satisfying (28), denoted by k^l, k^m, and k^h where $k^l < k^m < k^h$ (see Figure 1). Note that from the first property of (27) $\pi(k^l) > \pi(k^m) > \pi(k^h)$.

[Figure 1 around here]

Therefore, we obtain the following proposition:[9]

Proposition 1. *If θ and ϕ are large, there are three steady states. Moreover, if the instantaneous utility function has high relative risk aversion, the steady state where $k = k^l$ and the steady state where $k = k^h$ are saddle-path stable, and the steady state where $k = k^m$ is unstable.*

Proof. See Appendix A and B for the existence and the stability of the steady states respectively. □

Proposition 1 implies that the initial capital stock determines which steady state is reached. If the initial capital stock is less than k^m, production and deposits, and hence the demand for cash and reserves, are low. Therefore, the inflation tax base is small and a high inflation rate is required to finance the fiscal deficit (see the first properties of (25) and (27)). The high inflation rate makes the real rate of interest on deposits lower than the time preference rate. This induces the household to decumulate capital. Consequently, the economy converges to the steady state where the capital stock is low ($k = k^l$) and the inflation rate is high ($\pi = \pi(k^l)$). Alternatively, if the initial capital stock is more than k^m, the economy converges to the steady

[9] Many papers, e.g. Mankiw (1981), report that relative risk aversion is high (see also Romer, 2006, Chapter 7).

state where the capital stock is high ($k = k^h$) and the inflation rate is low ($\pi = \pi(k^h)$). The respective paths converging to the high-inflation steady state and the low-inflation steady state are uniquely determined. This is in contrast to Evans and Yarrow (1981), Sargent and Wallace (1981), Weil (1987), Bruno and Fischer (1990), and Evans (1995), where although there are a high-inflation steady state and a low-inflation steady state, the equilibrium paths are neither uniquely determined nor depend on the initial capital stock.

4.1 Policy Effects

This subsection investigates policies to bring the economy out of the poverty trap. From the third and fourth properties of (27), decreases in the fiscal deficit ϕ and the government's propensity to spend θ reduce the inflation rate. Hence, they increase the real rate of interest on deposits, as shown by partially differentiating $\psi(k)$ of (28) or (29) with respect to θ and ϕ:

$$\frac{\partial\psi(k)}{\partial\theta} < 0, \quad \frac{\partial\psi(k)}{\partial\phi} < 0.$$

Thus, if θ and ϕ are decreased, it is possible that the steady states where $k = k^l, k^m$ disappear and only the steady state where $k = k^h$ exists (see Figure 2).

[Figure 2 around here]

Furthermore, if they are sufficiently decreased, the poverty trap is completely removed because $\psi(k)$ is monotonically decreasing in k (see Appendix A).

The present analysis suggests that when an economy is on its way to reaching a steady state where the capital stock is low and the inflation rate is high, a reduction in fiscal deficits enables the economy to turn around and advance toward a steady state where the capital stock is high and the inflation rate is low. This consequence is somewhat similar to the case of Bolivia. In September 1985, a fiscal reform, including a reduction in fiscal deficits, stopped hyperinflation in Bolivia. Afterwards, the rate of real GDP growth in Bolivia changed from negative to positive.[10]

In contrast, the sign of $\partial\psi(k)/\partial\epsilon$ is ambiguous, that is, a reduction in ϵ does not necessarily help the economy escape from the poverty trap. This is due to the following opposing effects. While it positively affects the real rate of interest on deposits by improving the distortion caused by the reserve requirement, it negatively affects the real rate of interest on deposits by decreasing the inflation tax base and increasing the inflation rate. These effects are implied by (12) and the second property of (27).

4.2 The Independent Monetary Authority

This subsection shows that if the monetary authority is independent and not compelled to finance the fiscal deficit, the poverty trap is eliminated independently of the fiscal position. The monetary authority independently sets the money growth rate μ as follows:[11]

$$\dot{M}_t/M_t = \mu, \quad \mu > -\rho.$$

Since $\dot{m}_t/m_t = \mu - \pi_t$, the steady-state inflation rate equals the money growth rate:

$$\pi = \mu. \tag{30}$$

[10] See, e.g., Sachs (2005, Chapter 5) for details of the situation in the Bolivian economy.

[11] Under the assumption that $\mu > -\rho$, from (1), (23), and (30), $r^D > -\pi$ and $r > -\pi$. Thus, both the CIA constraint and the reserve requirement are binding.

The fiscal authority must cover spending not financed by seigniorage μm_t $(= \dot{M}_t/P_t)$:

$$g_t - \mu m_t = \tau_t,$$

where, as well as Subsection 2.4, $g_t = \theta f(k_t)$.

In this policy regime (28) is invalid. Instead, from (23) and (30), the steady-state capital stock is uniquely determined by the following equation:

$$\rho = (1-\epsilon) f'(k) - \epsilon\mu. \tag{31}$$

Thus, there exists a unique steady state, namely the poverty trap is eliminated. It is because the monetary authority can independently control the steady-state inflation rate by setting the money growth rate. This is similar to the elimination of the high inflation trap discussed in Bruno and Fischer (1990). In contrast to the previous case of a dependent monetary authority, from (31) we obtain

$$\frac{\mathrm{d}k}{\mathrm{d}\theta} = 0, \quad \frac{\mathrm{d}k}{\mathrm{d}\phi} = 0, \quad \frac{\mathrm{d}k}{\mathrm{d}\epsilon} < 0,$$

implying that only a reduction in the reserve requirement promotes capital accumulation.

5 Concluding Remarks

We have constructed a dynamic general equilibrium model where a household faces a CIA constraint, a commercial bank is required to hold money as reserves, all capital is intermediated by the commercial bank, a fiscal authority runs a deficit, and a monetary authority is compelled to finance the fiscal deficit through money creation. If the capital stock is low, production and deposits, and hence demand for cash and reserves, are low. Since the low demand for money reduces the inflation tax base, the fiscal deficit leads to a high inflation rate. Under the reserve requirement, the high inflation rate lowers the real rate of interest on deposits and decumulates capital. Therefore, an economy where the initial capital stock is less than a certain threshold converges to a steady state where the capital stock is low and the inflation rate is high.

Tight fiscal policies, such as decreases in the fiscal deficit and the government's propensity to spend, reduce inflation and consequently bring the economy out of the poverty trap. If they are sufficiently decreased, the poverty trap disappears. Moreover, an independent monetary authority removes the poverty trap independently of the fiscal position because it can control the steady-state inflation rate by setting the money growth rate. These results suggest that loose fiscal policies can create a poverty trap under low central bank independence.

Appendix A The Nonmonotonicity of $\psi(k)$ and the Existence of the Steady States

This appendix examines the condition for $\psi(k)$ to be not monotonic and shows that the nonmonotonicity of $\psi(k)$ generates the three steady states. From (29), differentiating $\psi(k)$ yields

$$\psi'(k) = \epsilon\phi k^{\alpha-2}\left[\chi(k) - \Phi\right],$$

where

$$\chi(k) \equiv \left[(1-\theta)Ak^{\alpha} + \frac{\epsilon}{1-\epsilon}k\right]^{-2}\left[\alpha(1-\theta)Ak + \frac{\epsilon}{1-\epsilon}k^{2-\alpha}\right] > 0 \tag{A1}$$

and

$$\Phi \equiv \frac{\alpha(1-\alpha)(1-\epsilon)A}{\epsilon\phi} > 0. \tag{A2}$$

Thus we obtain

$$\psi'(k) \gtreqless 0 \iff \chi(k) \gtreqless \Phi. \tag{A3}$$

Let us find the form of $\chi(k)$ to distinguish the sign of $\psi'(k)$. From (A1), $\chi(0) = 0$ and $\chi(\infty) = 0$.[12] Differentiating $\chi(k)$ gives

$$\chi'(k) = \frac{(1-2\alpha)k\left[\alpha(1-\theta)^2A^2k^{\alpha-1} + \dfrac{2\epsilon(1-\theta)A}{1-\epsilon} - \dfrac{\alpha\epsilon^2k^{1-\alpha}}{(1-2\alpha)(1-\epsilon)^2}\right]}{\left[(1-\theta)Ak^{\alpha} + \dfrac{\epsilon}{1-\epsilon}k\right]^3}.$$

[12] By arranging $\chi(k)$ as follows:

$$\chi(k) = \frac{\alpha(1-\theta)A + \frac{\epsilon}{1-\epsilon}k^{1-\alpha}}{\left[(1-\theta)Ak^{\alpha-\frac{1}{2}} + \frac{\epsilon}{1-\epsilon}k^{\frac{1}{2}}\right]^2} \quad \text{or} \quad \chi(k) = \frac{\alpha(1-\theta)Ak^{-1} + \frac{\epsilon}{1-\epsilon}k^{-\alpha}}{\left[(1-\theta)Ak^{\alpha-1} + \frac{\epsilon}{1-\epsilon}\right]^2},$$

we find

$$\chi(0) = \frac{\alpha(1-\theta)A}{\infty} = 0, \quad \chi(\infty) = \frac{0}{[\epsilon/(1-\epsilon)]^2} = 0.$$

When $\chi'(k) = 0$, we have

$$\alpha(1-\theta)^2 A^2 k^{\alpha-1} + \frac{2\epsilon(1-\theta)A}{1-\epsilon} - \frac{\alpha\epsilon^2 k^{1-\alpha}}{(1-2\alpha)(1-\epsilon)^2} = 0, \tag{A4}$$

where as k increases, the left-hand side decreases monotonically from infinity to minus infinity. Thus, k satisfying (A4), i.e., $\chi'(k) = 0$ is uniquely obtained. Letting $\tilde{k}$ denote it, we find

$$\chi'(k) > 0 \quad \text{if} \quad k < \tilde{k},$$
$$\chi'(k) < 0 \quad \text{if} \quad k > \tilde{k},$$

which implies that $\chi(k)$ is maximized when $k = \tilde{k}$.

Since from (A1) and (A2) an increase in θ shifts $\chi(k)$ upward and an increase in ϕ shifts Φ downward:

$$\frac{\partial \chi(k)}{\partial \theta} = \frac{k^2 \left[\alpha(1-\theta)A^2 k^{\alpha-1} + \dfrac{\epsilon A(2-\alpha)}{1-\epsilon}\right]}{\left[(1-\theta)Ak^\alpha + \dfrac{\epsilon}{1-\epsilon}k\right]^3} > 0, \quad \frac{\partial \Phi}{\partial \phi} < 0, \tag{A5}$$

$\chi(\tilde{k}) > \Phi$ if θ and ϕ are large. Hence, there are two values of k satisfying the equality of (A3), denoted by $\underline{k}$ and $\overline{k}$ in Figure 3.

[Figure 3 around here]

From (A3) and Figure 3, we find that $\psi(k)$ is not monotonic as follows:

$$\psi'(k) < 0 \quad \text{if} \quad k < \underline{k},$$
$$\psi'(k) \geq 0 \quad \text{if} \quad \underline{k} \leq k \leq \overline{k},$$
$$\psi'(k) < 0 \quad \text{if} \quad \overline{k} < k.$$

Because of this nonmonotonicity of $\psi(k)$, there are the three values of k satisfying (28), as illustrated in Figure 1, and hence the three steady states mentioned in Proposition 1.

In contrast, if θ and ϕ are low, from (A5) we obtain $\chi(\tilde{k}) < \Phi$, which implies $\chi(k) < \Phi$ for any k. Since from (A3) $\psi'(k) < 0$ for any k, there exists a unique steady state. That is, the poverty trap is eliminated.

Appendix B The Stability of the Steady States

We linearize (19), (21), and (22) in the neighborhood of the steady states where m and k are given by (24) and (28) respectively and, from (18), λ is

$$\lambda = \frac{u'(c)}{(1-\epsilon)\left[f'(k)+\pi\right]+1}. \tag{B1}$$

Note that in (B1) π is given by (26) and, from (17) and (24), $c = (1-\theta)f(k)$. The linearization yields

$$\begin{pmatrix} \dot{\lambda}_t \\ \dot{k}_t \\ \dot{m}_t \end{pmatrix} = \begin{pmatrix} \epsilon\lambda\pi_\lambda & -(1-\epsilon)\lambda f'' + \epsilon\lambda\pi_k & \epsilon\lambda\pi_m \\ 0 & (1-\theta)f' + \epsilon/(1-\epsilon) & -1 \\ -\pi_\lambda m & -\pi_k m & -\pi_m m - \pi \end{pmatrix} \begin{pmatrix} \lambda_t - \lambda \\ k_t - k \\ m_t - m \end{pmatrix},$$

where, from (18),

$$\pi_\lambda = -\frac{u'}{(1-\epsilon)\lambda^2} < 0, \quad \pi_k = -\frac{\epsilon u''}{(1-\epsilon)^2\lambda} - f'' > 0, \quad \pi_m = \frac{u''}{(1-\epsilon)\lambda} < 0. \tag{B2}$$

Therefore, we obtain the following characteristic equation:

$$-z^3 + q_2 z^2 + q_1 z + q_0 = 0, \tag{B3}$$

where

$$q_2 \equiv (1-\theta)f' + \frac{\epsilon}{1-\epsilon} + \epsilon\lambda\pi_\lambda - \pi_m m - \pi, \tag{B4}$$

$$q_1 \equiv -\left[(1-\theta)f' + \frac{\epsilon}{1-\epsilon}\right](\epsilon\lambda\pi_\lambda - \pi_m m - \pi) + \epsilon\lambda\pi\pi_\lambda + \pi_k m, \tag{B5}$$

$$q_0 \equiv -\pi_\lambda\lambda m\left[(1-\epsilon)f'' + \frac{\epsilon\pi}{m}\left\{(1-\theta)f' + \frac{\epsilon}{1-\epsilon}\right\}\right] = -\pi_\lambda\lambda m\psi'(k). \tag{B6}$$

Note that differentiating $\psi(k)$ of (28) and then applying (24) and (26) to the result we can show that the terms in the square brackets of (B6) equal $\psi'(k)$.

Let z_1, z_2, and z_3, where $z_1 \leq z_2 \leq z_3$, denote roots of (B3). Since $\psi'(k) < 0$ if $k = k^l, k^h$ and $\psi'(k) > 0$ if $k = k^m$ (see Figure 1), from (B6) we obtain

$$q_0 = z_1 z_2 z_3 < 0 \quad \text{if} \quad k = k^l, k^h, \tag{B7}$$

$$q_0 = z_1 z_2 z_3 > 0 \quad \text{if} \quad k = k^m. \tag{B8}$$

Substituting (B1) and the first and third properties of (B2) into (B4) so as to eliminate λ and then using (23), we find

$$q_2 = \left(f' + \pi + \frac{1}{1-\epsilon} \right) \left(-\frac{u''c}{u'} \right) \frac{m}{c} + \rho - \theta f' - \pi,$$

where m and π, as well as f' and c $(= (1-\theta)f)$, do not depend on a form of the instantaneous utility function u, as shown by (24) and (26), because from (28) k is independent of u, and where from (2) and (16) m/c is larger than one. Therefore, if relative risk aversion $-u''c/u'$ is higher than

$$\frac{[\theta f' + \pi - \rho]\, c}{\left[f' + \pi + \dfrac{1}{1-\epsilon} \right] m} \; (< 1),$$

we get

$$q_2 = z_1 + z_2 + z_3 > 0. \tag{B9}$$

From (B7) and (B9), we obtain

$$z_1 < 0, \quad z_2 > 0, \quad z_3 > 0 \quad \text{if} \quad k = k^l, k^h.$$

Since λ_t and m_t are jumpable and k_t is not, both the steady state where $k = k^l$ and the steady state where $k = k^h$ are saddle-path stable and the respective paths converging to the steady states are uniquely determined.

Substituting (B1) and (B2) into (B5) to eliminate λ, we derive

$$q_1 = -\left(f' + \pi + \frac{1}{1-\epsilon}\right)\left[\epsilon\pi + (1-\theta)f'\left(-\frac{u''c}{u'}\right)\frac{m}{c}\right] - f''m$$
$$+ \left[(1-\theta)f' + \frac{1}{1-\epsilon}\right]\left[\epsilon f' + (1+\epsilon)\pi + \frac{\epsilon}{1-\epsilon}\right],$$

where the first term is negative and the second and third terms are positive. Thus, if $-u''c/u'$ is high, we have

$$q_1 < 0. \tag{B10}$$

From (B8)-(B10), we find that there is no negative root satisfying (B3) and hence

$$z_1 > 0, \quad z_2 > 0, \quad z_3 > 0 \quad \text{if} \quad k = k^m,$$

implying that the steady state where $k = k^m$ is unstable.

References

[1] Aisen A., Veiga, F.J., 2008. The political economy of seigniorage. Journal of Development Economics 87, 29-50.

[2] Basu, P., 2001. Reserve ratio, seigniorage and growth. Journal of Macroeconomics 23, 397-416.

[3] Berument, H., 1998. Central bank independence and financing government spending. Journal of Macroeconomics 20, 133-151.

[4] Brock, P.L., 1989. Reserve requirements and the inflation tax. Journal of Money, Credit, and Banking 21, 106-121.

[5] Bruno, M., Fischer, S., 1990. Seigniorage, operating rules, and the high inflation trap. Quarterly Journal of Economics 108, 353-374.

[6] Catão, L.A.V., Terrones, M.E., 2005. Fiscal deficits and inflation. Journal of Monetary Economics 52, 529-554.

[7] Chari, V.V., Jones, L.E., Manuelli, R.E., 1995. The growth effects of monetary policy. Federal Reserve Bank of Minneapolis Quarterly Review 19, 18-32.

[8] Click, R.W., 1998. Seigniorage in a cross-section of countries. Journal of Money, Credit, and Banking 30, 154-171.

[9] Cukierman, A., Edwards, S., Tabellini, G., 1992. Seigniorage and political instability. American Economic Review 82, 537-555.

[10] Easterly, W., Schmidt-Hebbel, K., 1993. Fiscal deficits and macroeconomic performance in developing countries. World Bank Research Observer 8, 211-237.

[11] Evans, J.L., 1995. The demand for money: hyperinflation or high inflation traps. Manchester School of Economic and Social Studies Supplement, 40-56.

[12] Evans, J.L., Yarrow, G.K., 1981. Some implications of alternative expectations hypotheses in the monetary analysis of hyperinflations. Oxford Economic Papers 33, 61-80.

[13] Fischer, S., Sahay, R., Végh, C.A., 2002. Modern hyper- and high inflations. Journal of Economic Literature 40, 837-880.

[14] Freeman, S., Kydland, F.E., 2000. Monetary aggregates and output. American Economic Review 90, 1125-1135.

[15] Fry, M.J., 1995. Money, Interest, and Banking in Economic Development, Second Edition. Johns Hopkins University Press, Baltimore, MD.

[16] Haslag, J.H., 1998. Monetary policy, banking, and growth. Economic Inquiry 36, 489-500.

[17] Mankiw, N.G., 1981. The permanent income hypothesis and the real interest rate. Economics Letters 7, 307-311.

[18] Moser, G.G., 1995. The main determinants of inflation in Nigeria. IMF Staff Papers 42, 270-289.

[19] Romer, D., 1985. Financial intermediation, reserve requirements, and inside money. Journal of Monetary Economics 16, 175-194.

[20] Romer, D., 2006. Advanced Macroeconomics, Third Edition. McGraw-Hill Irwin, New York.

[21] Sachs, J.D., 2005. The End of Poverty: How We Can Make It Happen in Our Lifetime. Penguin Books, London.

[22] Sargent, T.J., 1986. Rational Expectations and Inflation. Harper and Row Publisher, New York.

[23] Sargent, T.J., Wallace, N., 1981. Some unpleasant monetarist arithmetic. Federal Reserve Bank of Minneapolis Quarterly Review 5, 1-17.

[24] Walsh, C.E., 1984. Optimal taxation by the monetary authority. NBER Working Paper No. 1375.

[25] Weil, P., 1987. Permanent budget deficits and inflation. Journal of Monetary Economics 20, 393-410.

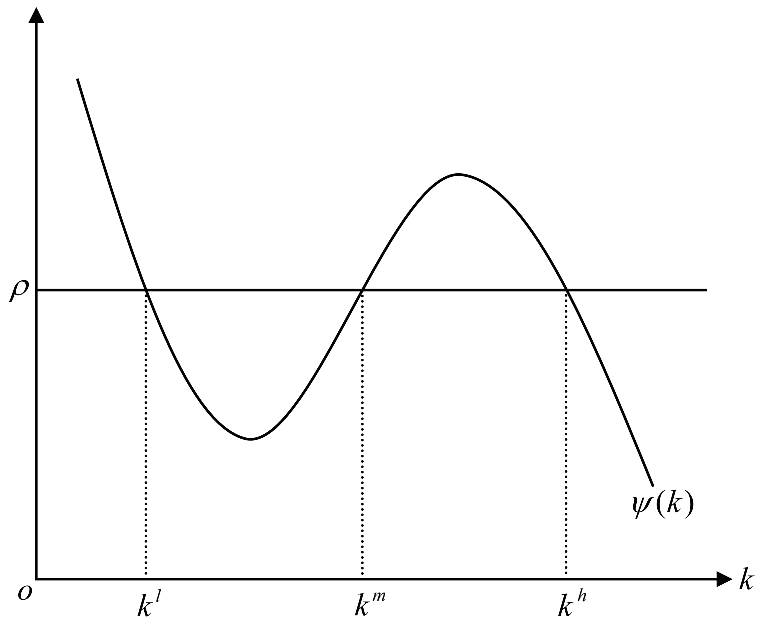

Figure 1: The existence of the three steady states

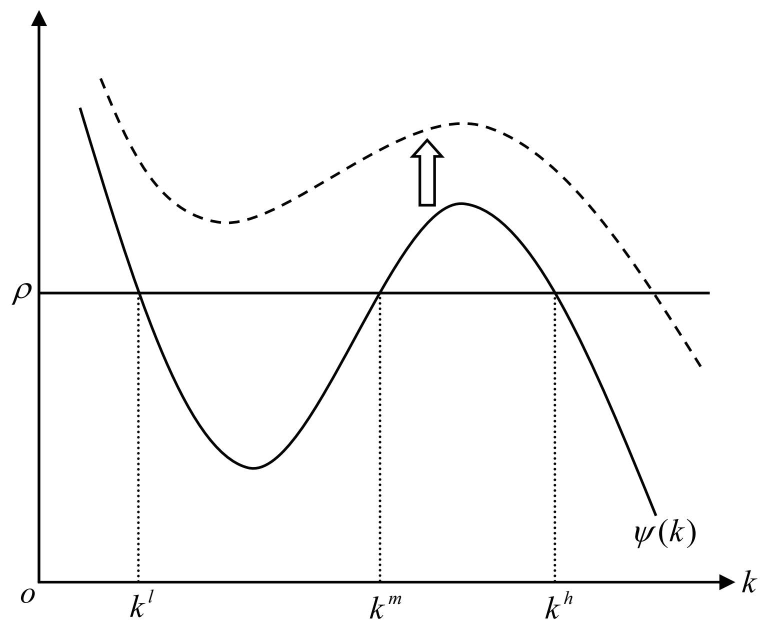

Figure 2: The effects of decreases in θ and ϕ

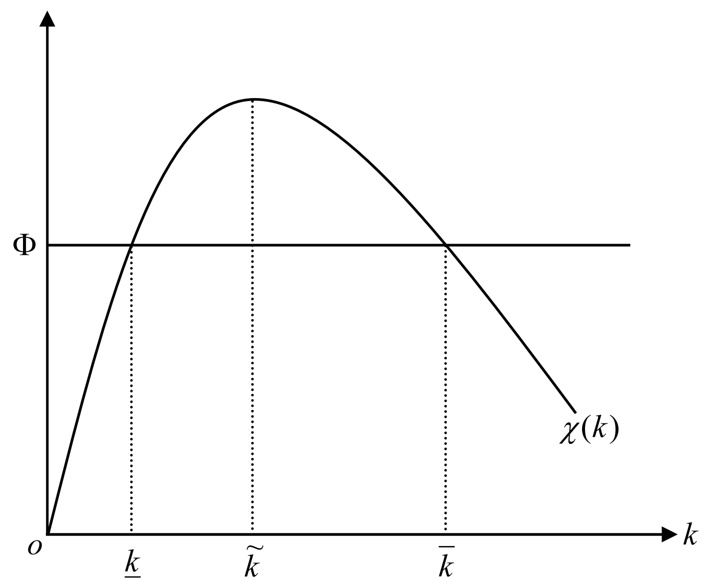

Figure 3: The relationship between $\chi(k)$ and Φ