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Abstract

This article describes Lowe's traverse employing a difference equations system and displays the traverse process with the aid of numerical analysis.

In order to investigate Lowe's model by a difference equations system, the distributive proportion of outputs between two sectors is given exogenously, and the fact that a proportional relation of this distributive proportion of output and gross saving ratio plays a crucial role is indicated.

Lowe described that the discontinuous traverse process, which serves to absorb an increase in the rate of labour supply, can be divided into four phases. To display the contortions, numerical examples of traverse are illustrated diagramatically with the aid of numerical analysis.

Keywords: Lowe, Numerical Analysis, Traverse, Simulation.

1 Introduction

The purpose of this paper is to describe Lowe's traverse by a difference equations system and to display the traverse process with the aid of numerical analysis.

First, with regard to the structure of production, Lowe's production industries is vertically divided into two basic sectors; equipment-good industries and consumer-good industries, described as sector 1 and sector 2 respectively, and equipment-good sector 1 is subdivided into sector 1a which produces the equipment, machine tool, applied to both sector 1a and 1b, and sector 1b which supplies equipment good to sector 2 [1]. In order to investigate such a traverse of Lowe's production industries by a difference equations system, the distributive proportion of outputs between sector 1a and sector 1b is given exogenously in this article.

Second, generally speaking, traverse path connects two dynamic equilibrium paths defined by different rates of change. Lowe was also concerned with the traverse that lead, under the impact of a change in the rate of increase or decrease of natural resources, from an initial dynamic equilibrium to a terminal equilibrium. This paper focuses on the consequences of an increase in the rate of labour supply.

Third, the traverse process is, in principle, discontinuous, and Lowe described that the discontinuous process, which serve to absorb an increase in the rate of labour supply, can be divided into four phases: partial liberation of existing capacity; augmentation of primary equipment output; augmentation of secondary equipment output; and augmentation of consumer goods output. With the aid of numerical analysis, numerical examples of capacity in Sector 1b, that must be

[1]The sectors are not concrete industries. See C.Gheruke[1]p.213.

freed in part from its original task of replacing and expanding capacity in Sector 2, are illustrated with diagrams and also the augmentation of output of secondary equipment and finally augmentation of consumer-good output are also illustrated with diagrams.

2 Reconstruction of Lowe's Core Model; Distributive proportion, Gross savings and a Similar Relation of Sectors

Lowe wrote ;

> It was then stated that, with our important exception, all growth process are, in principle, discontinuous. This was to emphasize the fact that, even if the growth stimuli - change in labour supply and productivity - are themselves continuous, the complementary real capital can be provided for only in discontinuous spurts [2].
>
> Because in the real world changes in the rate of change of the growth stimuli are the rule, actual growth processes present themselves not as continuous paths, in analogy with dynamic equilibrium, but as sequences of capital accumulation or capital decumulation, that is, as a chain of discrete short-term processes of which the long-term growth trend is no more than an abstraction [3].

Such a process, which proceeds step by step, can be expressed clearly by a discrete difference equations system [4].

2.1 Distributive Proportion

We reconstruct Lowe's core model by a difference equations system. Based on Lowe's model, the capital goods sector being '1a' and '1b', the consumption goods sector '2'; capital stock K; investment I; outputs Y; capital-output ratio (capital coefficient)v; rate of growth g; rate of depreciation for the capital stock d and saving ratio s. Right side suffix denotes the sector.

The capital stock at the next term is the capital stock at the present term minus the depreciation stock plus the incremental stock.

$$\begin{aligned} K_{1a}(t+1) + K_{1b}(t+1) &= (1-d)(K_{1a}(t) + K_{1b}(t)) + Y_{1a}(t) \\ K_2(t+1) &= (1-d)K_2(t) + Y_{1b}(t) \end{aligned}$$

Now, the distributive proportion $\sigma (0 < \sigma < 1)$ of final outputs between Sector 1a and Sector 1b is given exogenously. It is the proportion going to Sector 1a, and the following equations are obtained.

$$K_{1a}(t+1) = (1-d)K_{1a}(t) + \sigma Y_{1a}(t) \tag{1}$$

$$K_{1b}(t+1) = (1-d)K_{1b}(t) + (1-\sigma)Y_{1a}(t) \tag{2}$$

$$K_2(t+1) = (1-d)K_2(t) + Y_{1b}(t) \tag{3}$$

where, $K_{1a}(t+1) - K_{1a}(t) = \Delta K_{1a} = I_{1a}^{net}, K_{1b}(t+1) - K_{1b}(t) = \Delta K_{1a} = I_{1b}^{net}$.

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[2] Lowe [3] p.104.

[3] Lowe [3] p.104.

[4] See Hicks [2] p.63. and Taniguchi [4]

2.2 Gross savings and A Similar Relation of Sectors

Lowe defined the gross savings as follows [5]:

$$s^{gross} = \frac{Y_{1a}(t) + Y_{1b}(t)}{Y_{1a}(t) + Y_{1b}(t) + Y_2(t)}.$$

That is,

$$Y_{1a}(t) + Y_{1b}(t) = \frac{s^{gross}}{1 - s^{gross}} Y_2(t).$$

In case of the full capital utilization,

$$K_{1a}(t) + K_{1b}(t) = \frac{s^{gross}}{1 - s^{gross}} K_2(t)$$

is obtained. By using (1)$\sim$(3),

$$\frac{K(t+1)}{K(t)} - (1-d) = \frac{Y_{1a}(t) + Y_{1b}(t)}{K(t)}$$

is obtained. With the use of $K(t) = vY(t)$, this equation can be rewritten as

$$g + d = \frac{1}{v}\frac{Y_{1a}(t) + Y_{1b}(t)}{Y(t)}.$$

That is

$$s^{gross} = I^{gross} = v(g+d).$$

Certainly, the above equation is valid only in dynamic equilibrium.

With regard to the distributive proportion, by employing (1),

$$\begin{aligned} \sigma &= \left(\frac{K_{1a}(t+1)}{K_{1a}(t)} - 1 + d\right) v \\ &= v(g_{1a} + d) \qquad (4) \end{aligned}$$

is obtained. (4) is rewritten as follows:

$$g_{1a} = \frac{\sigma}{v} - d.$$

This equation means that the distributive proportion σ which is given exogenously determines the growth rate g_{1a}.

As $g_{1a} = g_{1b} = g_2$ is valid in the dynamic equilibrium, $\sigma = s^{gross}$ is obtained, therefore,

$$s^{gross} : 1 - s^{gross} = \sigma : 1 - \sigma$$

is realised.

This equation designates that the proportion of the equipment-good sector 1 to consumer-good sector 2 equals to the proportion of the equipment-machine tool sector 1a to sector 1b which supplies equipment good to sector 2. The similar relation exists between sectors in Lowe's model.

[5] The amounts of net savings plus amortization. See Lowe [3] p.127.

3 The Difference Equations System and the Numerical Examples

3.1 The Difference Equations System Model

With the use of (1)(2)(3), the difference equations system of the capital K is expressed as

$$K_{1a}(t+1) - (1-d+\frac{\sigma}{v})K_{1a}(t) = 0 \tag{5}$$

$$K_{1b}(t+1) - (1-d)K_{1b}(t) - \frac{1-\sigma}{v}K_{1a}(t) = 0 \tag{6}$$

$$K_2(t+1) - (1-d)K_2(t) - \frac{1}{v}K_{1b}(t) = 0. \tag{7}$$

From (5), $K_{1a}(t)$ is as follows:

$$K_{1a}(t) = K_{1a}(0)(1-d+\frac{\sigma}{v})^t. \tag{8}$$

By substituting the above equation to (6), K_{1b} is obtained as follows:

$$K_{1b}(t) \quad = \quad \Phi_1(1-d)^t + \frac{1-\sigma}{\sigma}K_{1a}(0)(1-d+\frac{\sigma}{v})^t. \tag{9}$$

Subsequently, by substituting $K_{1a}(t), K_{1b}(t)$ to (7) and by solving the difference equations, $K_2(t)$ becomes

$$K_2(t) = (\Phi_2 + \frac{1}{(1-d)v}\Phi_1 t)(1-d)^t + \frac{1-\sigma}{\sigma^2}K_{1a}(0)(1-d+\frac{\sigma}{v})^t \tag{10}$$

where

$$\begin{aligned} \Phi_1 &= K_{1b}(0) - \frac{1-\sigma}{\sigma}K_{1a}(0) \\ \Phi_2 &= K_2(0) - \frac{1-\sigma}{\sigma^2}K_{1a}(0). \end{aligned}$$

Thus, as the depreciation rate of capital is less than 1, the growth rate of capital stock of sector 1b and sector 2 approach to the growth rates $-d+\sigma/v$ of sector 1a from an arbitrary initial value by (8)(9)(10). Finally, the growth rate of the economy depends on the growth rate of sector 1a.

3.2 Numerical Examples

The coefficients are as follows; capital-output ratio (capital coefficient)$v = 4$, the rate of depreciation for the capital stock $d = 0.03$. At the initial dynamic equilibrium, $K_{1a}(0) = 1, K_{1b}(0) = 4, K_2(0) = 20$ are given as the initial capital stocks and the distributive proportion $\sigma = 0.2 (0 < \sigma < 1)$ of final outputs between Sector 1a and Sector 1b. Similarly, at the terminal equilibrium, $K_{1a}(0) = 1, K_{1b}(0) = 1.5, K_2(0) = 3.75$ are given and the distributive proportion $\sigma = 0.4 (0 < \sigma < 1)$. Since the rate of economic growth of dynamic equilibrium is written as $-d+\sigma/v$, the rate of economic growth of the initial equilibrium is 2% and 7% at the terminal equilibrium. For example, the rate of labour supply increases from 2% to 7%.

Since the economy begins to traverse at $t = 0$, the capital stock of the each sector is as follows:

$$\begin{aligned} K_{1a}(t) &= 1.07^t \\ K_{1b}(t) &= 2.5 \times 0.97^t + 1.5 \times 1.07^t \\ K_2(t) &= \left(16.25 + \frac{125}{194}t\right)0.97^t + 3.75 \times 1.07^t \end{aligned}$$

The differential capital stocks between the initial dynamic equilibrium and the traverse are illustrated in Figure 1. Negative area in Figure 1 designates the liberated capital which needs to traverse from the initial to terminal dynamic equilibrium.

The output of secondary equipment $Y_{1b}(t)(= K_{1b}(t)/4)$ and the consumer-good output $Y_2(t)(= K_2(t)/4)$ are illustrated in Figure 2 and Figure 3.

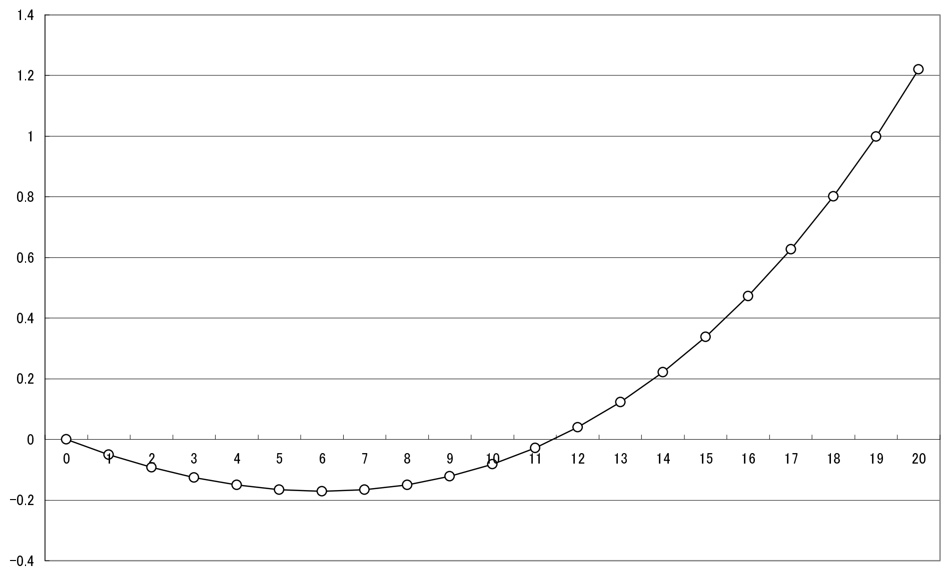

Figure 1: Differential capital K_{1b} between the initial dynamic equilibrium and the traverse (Negative area; the liberated capital stock)

4 Conclusion

First, the fact that a proportional relation of the distributive proportion of output and gross saving ratio exists should be mentioned. This means the proportion of the equipment-machine tool sector 1a to sector 1b equals to the proportion of the equipment-good sector 1 to consumer-good sector 2. Since this distributive proportion determines the rate of growth, the distributive proportion has a crucial role in Lowe's three sector model.

Second, there are several paths of traverse in Lowe's model. For example, one path maximizes the speed of adjustment, another minimizes the waste of resources, and a third minimizes the impact on consumption during the interval of capital formation. Some of these paths are mutually exclusive. In this paper, the capital stock is utilized fully, however, this traverse path does not maximize the speed of adjustment. The growth rate of the equipment-good sector 1a of the traverse is the same as the terminal dynamic equilibrium. At early phases of the maximum speed traverse path, the rate of growth of the equipment-good sector 1a is greater than that of the terminal dynamic equilibrium. It becomes slow gradually and approaches the terminal dynamic equilibrium path. There is a bottle neck of consumer-good output for the maximum speed traverse path, and the growth rate can not be determined without the amounts of consumer-good being given exogenously in the model. A maximum speed traverse path needs additional assumptions to illustrate.

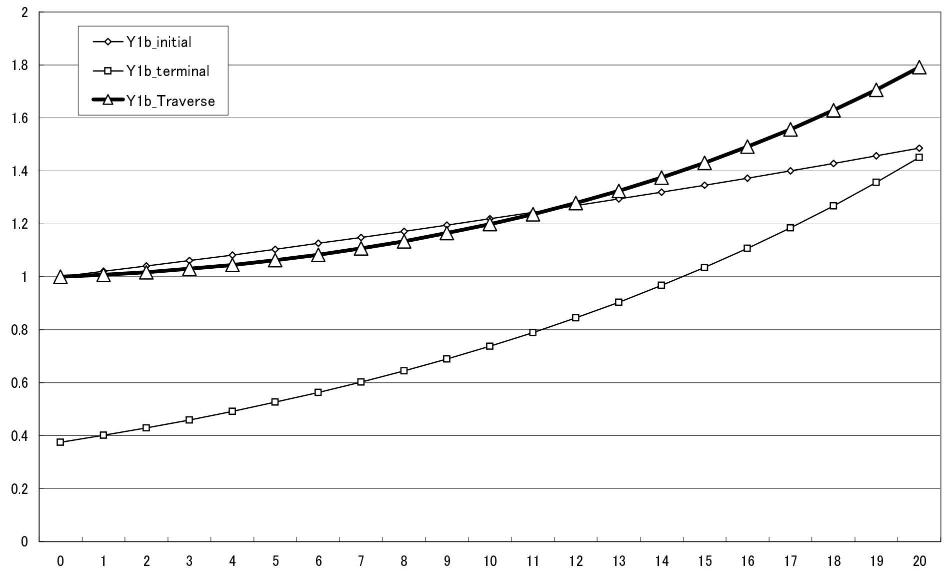

Figure 2: The output of secondary equipment Y_{1b}

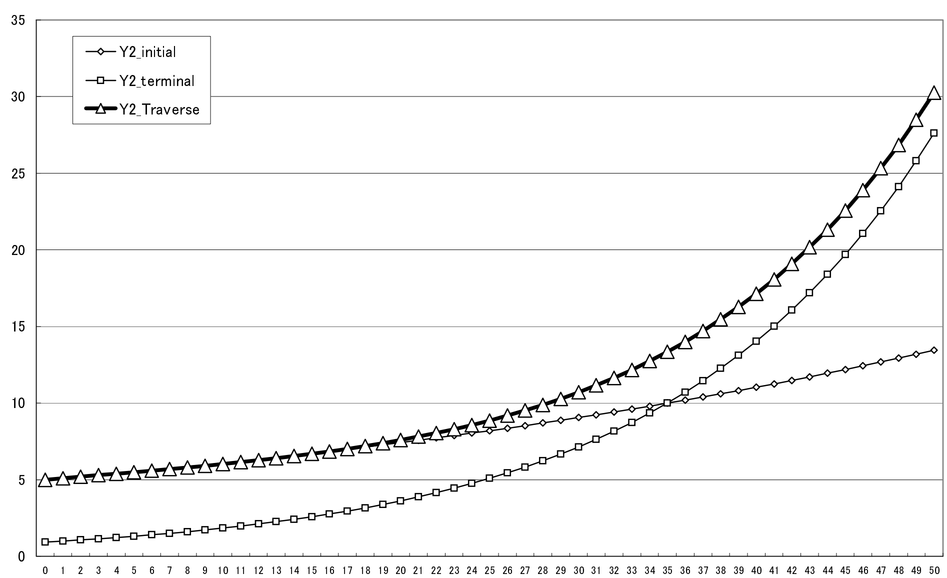

Figure 3: The output of consumer-good Y_2

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